Operations with Whole Numbers - Teaching Mathematics for Understanding Through Problem Solving, Discourse, and Practice

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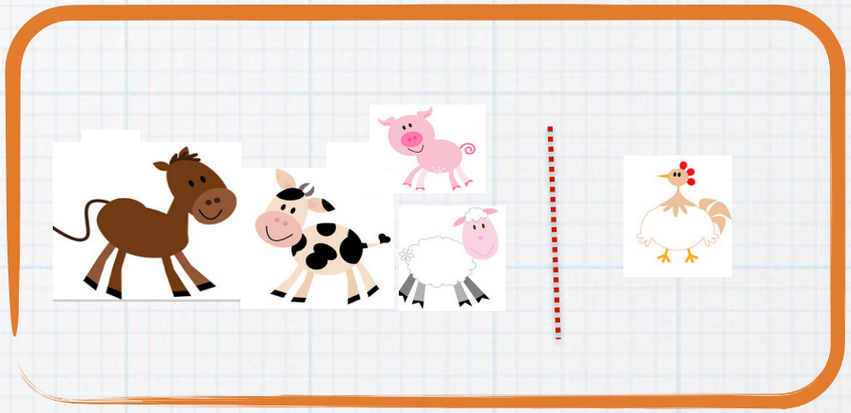
Session Goals

Illustrate how teaching Mathematics for Understanding through the TEKS Mathematical Process Standards is supported through

* Problem Solving
* Discourse
* Practice

"The brain is a pattern detector, constantly processing information from the senses."



"The chicken is different because the others have babies"

"The brain searches for meaning - looking for similarities between incoming patterns of information and information stored in memory."

(Ernst & Ryan, 2014, p.18)

The TEKS Mathematical Process Standards...

* Guide students to think about mathematics concepts
* Help students learn multiple strategies for solving problems
* Provide multiple types of models, tools, and representations for communicating mathematical thinking
* Help students to make connections between different topics and foster deeper understanding of mathematics concepts
* Weave other knowledge and skills together so that students may become successful problem solvers

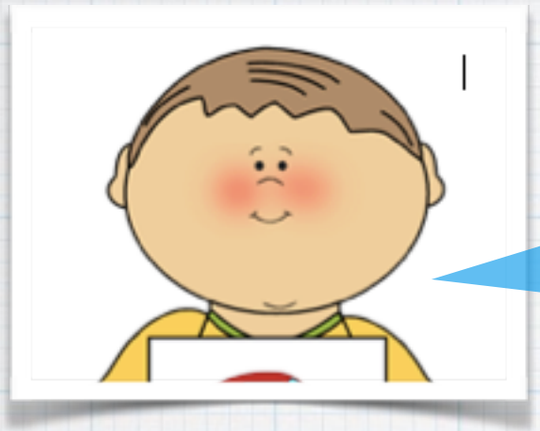
"Students build continually growing networks of connected mathematical ideas as they solve challenging mathematical problems, explain and justify their reasoning, get feedback from their peers and teachers, and revise their thinking."

(Ernst & Ryan, 2014, p.18)

What does a problem-solving mathematics lesson sound like?

Sandy had 46 baseball cards. Her brother gave her 37 baseball cards for her birthday. Now how many baseball cards does Sandy have?

Source: Sample vignette from Ernst & Ryan, 2014, p. 3-15.



Um.. first Sandy has 46 baseball cards.

Sandy has 46 cards. → Brother gave her 37 cards. → How many cards does she have?

Her brother gives her 37 more baseball cards.

How many cards does she have now?

Sandy has 46 cards.

Brother gave her 37 cards.

How many cards does she have?

