SMU

Engaging Models and Activities to Support Fraction Instruction

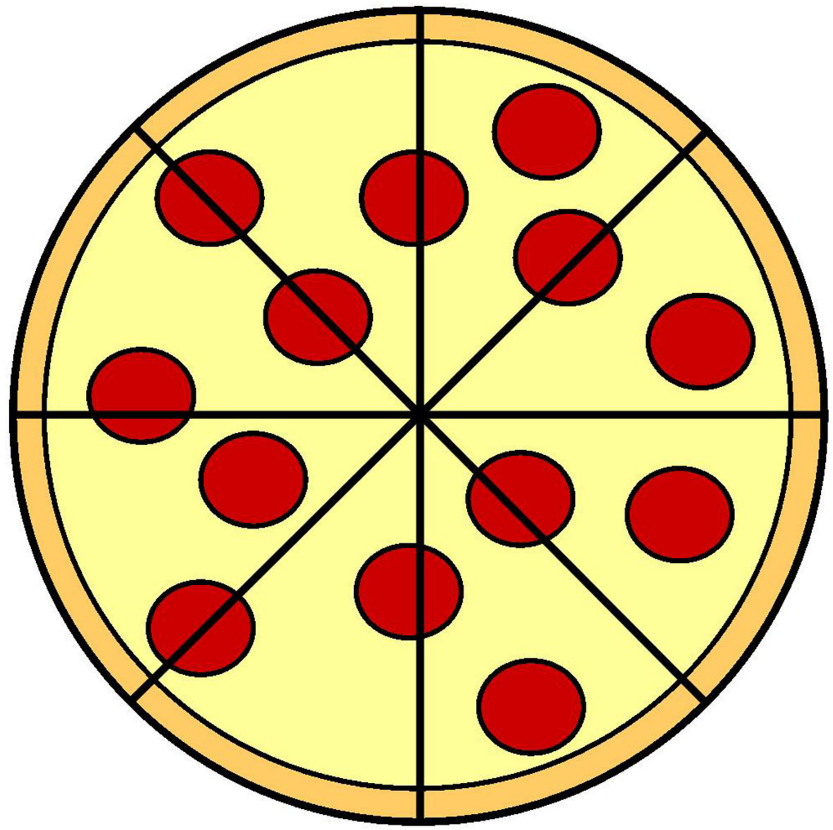
Presented by: Sharri Zachary & Dawn Woods

Abstract

- Discover how Research in Mathematics Education at Southern Methodist University applies research-based strategies to support the revised mathematics TEKS to promote student engagement in fraction instruction.
- Learn ways to support students' conceptual development of fractions with engaging activities and visual representations.

Developing Fraction Proficiency

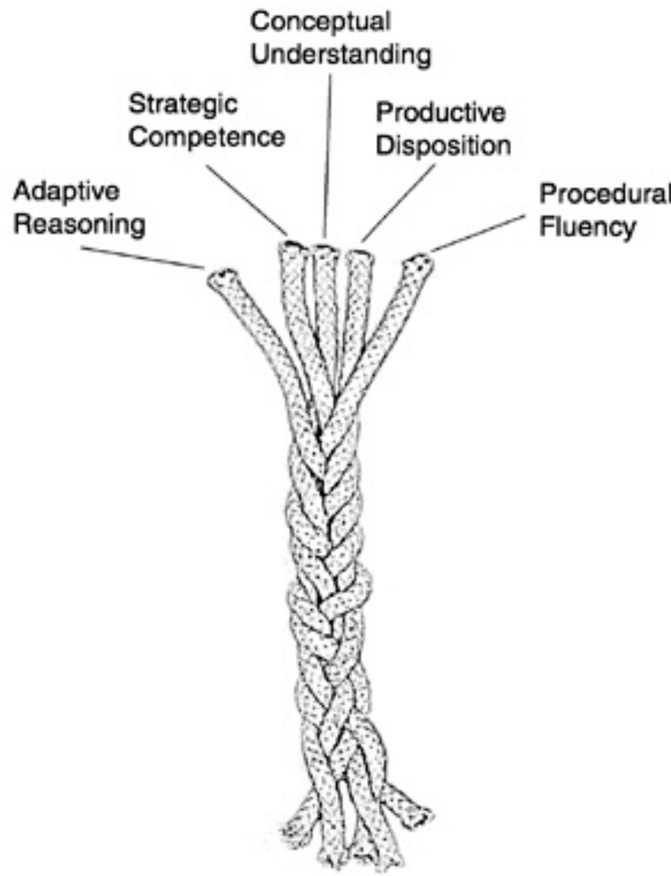
- Fraction proficiency is often developed with the use of pizzas and pies.
- However, it is important for students to recognize that **fractions are numbers** that expand the number system beyond whole number.
- Our challenge to you is to take away one idea that will expand your students' knowledge about the number system.



What Does Each Student Need to Know?

