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Spatial Reasoning Cognitive Interviews: Narrative Summary of Qualitative Analyses

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Spatial Reasoning Cognitive Interviews: Narrative Summary of Qualitative Analyses

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Abstract

The purpose of this technical report is to describe the process and outcomes of summarizing qualitative analyses of Cognitive Interviews (CI) into a narrative form. The interviews were conducted as one step in the empirical recovery and reconciliation of the Spatial Reasoning (SR) Learning Progression (LP) within the Measuring Early Mathematics and Reasoning Skills (MMaRS) project. This report details the outcomes of the qualitative analyses from CIs conducted with students in grades K-2 and the review process undertaken in creating a narrative style description of students' words and actions in those interviews. More information about the administration of the interviews can be found in the SR Cognitive Interview Administration technical report (Tech. Rep. No. 20-23). Details on the qualitative analyses can be found in the Spatial Reasoning Cognitive Interview: Qualitative Data Analysis technical report (Tech. Rep. No. 20-21).

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Spatial Reasoning Cognitive Interviews: Summaries of Qualitative Outcomes

Introduction

The purpose of this report is to describe the outcomes from the qualitative analyses of the Spatial Reasoning (SR) Cognitive Interviews (CIs) of the Measuring Early Mathematics and Reasoning Skills (MMaRS) project. After an iterative open-coding process through which the research team found emergent themes in student reasoning responses, manuscript style summaries were written to create a narrative of students' conceptions, including misconceptions and errors in thinking, and detail the interrelations of their knowledge skills and abilities. See the Spatial Reasoning Cognitive Interview: Qualitative Data Analysis technical report (Tech. Rep. No. 20-21) for details on the analysis methods and information related to the hypothesized Learning Progressions (LP) upon which the interviews were based. This report aims to report students' responses from our sample to reasoning probes and patterns of thinking that their responses held.

Research Questions

We designed the cognitive interviews to address four research questions related to empirically evaluating the SR learning progression. We included detailed sub-questions within each overarching research question. Questions 3 and 4 required information from the cognitive qualitative analyses which informed these manuscript style summaries. This report details the methods for writing and revising the narrative summaries and how this format answers Research Questions 3 and 4.

RQ 1: Developmental Appropriateness

- 1.1 Do the entry and exit KSAs align with teachers' expectations of pre-requisite and target skills?
- 1.2 Does teachers' frequency of teaching KSA align with progression?
- 1.3 Does student performance and engagement indicate floor or ceiling effects that align with entry and exit KSAs?

RQ 2: Ordering

- 2.1 Are teachers' perceptions of the appropriateness aligned with the hypothesized order?
- 2.2 Do students demonstrate increasingly sophisticated reasoning aligned with the hypothesized ordering?
- 2.3 Do students appear comfortable with tasks and task elements?

RQ 3: Conceptions

- 3.1 Do students demonstrate reasoning that is consistent with the hypothesized conceptions?
- 3.2 What misconceptions and/or errors do students make? Is there a pattern leading to greater competence?

RQ 4: Interconnectedness

4.1 In what ways are students' KSAs interconnected?

4.2 In what ways does prior impact students' responding?

Methods

Initial Summary Draft

A narrative summary was written for each subcomponent directly following the open coding of interview data to fully encapsulate trends in student reasoning. These were structured as single paragraphs for each subcomponent that briefly stated what the child was tasked to do and described the various ways that children responded to reasoning questions. The narrative format allowed nuanced details to emerge that followed the preliminary progression of student reasoning seen in the data. We included quotes from student interviews to best illustrate findings.

This writing was completed prior to bulleted summaries for the coder to appropriately order any strategies that indicated levels of sophisticated reasoning, while separating those that did not follow a progression of developmental skill. Through this process, the conceptualization of student skills within each subcomponent, potential interconnections between subcomponents or core concepts, and evident misconceptions and errors in student thinking became more evident for synthesized bulleted summaries. Bullet style summaries were drafted in tandem and revised separately to inform reconciliation of the LP (See Appendix A for detailed process and sample).

