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Research in Mathematics Education

Imagination Station (Istation):

Universal Screener Instrument Development for Grade 4

Technical Report 11-01

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Universal Screener Instrument Development for Grade 4

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Abstract

In this technical report, we describe the development of the Grade 4 Formative Assessment Item Bank for the Imagination Station (Istation). The formative assessment item bank will be used to deliver a computerized adaptive universal screening assessment to support teachers' instructional decisions. The construct underlying the items is mathematics skills and knowledge in Grade 4 as defined by state and national content standards. We include a description of the process used to identify and sample the content and levels of cognitive complexity assessed in the item bank. Next, we describe the item writing procedures. Finally, we describe the process and outcomes of an external item review to document content related evidence for validity.

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Imagination Station (Istation) Universal Screener Instrument Development for Grade 4

Introduction

The purpose of the Grade 4 Formative Assessment Item Bank for the Imagination Station (Istation) is to support teachers' instructional decisions. Specifically, the formative assessment item bank is designed to serve as a computerized adaptive universal screening assessment system. By administering this assessment system, teachers and administrators can use the results to answer two questions: (1) are students at risk of failure in Grade 4 mathematics, and (2) what is the degree of intensity of instructional support students need to be successful in Grade 4 mathematics? Because the universal screener is designed for administration in fall, winter, and early spring, these decisions can be applied over the course of Grade 4. The universal screener is intended to be administered to all students receiving grade-level instruction.

The purpose of this technical report is to describe the development of the formative assessment item bank. Included in this description is the process used to identify and sample the content and levels of cognitive complexity assessed in the item bank, the item writing process, and external review procedures and results. The test development steps used to create the Formative Assessment Item Bank are representative of the Test Standards published by the American Educational Research Association (AERA), American Psychological Association (APA), & National Council on Measurement in Education (NCME) (1999), and represent best practices in test development.

Construct Definition

The construct of an assessment is considered on two dimensions: content and cognitive engagement. The content of the Grade 4 Formative Assessment Item Bank is based on the Curriculum Focal Points (CFP) published in 2006 by National Council of Teachers ofMathematics (NCTM). Using the NCTM Curriculum Focal Points as the referent standard, we aligned the mathematics content standards published by the Common Core Standards Initiative as well as from the states of Texas, Florida, New York, California, and Virginia to the CFPs. See Appendix A for the referent documents for each state. For Grade 4, we created a fourth CFP that encompasses measurement and geometry standards assessed across the states. See Appendix B for an abbreviated description of the assessed content.

The cognitive engagement dimension of the construct refers to the level of cognitive processing through which students are expected to engage with the content. The Formative Assessment Item Bank in Mathematics relies on the taxonomy of cognitive engagement published by Kilpatrick, Swafford, and Findell (2001) for the National Research Council. The five interdependent components (or strands) of students' engagement with the content provide a framework to promote mathematical proficiency.

1. Conceptual understanding pertains to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves

being able to logically organize one's knowledge to integrate and understand concepts as part of a coherent whole.

2. Procedural fluency pertains to students' ability to accurately and appropriately carry out
skills, including being able to select efficient and flexible approaches.

3. Strategic competence involves one's ability to formulate a problem in mathematical terms, to
represent it strategically (verbally, symbolically, graphically, or numerically), as well as to
solve it effectively. It is similar to problem solving and problem formation.

4. Adaptive reasoning involves the student's capacity to think logically about a problem, which
requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.

5. Productive disposition refers to a student's overall ability to perceive mathematics as worthwhile and to maintain a personal belief in one's own efficacy in solving problems.

The Formative Assessment Item Bank incorporates four of the five strands; productive disposition is not assessed.

The four CFPs were assessed for each of the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to accurately reflect the relative emphasis in the state standards. Easy, medium, and difficult items were written for each CFP across the four levels of cognitive engagement. The content sampling matrix is presented in Figure 1.