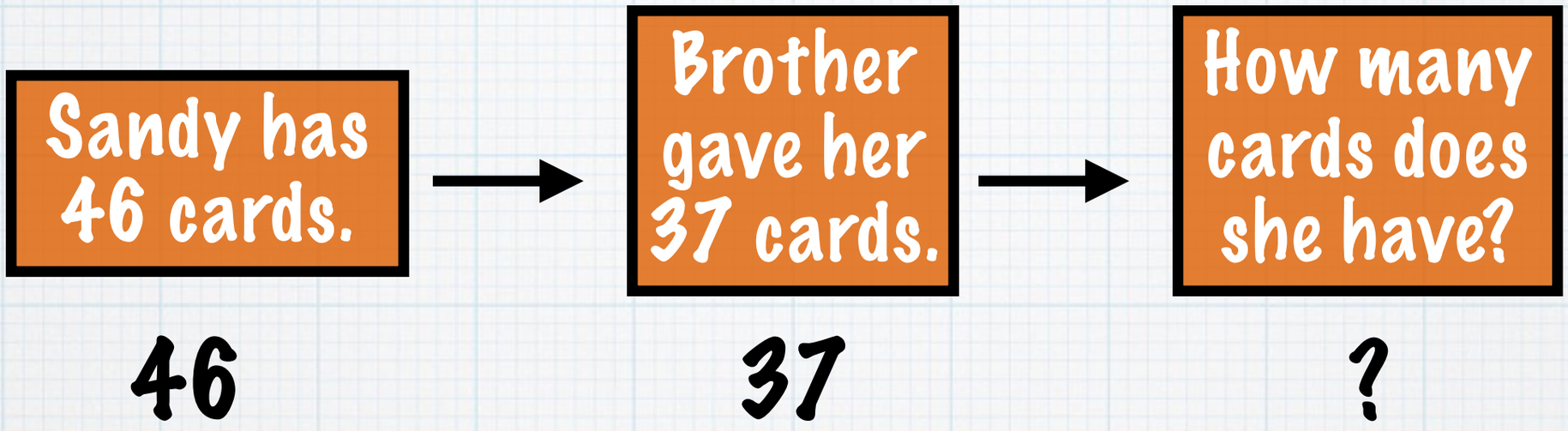
46

37

?

You have to add 37 cards to the 46 because now she has more.

Put a plus sign?



$$46 + 37 = ?$$

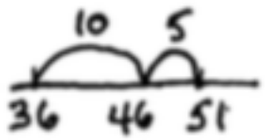
You have to find out how many baseball cards Sandy has now.

ADDITION STRATEGIES
36+15=

Adding on by a friendly number

36+15

10+5



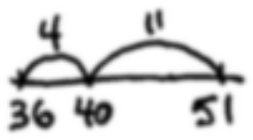
36+10=46

46+5=51

Adding on to a friendly number

36+15

4+11



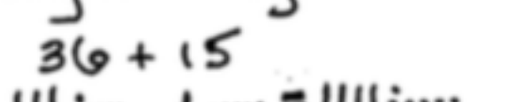
36+4=40

40+11=51

Adding tens and ones

Using a drawing:

36 + 15



30+6 10+5 40+11=51

Using equations:

36 + 15

30+6 10+5

40+11=51

30+10=40

6+5=11

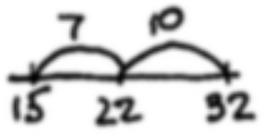
40+11=51

SUBTRACTION STRATEGIES
32-17=

Removing by a friendly number

32-17

10+7



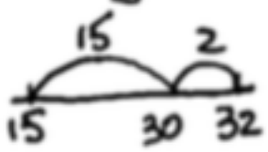
32-10=22

22-7=15

Removing to a friendly number

32-17

2+15



32-2=30

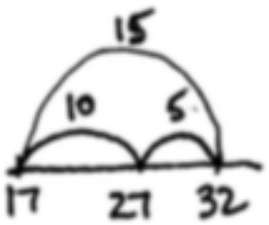
30-15=15

Adding on

17+__=32

17+10=27

27+5=32



17+15=32

32-17=15

46 + 37

40 + 6 30 + 7

I solved this problem by breaking the 46 into 40 and 6 and the 37 into 30 and 7

4 + 3 = 7, so 40 + 30 = 70

6 + 7 = 13

70 + 13 =

10 + 3

I added the 40 and 30 to get 70 and the 6 and 7 to get 13

70 + 10 = 80

80 + 3 = 83

then I added the 70 and 13 to get 83.

46 + 37 =

46 + 37

|||| ..... + ||| ..... = ||||||| ::::::.

I drew sticks of ten.
I made four sticks of ten and six ones,
and three sticks of ten and seven ones,
and then I counted the sticks
and counted the ones
and added them up.

46 + 37

|||| ..... + ||| ..... = ||||||| ::::::.

70 13

70 + 13 = 83

I got 83 too. We both got the same answer

But you made sticks of ten and I just used numbers.

How is Tiffany's solution similar to Michael's? different?

Michael
Adding tens and ones
(using drawings)

46 + 37

IIII ..... + III..... = IIIIIII :::::::.
70 13

70 + 13 =
10 + 3

70 + 10 = 80
80 + 3 = 83

Tiffany

46 + 37
40 + 6 30 + 7

40 + 30 = 70
6 + 7 = 13

70 + 13 =
10 + 3

70 + 10 = 80
80 + 3 = 83

They both did tens and ones.