External Review

After all summaries were drafted, we provided an external reviewer who had expertise in learning progressions with the manuscript-style narrative summaries for the Within Objects Targeted Learning Goal (TLG) Core Concepts (CC) 1 and 2; CC1 involves identifying and classifying two-dimensional shape and three-dimensional figures, and CC2 calls on children to demonstrate an understanding of transformations. The lead coder provided guiding questions that would elicit feedback for the purpose of verifying the accuracy and clarity in description of codes through manuscript style summary statements. The bulleted summaries of the first two concepts were reviewed externally, but the internal review produced a reconceptualization and the team chose to hold at that point on the review of bulleted summaries. We posed the following purpose, background, and guiding questions:

- Purpose: Verify the accuracy and clarity in description of codes through manuscript style summary statements and bulleted summaries.
- Background: After the codebook was finalized, the lead coder first wrote a narrative style summary for each subcomponent statement of the codes that emerged through open coding. Based on the codebook and that narrative, the lead coder further synthesized the information into the bullet style summaries that will be used through later reconciliation processes in the project.
- Questions:
    - Do the manuscript style summaries make sense?
    - Do they capture the essence of the codes?

- Are the examples clear? Do they need more or less detail?
  - I will include excerpts of the revision, and the table created to illustrate patterns within the data

To address responses to the guiding questions, the lead coder created a spreadsheet of all feedback that the reviewer provided to look for common themes across subcomponents (See Appendix B). Most notes related to the level of specificity that the reviews currently contained, though some were related to organization and inferred progression of skill. The coders determined that the lead should revise manuscript style summaries to incorporate additional detail before continuing the external review process. They also revised one subcomponent from each reviewed CC to illustrate how the feedback informed revisions, and delivered that document to Primary Investigator (PI) and Project Manager (PM) to review.

Summary Revisions

Based on the synthesis spreadsheet and the review of revisions, the PI and PM decided that the external reviewer provide feedback on one narrative summary from each of the remaining 4 core concepts. A similar cycle was followed in which the lead coder sent the prescribed subcomponent to the external reviewer with the same questions and tools. After review, the lead coder culled information from later reviews into the spreadsheet and found similar feedback patterns, such as requests for more clear definitions, counts or percentages of students by grade engaging in each strategy, and more illustrative language or inclusion of student transcripts to make sense of the information.

The lead coder applied feedback to specific subcomponents more generally across all subcomponents. They included writing an overall introduction for each core concept, then writing a single paragraph for each subcomponent within their respective core concepts. Those summaries are found in the results below.

Results

Summary Statements

We wrote initial drafts, followed by targeted revisions to increase precision in descriptions and evidence from interviews, for each subcomponent task from the Spatial Reasoning Cognitive Interviews. To structure the statements, we wrote a short introductory paragraph to each core concept and then the narrative style summary of reasoning used by students by subcomponent. The manuscript of the narrative-style summaries is forthcoming (Pinilla et al.).

RQ 3: Conceptions

RQ 3.1

One main purpose and outcome in writing these narrative style summaries was to organize student responses both sequentially and in clusters to represent how students understood

concepts and if their reasoning was consistent with those hypothesized conceptions. Using this intentional structure, we found that some student reasoning was consistent with the conceptions we hypothesized as necessary for each of the SR tasks. The careful analyses of their reasoning through this writing facilitated substantive findings that are located in the Spatial Reasoning Cognitive Interviews: Qualitative Data Analyses technical report (Tech. Rep. No. 20-21).

RQ 3.2

We did not account for correctness in student responses when writing these manuscript-style, narrative summaries of student strategies. Instead, these summaries focused on the reasoning strategies found through open-coding of interview data. This separation of the analyses makes answering research question 3.2 limited at this time; more concrete answers will be found in the reconciliation process.

While we could infer if students were using more or less sophisticated reasoning strategies, without accounting for correctness, those inferences may lead to false conclusions about reasoning processes if students were not arriving at correct or reasonable responses. In that, a major limitation of this process was the exclusion of correctness in determining if students demonstrated misconceptions and/or errors in their reasoning. In the bullet list summaries (See Appendix A), there are details of reasoning that were clearly misconceptions or errors in student thinking, but details therein are also subject to change when data sources are reconciled.

RQ 4: Interconnectedness

We found in the qualitative analyses, we did not have substantive results related to the interconnectedness of student spatial reasoning skills or Knowledge Skills and Abilities (KSAs) due to the ways in which we analyzed the data. To state findings, we need to reconcile this information with the quantitative analysis so that we can determine if students' KSAs are building appropriately in ways that align with the SR LP, or if there are misconceptions and errors that require reteaching or correcting.

There is some evidence of skills spanning subcomponents presented in the manuscript style summaries that align with the cross item analysis in the SR CI Qualitative Data Analyses technical report (Tech. Rep. No. 20-21).