Strands of Mathematical Proficiency

- Conceptual Understanding – Comprehension of mathematical concepts
- Procedural Fluency – Skill in carrying out procedures
- Strategic Competence – Ability to represent mathematical problems
- Adaptive Reasoning – Capacity for logical thought, explanation, & justification
- Productive Disposition – Inclination to see mathematics as sensible and useful

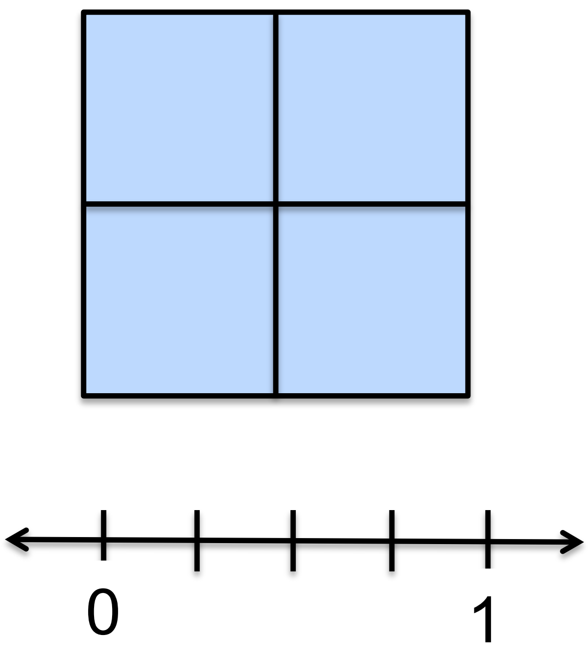


National Research Council (2001)

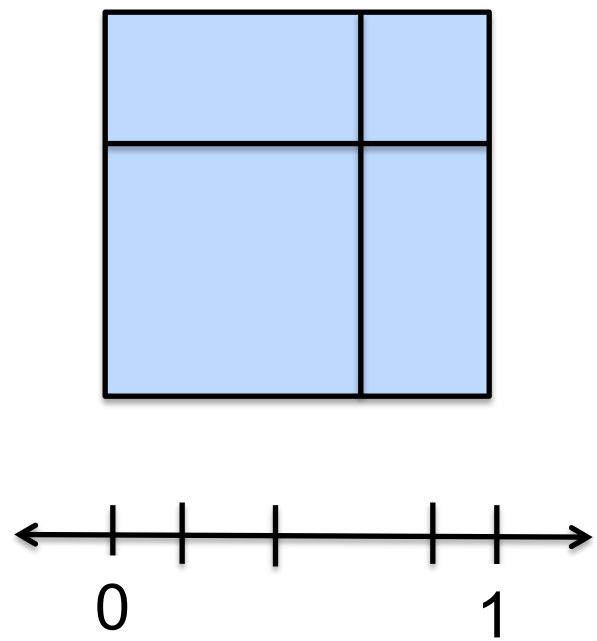
Activity: Models for Conceptual Understanding

Generate **examples** and **non-examples** of models that conceptually teach fractions.

Examples



Non-Examples



Conceptual Development of Fractions

Level 0	Understand Equal Partitioning
Level 1	Recognize Familiar Fraction Diagrams
Level 2	Understand Fractions as Counting All Parts and Shaded Parts
Level 3	Partition Shapes into Equal Parts and Select Parts
Level 4	Partition Quantities into Equal Parts and Select Some Parts
Level 5	Use Visual Representations of Fractions to Solve Simple Fraction Arithmetic Problems
Level 6	Understands Symbolic Computation of Fractions
Level 7	Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works

(Battista, 2012, p. 10)

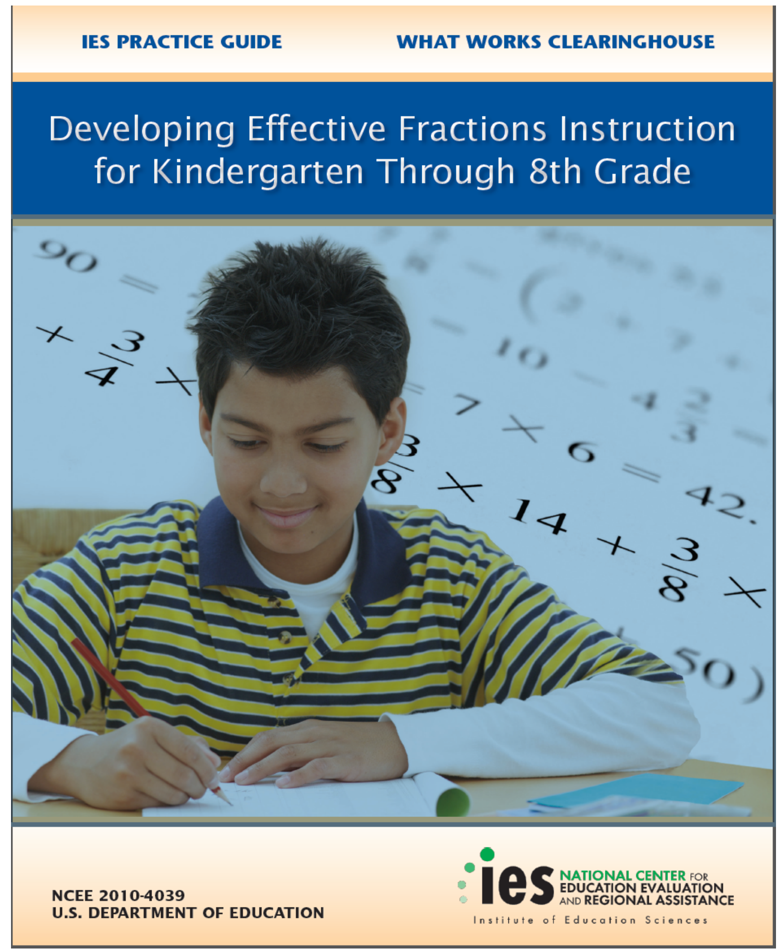
Conceptual Development of Fractions and the Revised TEKS