Item Writing

Item Specifications

Approximately 400 items were written for Grade 4. Multiple-choice items were created for efficiency in the computer delivery. Each item had three distractors and one correct answer. The distractors represented plausible misconceptions or errors. Errors included computational errors, conceptual errors, procedural errors, and strategy errors. The stem included text and/or graphics. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included. The reading level of the items was intentionally constrained to the 4th grade level; however, readability statistics were not calculated for each item. Whenever possible, plain language and simple, straight-forward statements were incorporated into the items.

Items are written following the principles of universal design for assessment (c.f., Ketterlin-Geller, 2005; 2008), and are amendable to accommodations. Items are scored dichotomously. The computerized adaptive test can be administered individually or in a group in an untimed setting.

Item Writers

Five item writers contributed items to the Grade 4 Formative Assessment Item Bank.

Item Writer 1. Item Writer 1 obtained a Bachelor of Public Administration degree from Texas State University – San Marcos. She worked as a long-term substitute on various high school campuses across the State of Texas teaching mathematics. Upon completing her degree she worked for the Office of the Attorney General for the State of Texas as a Child Support Officer. After almost four years working in the public sector, she was accepted into the New York City Teaching Fellows Program where she taught high school mathematics to special education students in an inner city school in Manhattan. During this time, Item Writer 1 completed a Master of Science degree in Special Education with Honors from the City College of New York. While teaching, her main focus was in a co-teaching environment instructing students with disabilities in the areas of Algebra and Geometry. After two years teaching in New York City, she left the classroom to focus on research, specifically in the area of test development for students with disabilities. She has worked on several projects and nationally funded grants in the area of educational research and is currently completing the requirements for a Ph.D. in Educational Research from Southern Methodist University.

Item Writer 2. Item Writer 2 holds a Bachelors of Science and a Masters degree in Special Education. He has been a research assistant, project coordinator and independent contractor for federally funded grants and state contracts since 2001. He assisted in the creation of a web-delivered math assessment, researching effectiveness of accommodations. Additionally, he was involved in the development of the alternate assessment for students with significant cognitive disabilities at the elementary, middle, and high school levels between 2002 and 2009. Item Writer 2 also was a member of an item writing team for creating items for math screening tests and assisted in development of accommodated versions of math items. He has been a part of several research teams conducting multi-state research projects examining comparability of performance on different alternate assessments.

Item Writer 3. Item Writer 3 earned a Master of Science in Mathematics Education and an Honors Bachelor of Science in Mathematics, both from Oregon State University. Throughout her six years of teaching, she has taught math at all levels from middle school through high school and community college. In addition to teaching, she currently works as a mathematics coach for her district. Within this position, she focuses on improving math instruction across the district by coordinating and developing curriculum that is aligned to state mathematics

standards. Her interest with assessments led her to writing mathematics assessment items.

Item Writer 4. Item Writer 4 has a Masters of Arts in Teaching from Oregon State University and a Bachelors of Arts in Mathematics from California State University at Fresno. Her thirteen years of teaching high school math have included courses at a variety of levels. The courses have varied in difficulty from foundational math to help transition students into high school through Advanced Placement (AP) and International Baccalaureate (IB) courses that offer students the option of earning college credit during high school. While teaching half-time, she is also currently a half-time math coach helping to coordinate and develop math instructional materials at all grade levels within the district. Over the last five years she has served on several state math panels that have worked to update our state math standards and review/align test items to our state standards.

Item Writer 5. Item Writer 5 is a school psychologist with expertise in mathematics education. She earned a Ph.D. in Educational Leadership with a focus on assessment and measurement. She has been the lead author on a district-wide mathematics formative assessment given to all first through eighth grade students three times per year. Her work on this project also included vertical equating and scaling tests. Since graduating, she worked for a nonprofit organization assisting in the design, development, and data collection of evaluations of education programs and improvement initiatives. Most recently, she served as a school psychologist where she conducted comprehensive psycho-educational evaluations to determine if students are eligible for special services and inform interventions.

Item Writing Training

All item writers were trained to write items that aligned with the content expectations and met the item specifications. The training included review of the Item Writing Training Manual, as well as participating in a training conference call with the researchers and project staff. The Item Writing Training Manual provides a detailed description of the principles of universal design for assessment. In addition, information was provided on the elements of high quality test design. Guidelines for writing selected response items were provided from recognized experts in item design. Moreover, sample items were provided to illustrate important components of effective items. A glossary of useful terms and a list of relevant websites were provided. The Item Writing Training Manual also included logistical information about formatting, reviewing, and submitting items.