Conclusion

The purpose of this technical report was to describe the process for drafting, soliciting feedback, developing a revision process, and ultimately writing a narrative summary of student strategies found through the SR CIs for the MMaRS project. The open-coding process and reconciliation informed descriptions of student strategies, as detailed in the SR CI Qualitative Data Analyses technical report (Tech. Rep. No. 20-21), which were expanded in the narrative format found herein. In conjunction with the quantitative data analyses (Tech. Rep. No. 20-08), full qualitative analyses, results from the SR Teacher Survey (Tech Rep. No. 20-10), and expert reviews, we will empirically recover the LPs and continue developing assessment tools for teachers.

References

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Appendix A – Bulleted Style Summary Descriptions & Example

Bulleted Summary. a bulleted style summary was created with axial codes of student reasoning and synthesized descriptions as related to the skill. This summary also included student misconceptions and errors that were discovered through the cognitive interview qualitative analysis to be considered for learning progression reconciliation and later assessment item writing. See Figure X for visual example, with details explained below for each field.

Naming Conventions. All naming conventions for the core concept name and number were included for continuity across documents, with an added field for a short description of the subcomponent. In this example, the subcomponent descriptor became "sorting".

Original subcomponent Statement and Elements That Vary. The subcomponent statement of student actions was included from the detailed learning progression descriptions, including the target grade band and developmental level. All elements that varied within the subcomponent based on hypothesized developmental appropriateness were included, delineated by grade band and level for further granularity in description.

Open Coding Themes. Synthesized themes captured in open coding were detailed with refined names. Some direct student examples were included if necessary to illustrate what students had done as reasoning. These codes served as the axial codes that aligned with all other steps in the bullet summaries.

Subcomponent Synthesized Description. In two steps, the lead coder first created synthesized descriptions of student actions that aligned with increasingly complex or mature ways of thinking about the given construct. Some skills evolved more linearly in progressions while others may develop simultaneously, while still others were not developmental in nature and were errors in student thinking or misconceptions that required academic feedback to facilitate student growth. For each synthesized statement, open coding themes were aligned and included to inform the statement and support the ordering or concurrent skill development.

Student Misconceptions or Errors. For those themes that did not represent developmental steps in a microprogression of the subcomponent, coders analyzed student transcripts to find misconceptions or errors in thinking. These were characterized by individual or common examples from transcripts and were further detailed in the descriptors of each. They often aligned with the levels of thinking that were listed in the synthesized descriptions, which could be later used by practitioners to facilitate academic feedback or scaffolded practice.

Learning Progression SR: Spatial Reasoning

	Within Objects						
	LEVEL	**SUBCOMPONENT**	**CODE**	**ORIGINAL SUBCOMPONENT STATEMENT & ELEMENTS THAT VARY**	**OPEN CODING THEMES**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION RECOMMENDATION (Cass/Robyn) *summarize/synthesize G (axial coding)***	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
Shape	**1**	**Sorting**	**SR.A.1.a**	Sort similar two- and three-dimensional shapes regardless of size, orientation, and dimensionality. [KF-KB] Elements that varied: Regular and irregular circles, squares, triangles, and rectangles [KF-KB] Regular or irregular hexagons, rhombus, cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids [KT-2F] Regular or irregular quadrilaterals [2B-2T]	(a) Match shapes: Students matched shapes, without naming them, to create groups - "Looked like that one" or "This one is the same as the other" (b) Identify 2D shapes: Used formal or informal shape names to define groups, (e.g., diamond for rhombus) - Used 2D shape name, such as a rectangle, to explain grouping (c) Apply 2D shape name to 3D solid: Categorized 3D solids with 2D shapes, but applied the name of a single face's 2D shape to the 3D solid as a whole - Triangular prism called triangle due to triangular-shaped face (d) Recognize Dimensionality: Attended to 2D versus 3D explicitly. Reasoned about sorting on the basis that a solid is 3D and a shape is 2D (e) Compare mathematical or non-mathematical visible attributes: Compared color or size of shapes, grouped blue shapes together based on color, or compared shapes using words such as "big", "little", or "middle" *All students who used size used at least one other theme*	i. The student sorts two-dimensional shapes into groups by matching a shape to one or more others with the same or similar attributes (a and e) ii.a. The student is able to sort similar two- and three-dimensional shapes regardless of size, orientation, and dimensionality. (b and c) ii.b. The student sorts shapes as two-dimensional versus three-dimensional. (d)	(1) Students attend to color (non-mathematical attribute) when sorting rather than focusing on a mathematical attribute; (2) Students sort based on size, or reason using size comparatives within groups. ii.a (3) groups shapes on commonality rather than sorting based on attribute (e.g., cylinders have a circular face and therefore go with circles; a group of blue shapes and a group of green shapes) ii.b.(4) sorts based on dimensionality without naming and as deepest level of grouping;
Core concept	Core concept number	Simple description of subcomponent	Naming convention within LP	Subcomponent statement from originally defined core concept and targeted learning goal with defined elements that vary within it.	Themes from open coding with student examples.	Synthesized descriptions after coding team reconciliation. Ordering is indicated through I, ii, iii progression, but further letter designations are not necessarily ordered.	errors and misconceptions seen by students at each level of skill