Level 0	Understand Equal Partitioning	1(6)(G); 1(6)(H); 2(3)(A)
Level 1	Recognize Familiar Fraction Diagrams	2(3)(B)
Level 2	Understand Fractions as Counting All Parts and Shaded Parts	2(3)(C)
Level 3	Partition Shapes into Equal Parts and Select Parts	2(3)(D): 2(6)B
Level 4	Partition Quantities into Equal Parts and Select Some Parts	2(6)B; 3(3)(A); 3(3)(B); 3(3)(C); 3(3)(E); 3(3)(F); 3(7)(A); 4(3)(C); 4(3)(D)
Level 5	Use Visual Representations of Fractions to Solve Simple Fraction Arithmetic Problems	3(3)(G); 3(3)(H); 4(3)(A); 4(3)(G)
Level 6	Understands Symbolic Computation of Fractions	3(3)(D); 4(3)(B); 4(3)(E); 5(3)(K); 5(3)(L)
Level 7	Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works	3(3)(E);4(3)(F); 5(3)(C); 5(3)(D); 5(3)(H); 5(3)(I); 5(3)(J)

(Battista, 2012, p. 10: Texas Essential Knowledge and Skills for Mathematics)

Developing Effective Fraction Instruction

1	Build informal understanding of sharing and proportionality.
2	Use number lines to develop the understanding that fractions are numbers that expand the number system beyond whole numbers.
3	Develop the “why” for computational procedures.
4	Develop conceptual understanding of strategies for solving ratio, rate, & proportional problems.
5	Professional development programs should improve teacher’s understanding of fractions and how to teach them.



(Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, Thompson, & Wray, 2010).

Build on Informal Understanding of Sharing

- Learning is most effective when it builds on existing knowledge.
- Most children have developed a basic understanding of sharing that allows them to divide a region or set of objects among two or more people.
- Early understanding of proportions can help young children compare how ½ of the area of a square is different than ½ of the area of a circle.

(Siegler, et al., 2010)

Developing Initial Fraction Concepts

- Use equal-sharing activities to introduce fractions.
  - Divide sets of objects and single whole objects.
- Extend equal-sharing activities to develop ordering and equivalence.
- Build on informal understanding to develop advanced concepts.
  - Begin with similar proportions and move toward ordering different proportions.

(Siegler, et al., 2010)

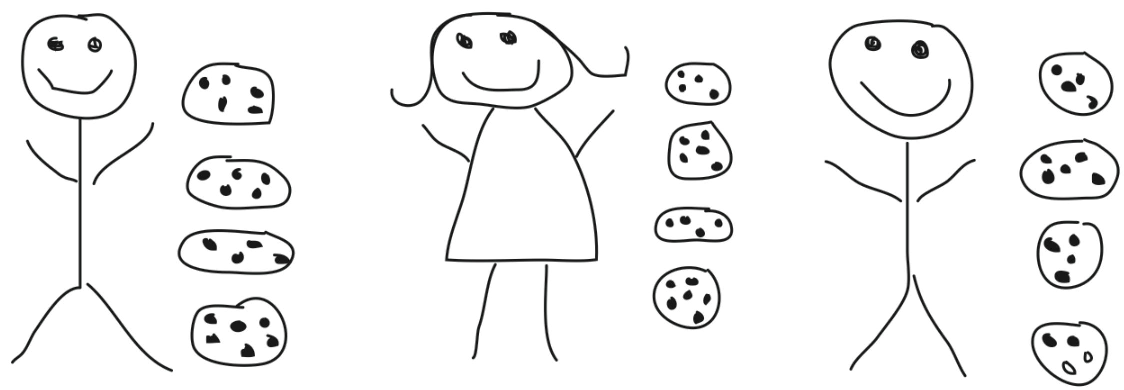
Sharing a Set of Objects Looks Like

Problem

Three children want to share 12 cookies so that each child receives the same number of cookies. How many cookies should each child get?

Examples of Solution Strategies

Students can solve this problem by drawing three figures to represent the children and then drawing cookies by each figure, giving one cookie to the first child, one to the second, and one to the third, continuing until they have distributed 12 cookies to the three children, and then counting the number of cookies distributed to each child. Other students may solve the problem by simply dealing the cookies into three piles, as if they were dealing cards.



(Siegler et al., 2010)

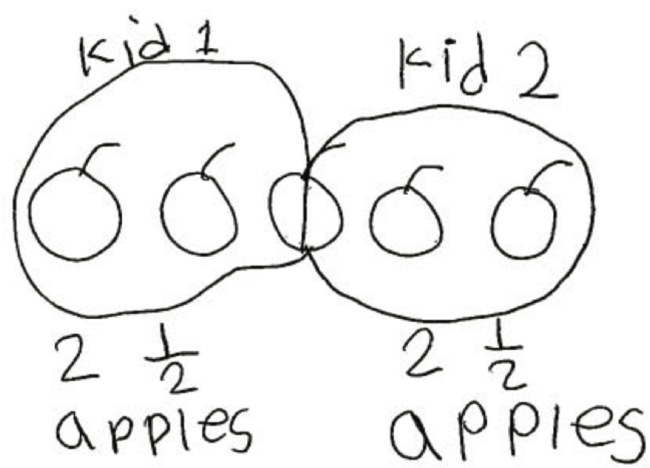
Partitioning Multiple Objects Looks Like

Problem

Two children want to share five apples that are the same size so that both have the same amount to eat. Draw a picture to show what each child should receive.