A training conference call was conducted to review the content standards and levels of cognitive complexity for Grade 4. Project staff provided a detailed description of the content. Example items for each level of cognitive complexity were disseminated and discussed. The material in the Item Writing Training Manual was reviewed and discussed until the item writers were confident they understood the content and objectives of the project.

Item Writing Process

After completing the training and attending a project conference call, item writers were provided with the item writing template for creating items. Items were submitted and reviewed by the researchers and project staff. At least two internal reviewers provided feedback for each item. Dimensions of the review included the mathematical accuracy of the item, alignment with the content standards, appropriateness of language and graphics for students in Grade 4, and compliance with the principles of universal design. Comments were returned to the item writers; revisions were made and resubmitted for approval.

Once items were accepted, item level information was entered into an Item Database. Graphics were created by the Istation graphic design team. The finalized items were copy edited and reviewed by SMU researchers and Istation staff.

Content-Related Evidence for Validity

To evaluate the accuracy and appropriateness of the content of the Formative Assessment Item Bank for students in Grade 4, all items were reviewed by mathematicians and teachers.

Mathematician Review

Three mathematicians reviewed all items in Grade 4. Two reviewers were professors of mathematics at a university in Texas. One reviewer was a post-doctoral fellow in mathematics at a university in California working on a project funded through the National Science Foundation. All reviewers had undergraduate and graduate degrees in mathematics. The years teaching and researching in mathematics ranged from 5-17 years. All reviewers were female. Two reviewers were Caucasian and one reviewer was Filipino and Caucasian.

The mathematicians were asked to review each item and evaluate the accuracy of the content, precision of the vocabulary, and effectiveness of distractors. The criteria were furtherdescribed as follows:

- Mathematical accuracy of content: Each item was written to reflect an integration of knowledge and skills identified by the NCTM Curriculum Focal Points. Is the item mathematically accurate?

- Precision of mathematical vocabulary: Is the mathematical vocabulary used accurately? Is the mathematical vocabulary precise?

- Appropriateness of the distractors: Most students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not accurate, precise, or the distractors were not effective; a rating of 2 indicated that the item was somewhat accurate, precise, or the distractors were somewhat effective; rating of 3 indicated that the item was somewhat accurate, precise, or the distractors were mostly effective; and a rating of 4 indicated the item was somewhat accurate, precise, or the distractors were extremely effective. In instances where the reviewer assigned a score of 1 or 2 on a category, recommendations were solicited that would aid in revision.

Overall, the mathematicians rated the items as mostly to always accurate, precise, and effective. For 57 items, the mathematicians recommended revisions. The primary consideration for revision was the mathematical precision of the language and graphics. For some items, the mathematicians identified multiple plausible answer choices. We revised all items in response to the recommendations. In instances where the mathematician did not provide a suitable suggestion, we revised the item and requested an additional review from an independent mathematician.

Teacher Review

Two teachers with experience teaching Grade 4 mathematics reviewed the items. One reviewer had been teaching in the state of Texas for more than 20 years. The other reviewer was from the state of Oregon and had 4 years of teaching experience. Both reviewers had Master's degrees in education and had experience working with students with disabilities. Both teachers were Caucasian females.

Teachers analyzed each item for grade-level appropriateness in terms of understandability of language and vocabulary, content or concepts, graphics, potential bias in language and/or content, clarity of directions and answers, and effectiveness of distractors. The criteria were further described as follows:

- Appropriateness of language: Is the language used in the item appropriate for students in your grade level? Are the question and response options written so that students in your grade level can understand the meaning of the problem?

- Appropriateness of mathematical vocabulary: Is the mathematical vocabulary representative of pre-requisite or instructional expectations in your grade level?

- Appropriateness of content or concepts: Is the task representative of pre-requisite or instructional expectations in your grade level?

- Appropriateness of visual representation: Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in your grade level? Can students in your grade level understand the meaning of the visual representation? Is the visual representation of the item clear?