Appendix B – External Feedback on Core Concepts 1 & 2

Question	Do the manuscript style summaries make sense?	Do they capture the essence of the codes?	Are examples clear? Do they need more or less detail?	Other comments
A.1.a	If your hypothesized higher level has only one student supporting it. I would suggest, in a future manuscript to justify keeping that level using research to support your choice. Having one student do something usually isn't enough to add or keep a level in a progression. Unless you can justify this addition or keeping the level using the previous research in math education, any evidence that shows that experts recognize that 2D vs 3D grouping is an important skill and that is hard to develop. I suggest you include the transcript for each example in the text to make it easier to understand to the reader (and that way they don't have to go look for it)	Yes, in terms of organization I suggest you follow a similar pattern of organization that I have described below. Idea discussed : i. Lower level (Describe) (or code), Show examples, explain give percentages. ii. Middle levels (Describe) (code) Show examples, explain give percentages. iii. Upper level (Describe) Show examples, explain give percentages. 2.Justify with research if you don't have enough example.	Show the actual transcript you are discussing underlining where you what you want the reader to attend to. What is the difference if any, among the students of different grades? Is there any difference in how a kindergartener answers these questions vs a 2nd grader? I suggest also adding more detail about how many students and/or the percentage of students that used the particular skill you are describing?	

A.1.b	Your examples of student work are great. I would add them to your summaries. Also, I would add percentages or counts if you have them. Otherwise in the future.	A.1.b.3D: Is the outside force, the students themselves?		
A.1.c & d	In the text you talk about a content knowledge skill you believe the students needed. I would describe that skill, how they use it, and why you think they need it. Otherwise the reader fees out of context and is hard to understand what you mean there	A.1.c (2D) (SR.A.1.d together) Do you feel like eventually you might have to expand these to identify the different ways they talk about the shape attributes? There might be groups within that may be interesting.		
A.2.a	The SR.A2 manuscript was a little bit hard to follow (the bullet points weren't) I think is more an organizational issue. I would give the context of the task you were describing and then describe what the students did starting from the lowest level to the highest level mentioning which grade they are from and what they did. I think that would make it easier to follow.	In terms of capturing the essence, I feel like you do is just sometimes hard to follow through because of the organization of the paragraph.	The examples are clear, I would add some quotes or transcripts to actually help the reader see what the students did and then explain what you understand they are doing.	what do you do when the kids use language like turn rotation, to mean something else? Or if they say I flipped it to mean they turned it? Or a reflection? I remember noticing that in the videos.

A.2.b	Yes, you capture everything and have really good explanations in there. However, I think they would be easier to understand if you broke down by code and organized it like I describe below. You can break it down with subheadings and explanations with examples and this will make it easier for the reader to follow your thoughts on the codes.	Yes, but a change in organization will help to be able to understand. o I feel these codes as listed in the bullet list capture well the data.	I would include the transcript, the grade and the percentages or counts of each thing you are describing.	I actually like how you organized it in the bullet points (column H) starting from the lowest level to the highest. Also, I like your commentaries. It makes it easier to understand. I would use a similar organization in the manuscript. I think you changed the order in the manuscript and is easier to understand in the order that is written in the bullet points.

A.2.c	It does, I would do as I suggested before in terms of organization. Also try not to sound like you are just writing bullet points, connect your sentences.	Yes, I like how you explain your findings here	As I mentioned I would show your actual examples and give numbers for each time you say some or many etc. and also would expand on the context.	I like these codes a lot and that you included the show me and the physical description of the things they are doing. This is great, often the students may not have the language but can physically explain things and just because they don't have the language doesn't mean they don't know or understand some aspects.
A.2.d	Shape: This one was harder to understand because it sounded like bullet points and some of the sentences were a little bit too long. • I would re-organize like I have suggested before adding examples and including numbers.	Yes	I would add numbers and transcripts like I mentioned before	