Examples of Solution Strategies

Students might solve this problem by drawing five circles to represent the five apples and two figures to represent the two children. Students then might draw lines connecting each child to two apples. Finally, they might draw a line partitioning the final apple into two approximately equal parts and draw a line from each part to the two children. Alternatively, as in the picture to the right, children might draw a large circle representing each child, two apples within each circle, and a fifth apple straddling the circles representing the two children. In yet another possibility, children might divide each apple into two parts and then connect five half apples to the representation of each figure.



(Siegler et al., 2010)

Fractions are Numbers

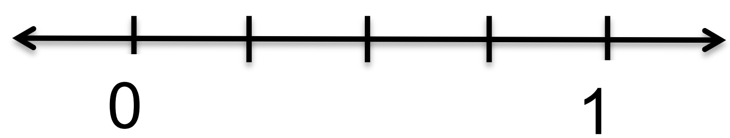
- Fractions are numbers with magnitudes (values) that can be ordered or considered equivalent.
- Many misconceptions stem from not understanding fractions are numbers with magnitude.
- Fractions provide a unit of measure that allows for more precise measurement than whole numbers.
- Reliance on part-whole instruction leaves unclear how fractions are related whole numbers.

(Siegler, et al., 2010)

Number Line as the Central Representational Tool

The use of number lines can help students visualize and understand:

- Part-whole relationship
- Relationship between fractions and whole numbers
- Relationship between fractions, decimals, and percent
- Magnitude of Fractions
- Fractions as a measure



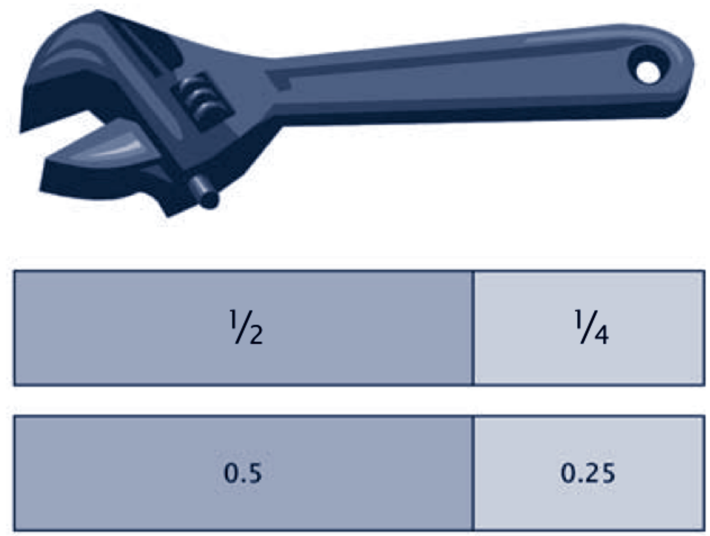
(Siegler et al., 2010)

Fraction Strips to Measure

Teachers can use fraction strips as the basis for measurement activities to reinforce the concept that fractions are numbers that represent quantities.[64]

To start, students can take a strip of card stock or construction paper that represents the initial unit of measure (i.e., a whole) and use that strip to measure objects in the classroom (desk, chalkboard, book, etc.). When the length of an object is not equal to a whole number of strips, teachers can provide students with strips that represent fractional amounts of the original strip. For example, a student might use three whole strips and a half strip to measure a desk.

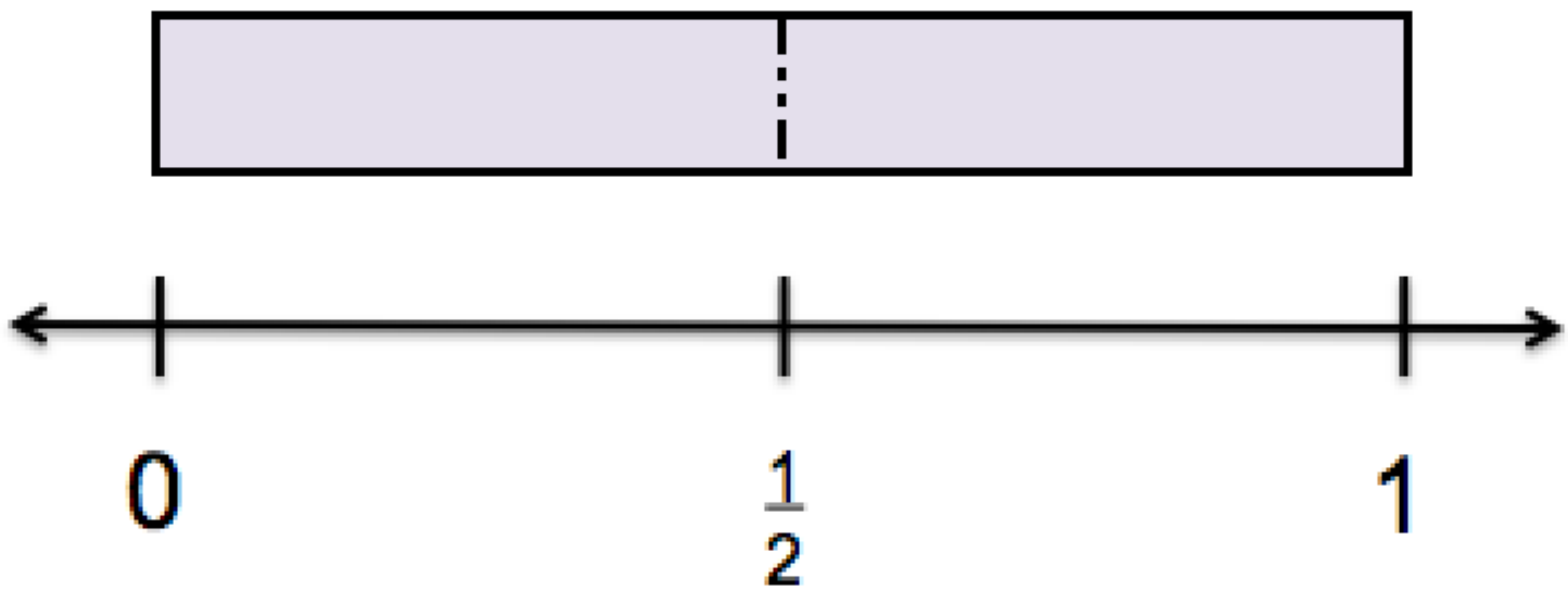
Using fraction strips to measure an object