Michael used the hundred chart and Tiffany didn't

They both used addition.

Most of it is the same. They mostly have the same numbers. But Michael drew tens and ones and Tiffany just did numbers.

We've been talking about choosing efficient ways to solve problems - ways that make it easy for you to keep track of the problem and all the steps.

Which solution was more efficient, adding tens and ones by using drawings or by using equations? Think, and then talk to your partner about which one you think is most efficient and why.

I think you might make a mistake when you are drawing. You might draw the wrong number.

Using equations is more efficient because it has fewer steps. You just have to write the numbers. You can add 6 and 7 in your head instead of counting.

Polya's Problem Solving Technique

* 1. Understand the problem
* 2. Devise a plan (strategy)
* 3. Carry out the plan (solve it)
* 4. Look back (reflection)

Which Process Standards did you notice in the sample lesson?

* apply mathematics
* use a problem-solving model
* select appropriate tools and techniques
* communicate mathematical ideas, reasoning, and their implications
* Create and use representations
* analyze mathematical relationships
* justify mathematical ideas and arguments

Collaboration: There's strength in numbers

* Ms. Santos owns a neighborhood grocery store. She has 56 apples to arrange in rows for her window display. She has room for 4 rows in her window. How many apples will there be in each row if she puts the same number in each row?

* Teacher Guide Stated Goals: 1) solving division story problems, and 2) using and interpreting division notation

Source: Sample vignette from Ernst & Ryan, 2014, p. 133-149.

Students have experience representing multiplication with arrays.

Students know how to break larger arrays into smaller arrays.

Students know multiplication as groups of things.

Learning Goals:

1. Solve a division word problem
2. Understand the meaning of division
3. Interpret division notation
4. Develop problem solving skills

1/3 of the class will say they don't know how to get started.

Students have not formally studied division and may be anxious.

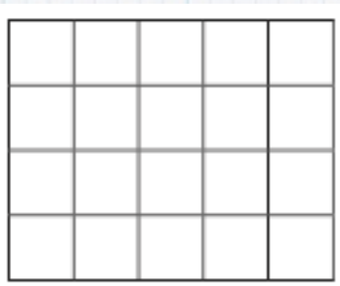
Anticipate Student Responses/Strategies

Tools: Model with tiles; Draw pictures; Use equations

Deals tiles by 1's

1	5
2	6
3	7
4	8

Arrays with friendly #s then reasons



$56 - 20 = 36$

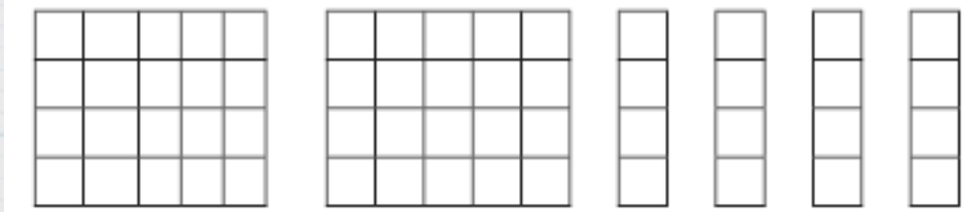
$36 \div 4 = 9$

I can add 9 more apples to each row.

Each row has 14 apples.

