Operations with Whole Numbers – Teaching Mathematics for Understanding
Through Problem Solving, Discourse, and Practice

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Session Goals: Illustrate how teaching mathematics for understanding through the TEKS Process Standards is supported through problem solving, discourse, and practice.

The TEKS Mathematical Process Standards
- Guide students to think about mathematics concepts
- Help students learn multiple strategies for solving problems
- Provide multiple types of models, tools, and representations for communicating mathematical thinking
- Help students to make connections between different topics and foster deeper understanding of mathematics concepts
- Weave other knowledge and skills together so that students may become successful problem solvers

"Students build continually growing networks or connected mathematical ideas as they
- solve challenging mathematical problems,
- explain and justify their reasoning,
- get feedback from their peers and teachers, and
- revise their thinking."   (Ernst & Ryan, 2014, p.18)

What does a problem-solving mathematics lesson sound like?
> Sandy had 46 baseball cards. Her brother gave her 37 baseball cards for her birthday. Now how many baseball cards does Sandy have?

Which Process Standards did you notice in the sample lesson?
- apply mathematics
- use a problem-solving model
- select appropriate tools and techniques
- communicate mathematical ideas, reasoning, and their implications
- create and use representations
- analyze mathematical relationships
- justify mathematical ideas and arguments

References and Resources:

Ernst, K., & Ryan, S. (2014). Success from the start: Your first years teaching elementary mathematics. Reston, VA: National Council of Teachers of Mathematics.

Smith, M. S., & Stein, M. K. (2011). Five practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.

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Your Turn: Textbook Problem Review, Student Responses, & Sequencing

1. *Work in small groups – organize by grade levels: K-2, 3-4, 5-6*
2. *Volunteer to be the recorder in your group*
3. *Read the sample textbook problems and modify if needed. What is the learning goal?*
4. *Record all of the all possible student responses you anticipate*
5. *Order the responses according to the learning goal (e.g. level of sophistication, efficiency, CRA).*

Grade K-2 Problems
1. Noah has 7 stickers. If Noah lost 4 stickers, how many stickers would Noah have left?

2. Emma planted 139 flowers. Liam planted 122 flowers. How many flowers did Emma and Liam plant altogether?

3. Ethan took 28 pictures at the zoo. Ava took 15 pictures at the zoo. How many more pictures did Ethan take than Ava?

Grades 3-4 Problems
4. Lucas is in charge of planting trees at the park. He gives 32 trees to a team of 4 helpers. Each helper plants an equal number of trees. How many trees will each helper plant?

5. At the school store, Isabella buys 4 notebooks. Each notebook contains 51 sheets of paper. How many pieces of paper did Isabella buy?

6. Gabriel has room for 6 passengers in his van. He needs to take 38 people to the game. How many trips does Gabriel need to make in order to drive everyone to the game?

Grades 5-6 Problems
7. Zoey bought a football and a t-shirt at the store. She paid $15.35 for the football. She paid a total of $24.02 for the football and the t-shirt. How much did Zoey pay for the t-shirt?

8. A water tank contains 512 ounces of water. What is the total number of 8 ounce bottles of water that can be filled from the tank?