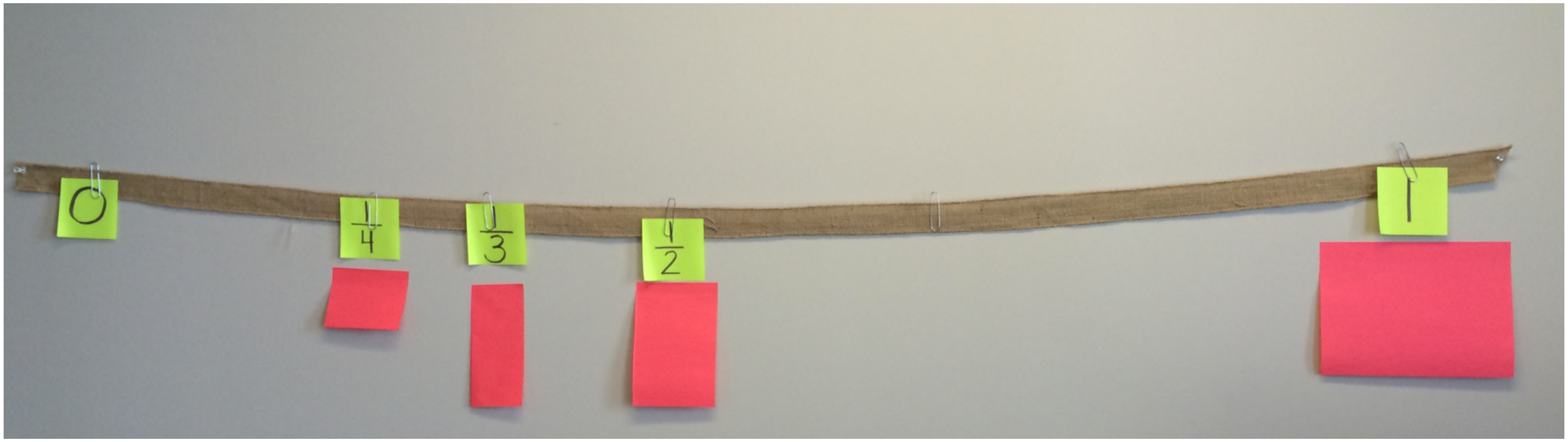
Teachers should emphasize that fraction strips represent different units of measure and should have students measure the same object first using only whole strips and then using a fractional strip. Teachers should discuss how the length of the object remains the same but how different units of measure allow for better precision in describing it. Students should realize that the size of the subsequently presented fraction strips is defined by the size of the original strip (i.e., a half strip is equal to one-half the length of the original strip).

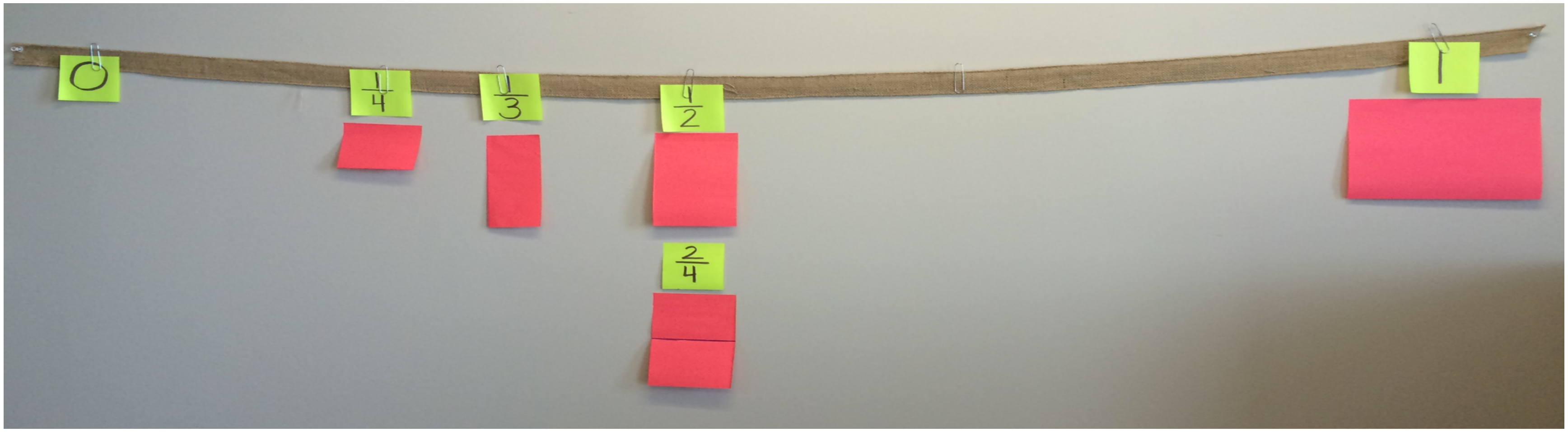
(Siegler et al., 2010)