$4 \times 5 = 20$

Deals by 1's then chunks by friendly #s



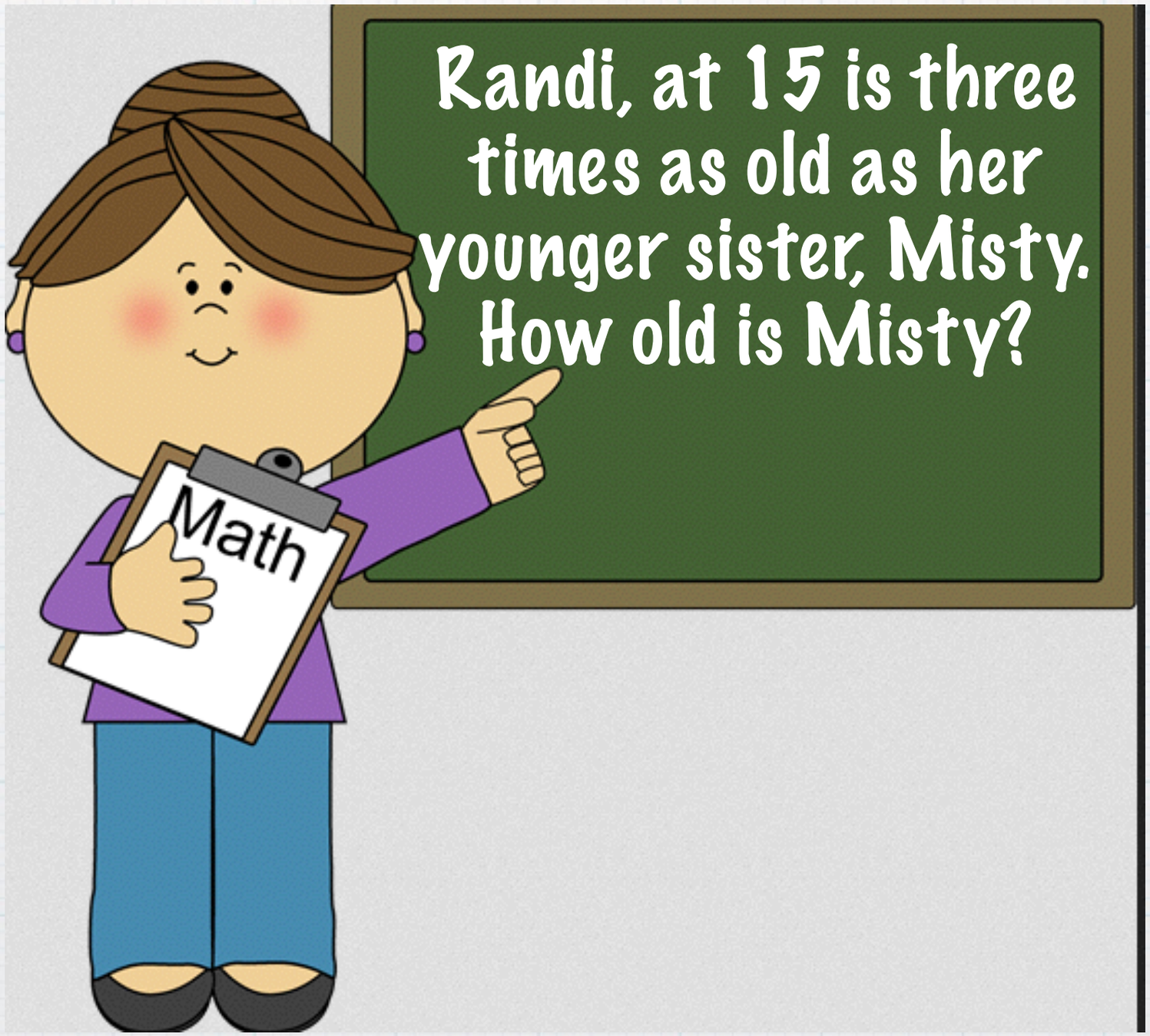
Multiples of 10

$4 \times 10 = 40 \rightarrow 56 - 40 = ? \rightarrow 56 - 40 = 16$

$4 \times ? = 16$

Is this problem division or a multiplication? Why? What equation would match the problem?

Let's starts here...



TEKS

(4.4) Number, operation, and quantitative reasoning. The student multiplies and divides to solve meaningful problems involving whole numbers. The student is expected to:

(B) represent multiplication and division situations in picture, word, and number form

(C) recall and apply multiplication facts through 12 x 12

(D) Use multiplication to solve problems

(E) Use division to solve problems

Division Strategies

Repeated Subtraction

Skip Counting

Equal Groups

Fact Families

What strategies would you anticipate 4th grade students using in the Fall?

Fact Family

$15 = 3 \times 5$

5 years old.

Equal Groups

Randi = 3 x Misty

Misty = Randi ÷ 3

Misty = 5

$3\overline{)15}\ 5$

15

Misty is 5 years old.

Randi 15

M	M	M
5	5	5

Misty is 5.

Relationship between multiplication and division, equations

Your Turn...

* Small Groups - Find 4 other people in your same grade level: K-2, 3-4, or 5-6
* Volunteer to be the recorder for your group
* In the next 10 minutes, read textbook problems and modify if needed
* Brainstorm as many student responses as possible. Recorder, write all responses on one page.
* Order them from least to most sophisticated
* Now, let's share!

Summing It Up

* Engaging students in mathematics content through the mathematical process standards provides opportunities to develop complex thinking, and are integral in learning mathematics with understanding

* Start Small, but Start

References & Resources

Ernst, K., & Ryan, S. (2014). Success from the start: Your first years teaching elementary mathematics. Reston, VA: National Council of Teachers of Mathematics.

Smith, M. S., & Stein, M. K. (2011). 5 Practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.

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