9. Jack has 480 baseball cards in his collection. He puts these cards into boxes. Each box has 15 cards. How many boxes does Jack need for his baseball card collection?
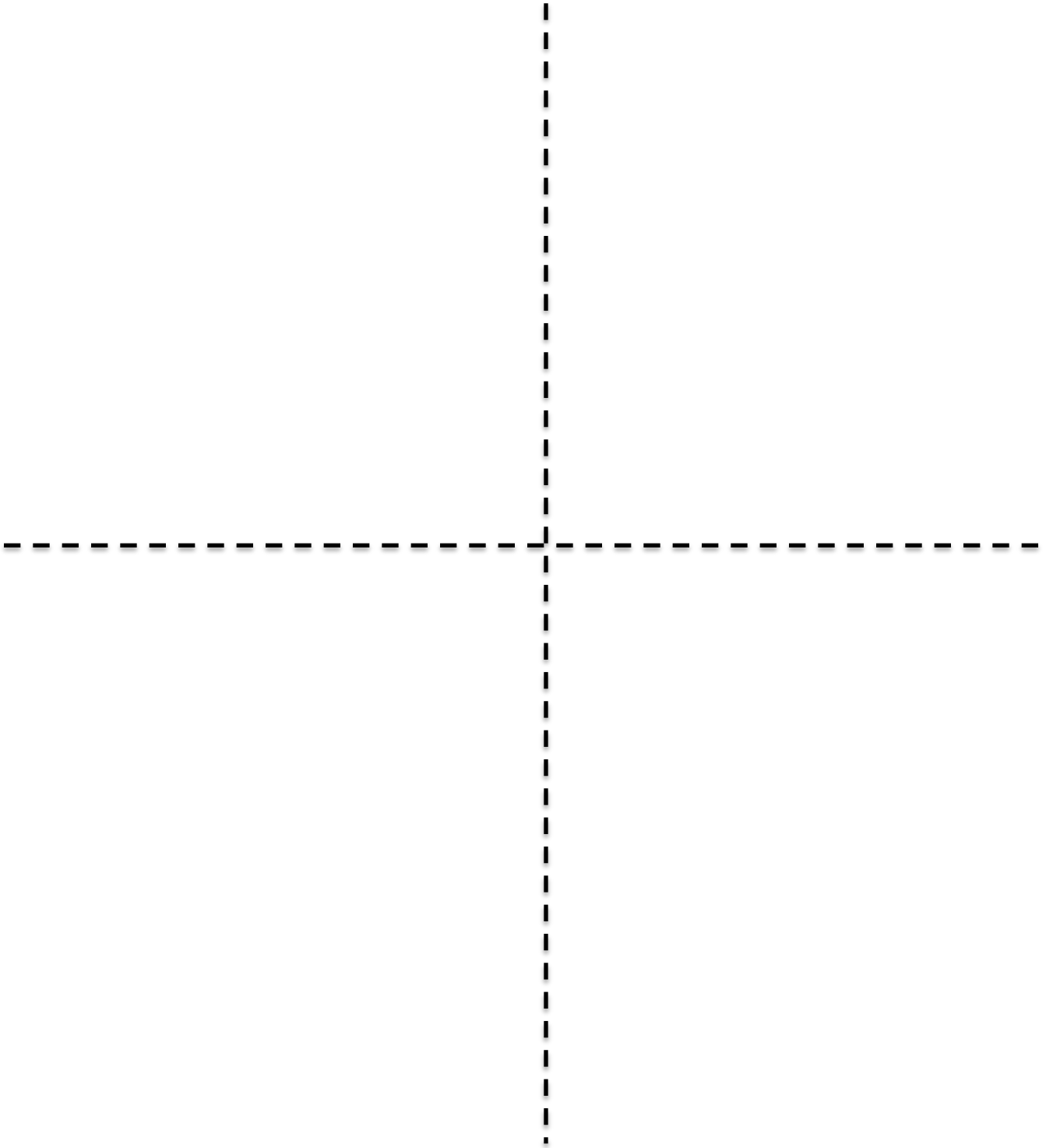
Problem # _____   Grade Level: _____

Learning Goal: _____

Modifications to problem? _____

Possible Student Responses & Sequence

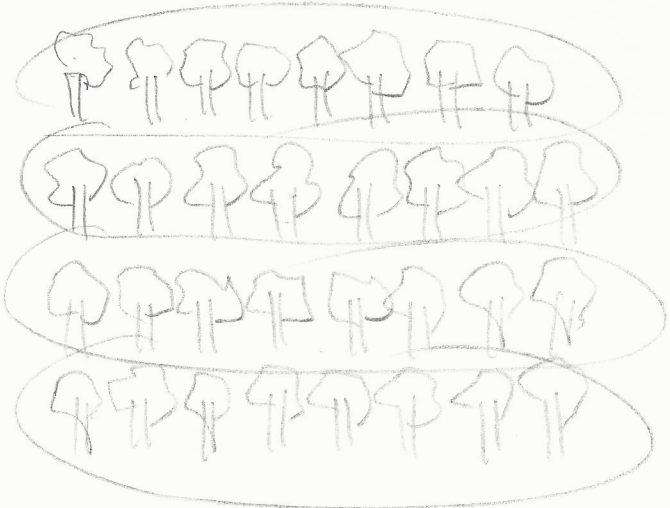Problem # ___2___   Grade Level: _2nd_   Group Recorder Page