Activity: Connecting Fraction Strips to the Number Line



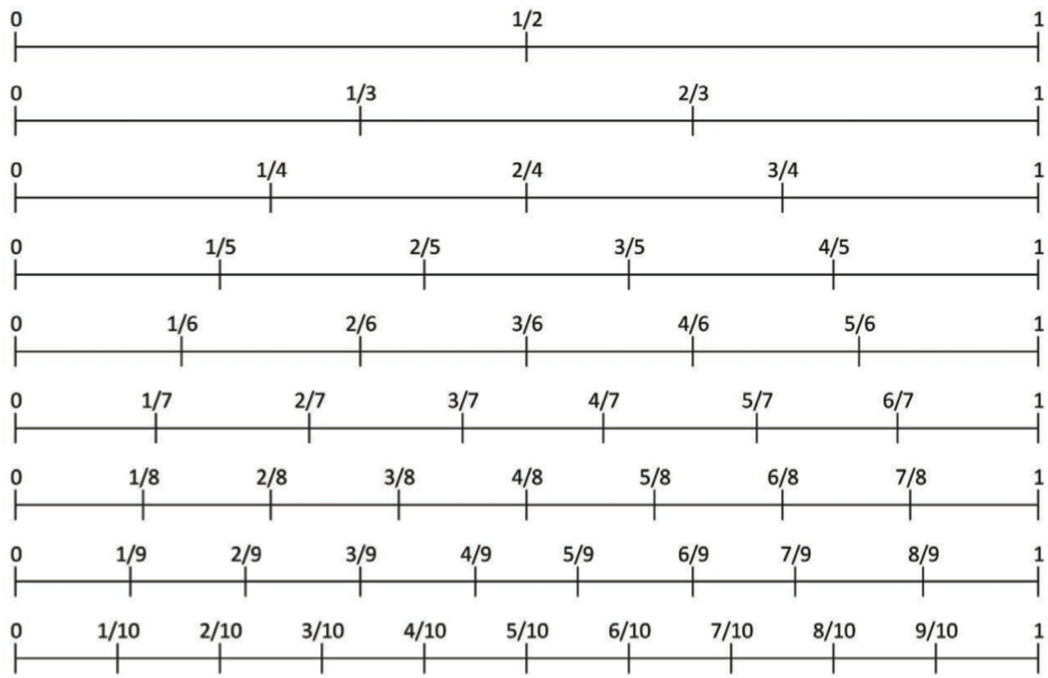
Fractions on a Number Line





Finding Equivalence on Number Lines

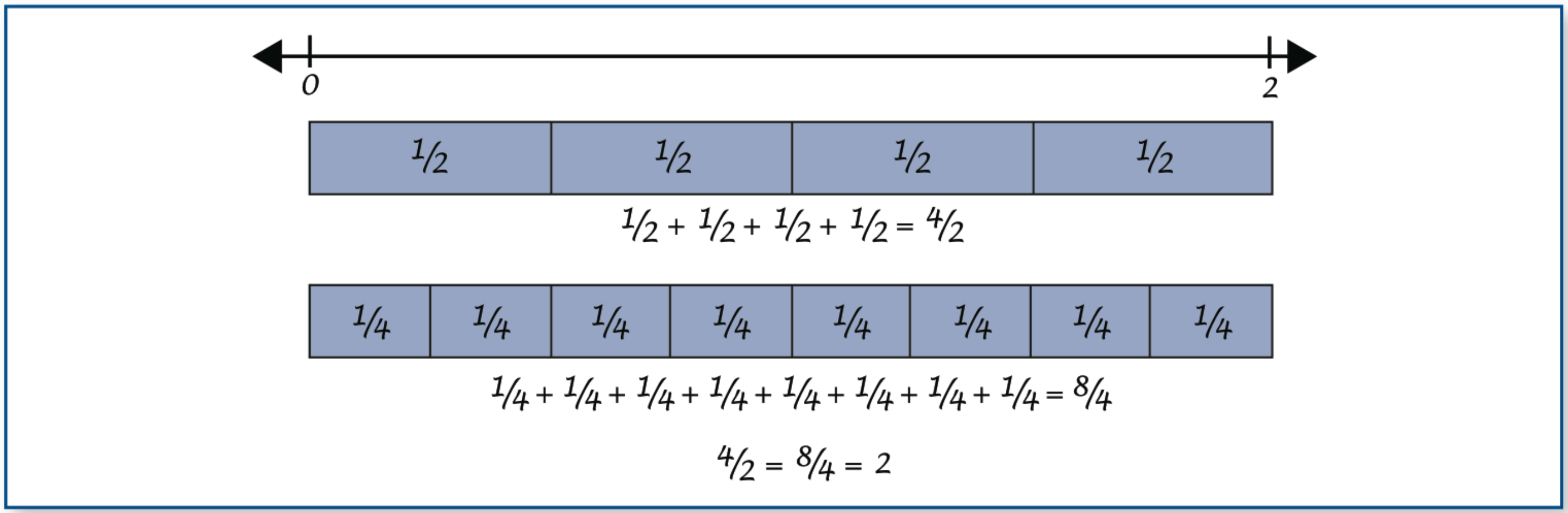
Use of number lines to teach equivalence of fractions in a Japanese curriculum



Source: Adapted from Shoseki (2010).