- Bias in language or content: Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs? Example: "What *is the most appropriate measurement unit for the length of a sub or hoagie?"* may be unfair for students in certain geographic regions and students with diverse background who are unfamiliar with the terms "sub or hoagie."

- Effectiveness of the distractors: Some students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Do the distractors discriminate between students with specific misconceptions?

The items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 indicated that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); rating of 3 indicated that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 indicated that the item/distractors were extremely appropriate based on the criterion (or not biased *and* has multi-cultural components to it). In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Overall, the teachers rated the items as mostly appropriate in regards to language, vocabulary content, visual representation, bias, and effectiveness of distractors. The teachers recommended revising 33 items, primarily due to language. Most of the comments suggested using common language for students in Grade 4, such as using the term "garden" as opposed to the term "flower bed." Several comments referenced the clarity of the visual/graphics. The research team reviewed all suggestions and made revisions based on teacher feedback.

Conclusions

The purpose of this technical report was to describe the development of the formative assessment item bank. We described the construct underlying the items in reference to the content standards and levels of cognitive complexity and described the process for sampling the content assessed in the item bank. Next, we described the item writing procedures and provided the qualifications for the item writers. Finally, we documented the process and outcomes of an external item review by mathematicians and teachers to document content related evidence for validity.

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Figure 1

	Procedural fluency			Conceptual understanding			Strategic competence			Adaptive reasoning		
CFP	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult
1	10	10	10	10	10	10	7	6	7	7	6	7
2	10	10	10	10	10	10	7	6	7	7	6	7
3	10	10	10	10	10	10	7	6	7	7	6	7
4	10	10	10	10	10	10	7	6	7	7	6	7
Total By Difficulty	40	40	40	40	40	40	28	24	28	28	24	28

Appendix A - State Content Standards Referent Sources

National Council of Teachers of Mathematics (NCTM) Curricular Focal Points

The National Council of Teachers of Mathematics (NCTM) Curricular Focal Points were retrieved from http://www.nctmmedia.org/cfp/front_matter.pdf on 4/20/2010. Additional information was also retrieved on 4/20/2010 from: www.nctm.org/focalpoints . The coding system for the NCTM Critical Focal Points can be found under Part II.

Florida

Florida's Next Generation Sunshine State Math Standards (adopted 2007) were retrieved on 4/20/2010 from http://www.floridastandards.org/Standards/FLStandardSearch.aspx. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from Florida Department of Education. Big Ideas for each of the grade levels were also verified.

California

California's Math Content Standards (adopted 1997) were retrieved on 4/24/2010 from http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf . California Green Dot Standards are the selected standards (as of 2006) that appear 85% of the time on California state tests. These green dot standards were retrieved on 4/24/2010 from http://caworldclassmath.com/high_ca_standards.html and etc.usf.edu/flstandards/math/california.ppt . Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the California State Board of Education.

New York

The New York State Standards (revised on March 15, 2005) were retrieved on 4/21/2010 from: http://www.bootstrapworld.org/standards/ny/NYMathematicsCoreCurriculum.pdf .Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the New York State Board of Education.

Texas

The Texas State Standards for Math (Version 2.1; revised 2010) were retrieved on 4/21/2010 from: http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the Texas State Board of Education. The Texas Education Agency (TEA) released a 2010 document entitled *Texas Response to Curriculum Focal Points: Kindergarten through Grade 8 Mathematics* that included coordinating TEKS.

Common Core Standards

The Common Core Standards in Mathematics were retrieved on June 10, 2011 from http://www.corestandards.org/the-standards/mathematics . These standards were published in 2010.

They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

Virginia

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved on June 10, 2011 from www.doe.virginia.gov/testing/sol/standards_docs/mathematics/review.shtml . Verification of accuracy and currency of the standards was obtained from Istation on June 10, 2011. The Curriculum Frameworks documents were referenced to determine the essential knowledge and skills students are expected to learn for each grade.