Learning Goal: _add three-digit numbers_

Modifications to problem? _None_

Possible Student Responses & Sequence

Emma planted 139 flowers. Liam planted 122 flowers. How many flowers did Emma and Liam plant altogether?

Use pictures

$$2\square\ 5|\ \underline{||}^{\circ} = 261$$

Used expanded-form algorithm

$$100 + 30 + 9$$
$$100 + 20 + 2$$
$$\overline{200 + 50 + (11)}$$
$$\qquad\qquad\qquad 10+1$$
$$200 + 60 + 1 = 261$$

Can use open # line
(some use w/ 100's chart)

Some use trad'l algorithm

$$\begin{array}{c} H\ T\ O \\ 1\ 3\ 9 \\ +\ 1\ 2\ 2 \\ \hline 2\ 6\ 1 \end{array}$$

Problem # ____4____   Grade Level: _3-4_

Learning Goal: _Division_____

Modifications to problem? _None_____

Possible Student Responses & Sequence

32 trees
4 helpers
8 trees for each helper!

Helper 1 H2 H3 H4

| 8 | 8 | 8 | 8 |

32 trees

8 trees to each helper

H₁   H₂   H₃   H₄

0   8   16   24   32 trees

$$8\overline{)32}$$
  4
  -32
  00
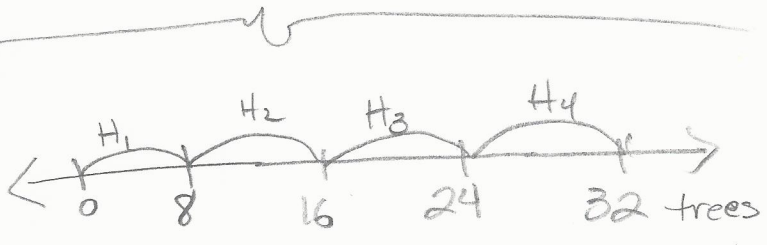
8 trees for each helper

because:
    8 → trees
  × 4 → helpers
   32 → trees

Fact family

$4 \times 8 = 32$

$8 \times 4 = 32$

$32 \div 4 = 8$

$32 \div 8 = 4$

Problem # _____4_____

Learning Goal: _____

Modifications to problem? _____

Lucas is in charge of planting trees at the park.
He gives 32 trees to a team of 4 helpers.
Each helper plants an equal number of trees.
How many trees will each helper plant?

Possible Student Responses & Sequence

32

| 32 | | | |
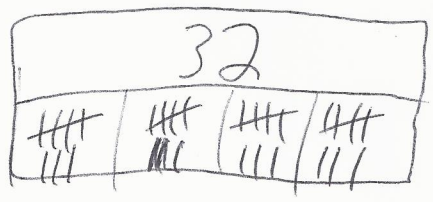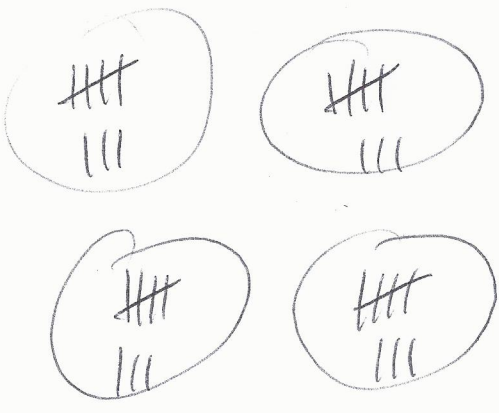
$32 \div 4 = 8$