(Siegler et al., 2010)

Fraction Strips and Number Lines to Show Equivalence



(Siegler et al., 2010)

Develop the How and WHY for procedures

- Computational procedures by rote can quickly become forgotten.
- When students understand *HOW* and *WHY* procedures work students build conceptual understanding.

(Siegler, et al., 2010)

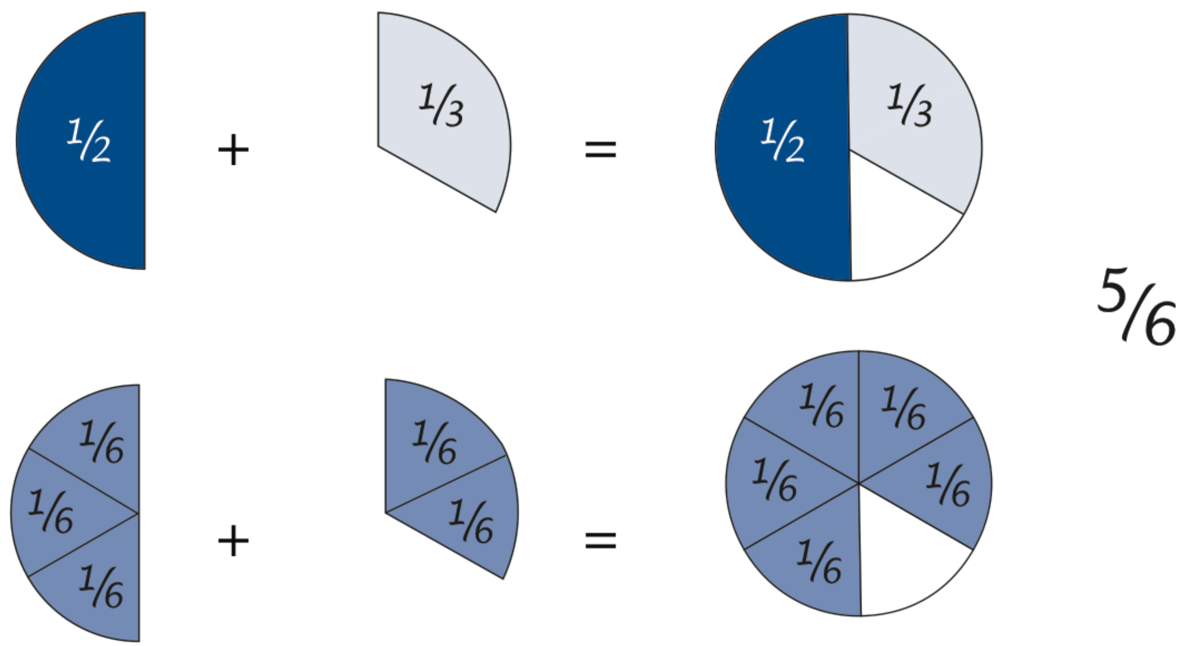
Developing the How and Why for Procedures

- Use visual representations such are area models and number lines to build conceptual understanding.
- Provide opportunities to predict or judge reasonableness of answers.
- Address common misconceptions.
- Present real-world contexts with plausible numbers.

(Siegler, et al., 2010)

Add and Subtract with Fraction Circles

Adding ½ + ⅓ using fraction circles

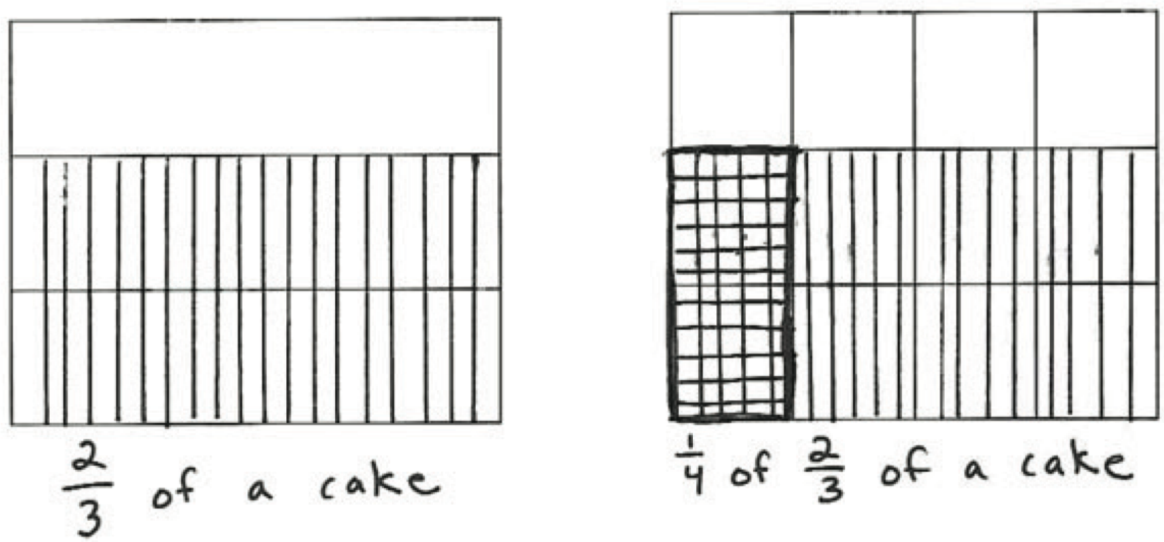


Source: Adapted from Cramer and Wyberg (2009).