Appendix B - Content Description

GRADE 4 MATHEMATICS CURRICULUM FOCAL POINTS	
CFP 1: Number and Operations* and *Algebra **Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication.**	
4.1A.1	Students use understandings of multiplication to develop **quick recall of the basic multiplication facts** and related division facts.
4.1B.1	Students apply their understanding of **models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line)** as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers.
4.1B.2	Students apply their understanding of models for **place value** as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers.
4.1B.3	Students apply their understanding of models for **properties of operations** (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers.
4.1C.1	Students select appropriate methods and apply them accurately to **estimate products or calculate them mentally**, depending on the context and numbers involved.
4.1D.1	Students develop **fluency with efficient procedures,** including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.
4.1E.1	Students continue identifying, describing, and extending **numeric patterns** involving all operations **and nonnumeric** growing or repeating patterns.
A4.CFP1.1	Evaluate and express relationships using open sentences with one operation

A4.CFP1.2	Use the symbols <, >, =, and ≠ (with and without the use of a number line) to compare whole numbers and unit fractions and decimals (up to hundredths)
A4.CFP1.3	Find the value or values that will make an open sentence true, if it contains < or >
A4.CFP1.4	Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers
A4.CFP1.5	Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable)
A4.CFP1.6	Understand that an equation such as y = 3 x + 5 is a prescription for determining a second number when a first number is given
A4.CFP1.7	Know and understand that equals added to equals are equal
A4.CFP1.8	Know and understand that equals multiplied to equals are equal
CFP 2: Number and Operations **Developing an understanding of decimals, including the connections between fractions and decimals**	
4.2A.1	Students **understand decimal notation** as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on.
4.2B.1	Students **read and write fractions or decimals that are greater than or less than 1** in problem solving.
4.2B.2	Students **identify equivalent fractions or decimals** in problem solving.
4.2B.3	Students **compare and order fractions or decimals** in problem solving.
4.2B.4	Students **estimate decimal or fractional amounts** in problem solving.
4.2C.1	Students **connect equivalent fractions and decimals** by comparing models to symbols and locating equivalent symbols on the number line.

4.2D.1	Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts.
4.2E.1	Students use estimation in determining the relative sizes of amounts or distances.
4.2F.1	Students develop understandings of strategies for multi-digit division by using models that represent division as the **inverse of multiplication.** 4 x 45 = 180, 180 ÷ 4 = 45, and 180 ÷ 45 = 4.
4.2F.2	Students develop understandings of strategies for multi-digit division by using models that represent division as **partitioning.** We can share 180 things (possibly represented by base-ten blocks) evenly among 4 groups and determine the number of items in each group.
4.2F.3	Students develop understandings of strategies for multi-digit division by using models that represent division as **successive subtraction**. We can find the quotient of 180 ÷45 by repeatedly subtracting 45 and counting the number of groups of 45 subtracted before reaching zero.
4.2G.1	By working with decimals, students extend their ability to recognize equivalent fractions.
A4.CFP2.1	Skip count by 1,000's
A4.CFP2.2	Compare and order numbers to 10,000
A4.CFP2.3	Develop an understanding of the properties of odd/even numbers as a result of multiplication
A4.CFP2.4	Use a variety of strategies to add and subtract numbers up to 10,000
A4.CFP2.5	Select appropriate computational and operational methods to solve problems
A4.CFP2.6	Interpret the meaning of remainders
A4.CFP2.7	Add and subtract proper fractions with common denominators
A4.CFP2.8	Express decimals as an equivalent form of fractions to tenths and hundredths
A4.CFP2.9	Add and subtract decimals to tenths and hundredths using a hundreds chart
A4.CFP2.10	Round numbers less than 1,000 to the nearest tens and hundreds