lowest

$32 \div 4 = \boxed{8}$

$4 \times \boxed{8} = 32$

8

highest

repeated addition

4
8
12
16
20
24
28
32

0
1
2
3
4
5
6
7
8

equal groups on a number line

2nd highest

32

| 32 | | | |
| 5 2 | 5 2 | 5 2 | 5 2 |
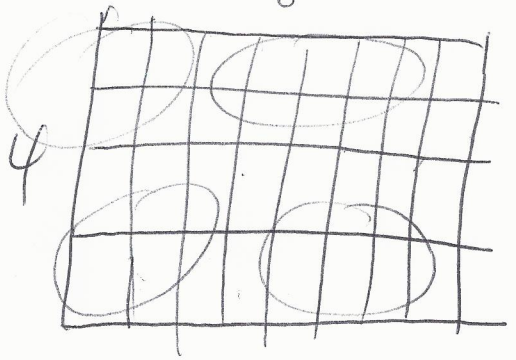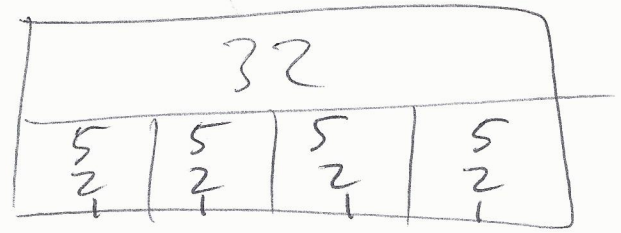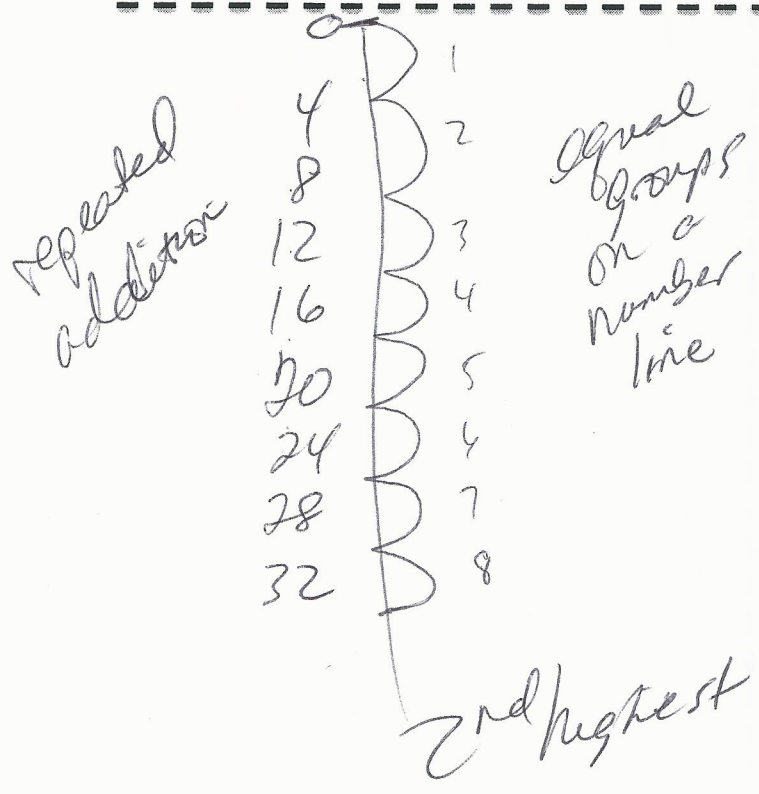
10 is too high to split 4 ways

$5 \times 4 = 20$, $2 \times 4 = 8$

2nd lowest

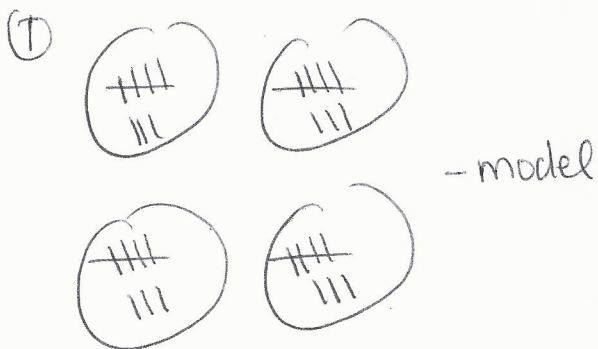Problem # ___4___   Grade Level: ___3___   Group Recorder Page

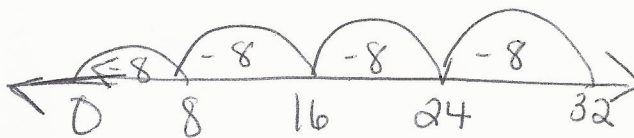Learning Goal: _I can use a strategy to solve a division problem._

Modifications to problem? _____

Possible Student Responses & Sequence

Lucas is in charge of planting trees at the park.
He gives 32 trees to a team of 4 helpers.
Each helper plants an equal number of trees.
How many trees will each helper plant?

① — model

② $32 \div 8 =$ repeated subtraction

4 groups of 8

③ $\underline{8} \times 4 = 32$ trees
groups

$5 \times 4 = 20$
$3 \times 4 = \underline{+ 12}$
$ 32$
8 partial products

so

$32 \div 4 = 8$

④ $32 \div 8 = 4$ related facts

I know that $8 \times 4 = 32$
so $32 \div 8 = 4$

Problem # ___5___   Grade Level: 3rd/4th   Group Recorder Page

Learning Goal: Single-digit by double-digit multiplication

Modifications to problem? _____

Possible Student Responses & Sequence

④ Standard Algorithim: $\begin{array}{r} 51 \\ \times\ 4 \\ \hline 204 \end{array}$

At the school store, Isabella buys 4 notebooks. Each notebook contains 51 sheets of paper. How many pieces of paper did Isabella buy?

② Repetitive Addition
   51+51+51+51

③ Base 10 Blocks / Models

③ Strip Diagram

③ Area Models

② Equal Groups

③ $50 \times 4 = 200$
   $\begin{array}{r} 1 \times 4 =+\ \ 4 \\ \hline 204 \end{array}$

① Draw Picture

Problem # ___6___   Grade Level: ___3-4___

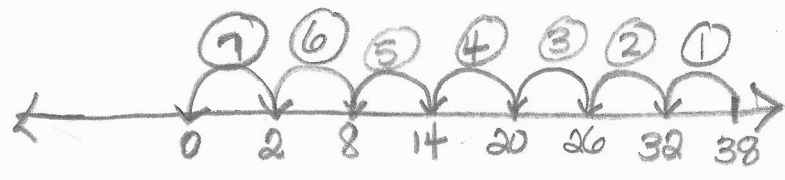Learning Goal: _SW solve division using multiple representations_

Modifications to problem? ___N/a___

Possible Student Responses & Se

Gabriel has room for 6 passengers in his van. He needs to take 38 people to the game. How many trips does Gabriel need to make in order to drive everyone to the game?

7 trips

7 → 6 → 5 → 4 → 3 → 2 → 1
0   2   8   14 20 26 32 38
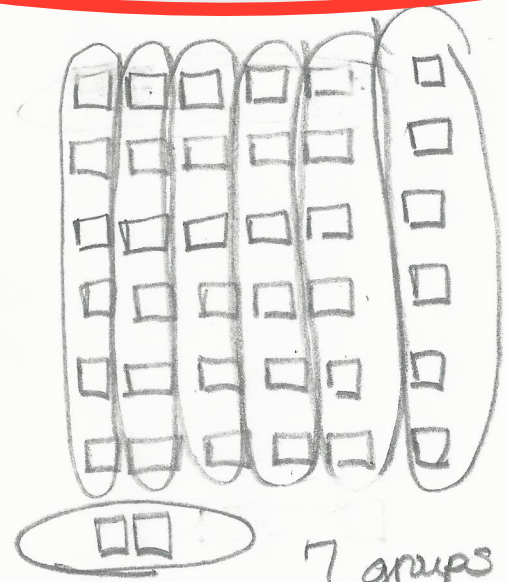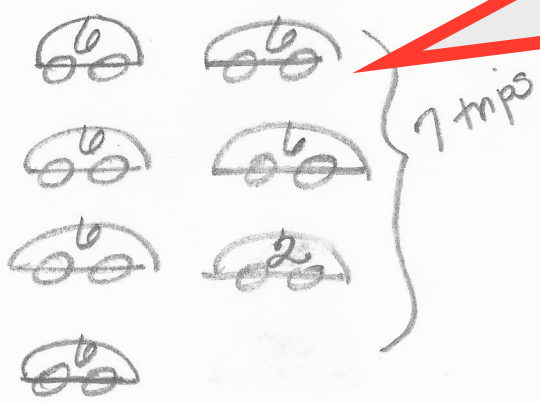
7 groups → 7 trips
6 R 2