A4.CFP2.11	Read and write whole numbers in the millions.
A4.CFP2.12	Decide when a rounded solution is called for and explain why such a solution may be appropriate.
A4.CFP2.13	Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalents of fractions (see Standard 4.0).
A4.CFP2.14	Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").
A4.CFP2.15	Estimate and compute the sum or difference of whole numbers and positive decimals to two places
A4.CFP2.16	Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer
A4.CFP2.17	Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multi-digit numbers
A4.CFP2.18	Solve problems involving division of multi-digit numbers by one-digit numbers
A4.CFP2.19	Understand that many whole numbers break down in different ways (e.g., 12 = 4 x 3 = 2 x 6 = 2 x 2 x 3)
CFP 3: Measurement **Developing an understanding of area and determining the areas of two-dimensional shapes** ***Data Analysis Connections to the Focal Point*** **includes students making frequency tables, bar graphs, picture graphs, line plots, and stem-and-leaf plots.**	
4.3A.1	Students recognize **area as an attribute** of two-dimensional regions.
4.3B.1	Students learn that they can **quantify area** by finding the total number of same-sized units of area that cover the shape without gaps or overlaps.
4.3C.1	Students understand that a **square**, which is 1 unit on a side, is the **standard unit** for measuring area.

4.3D.1	Students select appropriate **units, strategies** (e.g., decomposing shapes), **and tools** for solving problems that involve **estimating or measuring** area.
4.3E.1	Students **connect area measure to the area model** that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.
4.3F.1	Students extend their understanding of properties of two-dimensional shapes as they find the **areas of polygons**.
4.3G.1	Students build on their earlier work with symmetry and congruence in grade 3 to encompass **transformations**, including those that produce line and rotational symmetry.
4.3H.1	By using transformations to **design and analyze simple tilings and tessellations**, students deepen their understanding of two-dimensional space.
4.3I.1	As part of understanding two-dimensional shapes, students **measure and classify angles**.
4.3J.1	Students apply their understanding of place value to **develop and use stem-and-leaf plots**.
A4.CFP3.26	Identify the mode(s) for sets of categorical data and the mode(s), median, and any apparent outliers for numerical data sets
A4.S.1	Design investigations to address a question from given data
A4.S.2	Collect data using observations, surveys, and experiments and record appropriately
A4.S.3	Represent data using tables, bar graphs, and pictographs
A4.S.4	Read and interpret line graphs
A4.S.5	Develop and make predictions that are based on data
A4.S.6	Formulate conclusions and make predictions from graphs
A4.S.7	Represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams)
A4.S.8	Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; 3 /4)

Measurement and Geometry Connections to the Focal Point

Students build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Measurement	
A4.CFP3.1	Students select appropriate units, strategies, and tools for solving problems that involve estimating measurements or measuring various attributes.
A4.CFP3.8	Use a ruler to measure to the nearest standard unit (whole, ½ and ¼ inches, whole feet, whole yards, whole centimeters, and whole meters)
A4.CFP3.9	Know and understand equivalent standard units of length: 12 inches = 1 foot 3 feet = 1 yard
A4.CFP3.10	Select tools and units appropriate to the mass of the object being measured (grams and kilograms)
A4.CFP3.11	Measure mass, using grams
A4.CFP3.12	Select tools and units appropriate to the capacity being measured (milliliters and liters)
A4.CFP3.13	Measure capacity, using milliliters and liters
A4.CFP3.14	Make change, using combined coins and dollar amounts
A4.CFP3.15	Calculate elapsed time in days and weeks, using a calendar
Geometry	
A4.CFP3.17	Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation y = 3 x and connect them by using a straight line)
A4.CFP3.18	Understand that the length of a horizontal line segment equals the difference of the x- coordinate

A4.CFP3.19	Understand that the length of a vertical line segment equals the difference of the y- coordinates
A4.CFP3.20	Identify lines that are parallel and perpendicular
A4.CFP3.21	Identify the radius and diameter of a circle
A4.CFP3.22	Identify congruent figures
A4.CFP3.23	Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4, and full turns
A4.CFP3.24	Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their attributes
A4.CFP3.25	Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid)
A4.CFP3.16	Recognize that rectangles that have the same area can have different perimeters
A4.CFP3.6	Find perimeter of polygons by adding sides
A4.CFP3.4	Identify and build a three-dimensional object from a two-dimensional representation of that object and vice versa.
A4.CFP3.5	Identify points and line segments when drawing a plane figure
A4.CFP3.7	Define and identify vertices, faces, and edges of three-dimensional shapes
A4.CFP3.2	Students use concrete models of standard cubic units to estimate and measure volume.
A4.CFP3.3	Students use tools to measure temperature, temperature change, and time change.