6 R 2
6) 38
   36
    2   ⟶ 7 Trips to take last 2 people

‖‖‖ |
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Problem # <u>7</u>

Learning Goal: _____

Modifications to problem? _____

Zoey bought a football and a t-shirt at the store. She paid $15.35 for the football. She paid a total of $24.02 for the football and the t-shirt. How much did Zoey pay for the t-shirt?

Possible Student Responses & Sequence

(7) $24.02 $-15.35

$$24.02 \quad -15.35$$

35   15

[number line: 8.65 8.67 9 .... 24]
24
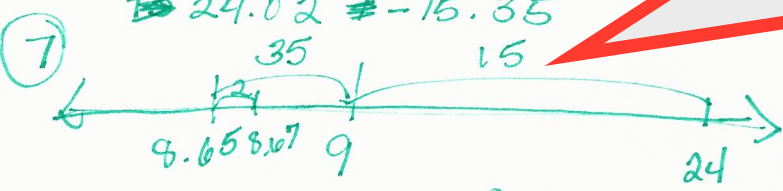
Subtracting in Parts
$24.02 - 15 = 9.02$
$9.02 - .02 = 9$
$9.00 - .33 = 8.67$

$15. + 5 = 20$
$20 + 4$

$15.35 + 65 = 16.00$
$16.00 .02 = 16.02$
$16.02 + 8 = 24.02$
_____
8.67

Subtract Back
$24.02 - .02 = 24$
$24 - .$

Problem # ___8___   Grade Level: ___5-6___   Group Recorder Page

Learning Goal: _____

Modifications to problem? _____

Possible Student Responses & Sequence

A water tank contains 512 ounces of water. What is the total number of 8-ounce bottles that can be filled from the tank?

8

$8\overline{)512}$ Long Div.

$8 \times 10 = 80$
$8 \times 50 = 400$ Mult. Up.
$8 \times 4 = 32$ "Partial Product"
        $\overline{512}$
64

⑩ ⑩ ⑩ ◯◯ ◯ etc.
◯◯

Using Base 10

Problem # ___8/9___   Grade Level: __5-6__   **Group Recorder Page**

Learning Goal: _____

Modifications to problem? _____

Possible Student Responses & Sequence

#8 A water tank contains 512 ounces of water. What is the total number of 8-ounce bottles that can be filled from the tank?

#8 Partial Quotient   ②

$$8 \overline{)512}$$

64

$$-80$$ 8x10 groups

32
-32 = 8x4 grps

4 8 2
-80   8x10 groups

35 2
-80   8x 10 groups

27 2
-80   8x 10 groups

19 2
-80   8x0 groups

11 2
-80   8x 10 groups

3 2

③ Most Soph-

#8 Area Model

$$8 \overline{)500 + 10, 8}$$

Repeated subtraction: subtract 8 as many times as possible. Then count how many 8's there were.

Repeated
Sub

① Least

$$512$$
$$- 8$$
_____

Strip diagram

#9

480

| 15 | ⟩ ?

Trad. Al.

$$15 \overline{)480}$$

#9 Jack has 480 baseball cards in his collection. He puts these cards into boxes. Each box has 15 cards. How many boxes does Jack need for his baseball card collection?