(Siegler, et al., 2010)

Redefining Unit When Multiplying Fractions

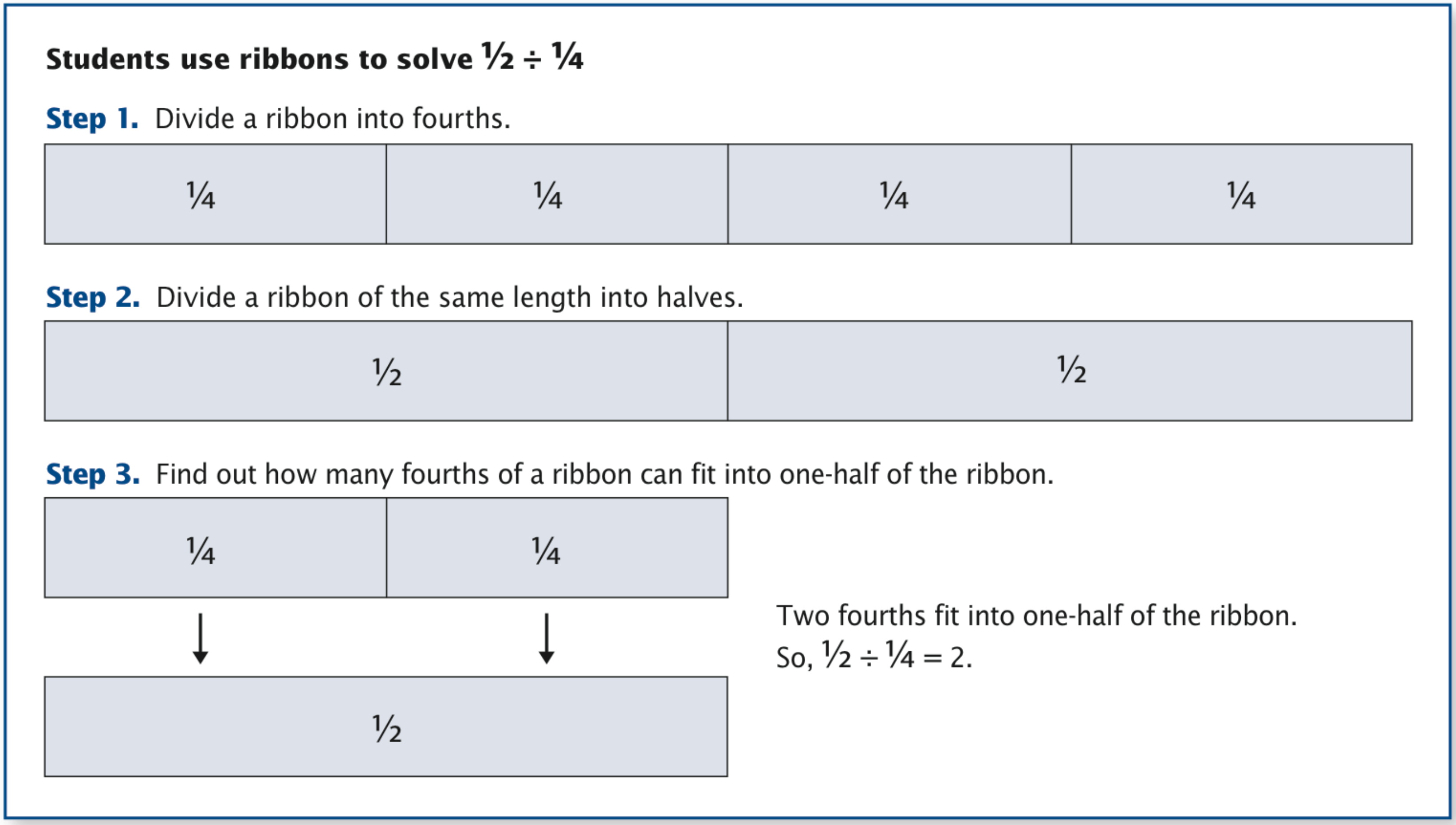
Lori is icing a cake. She knows that 1 cup of icing will cover ⅔ of a cake. How much cake can she cover with ¼ cup of icing?



$$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} \text{ of a cake}$$

(Siegler, et al., 2010)

Ribbon Models to Divide Fractions



(Siegler, et al., 2010)

How Will We Know When Each Student Has Learned It?

Modeling Math: Assessing What Students Know

Concept: ____

I can use numbers and symbols to show it.	I can draw a picture of it.
I can create a story problem about it.	I can explain it using math models.

Adapted from: Suh, J.M. (2007). Tying it all together. *Teaching Children Mathematics 14*(3), 164-169

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Activity: Gallery Walk

Research In Mathematics Education (RME) CAMT Presentations

- **MSTAR: Understanding the Value of an Assessment Plan** - Monday, July 21, 1:00 - 2:00, Omni FW 5
- **PreCal 911: Engaging Activities to Save the Day!** Tuesday, July 22, 10:00 - 11:00, CC 201C
- **Teacher T.O.M. - A Strategy for Reflective Practice** - Tuesday, July 22, 11:30 - 12:30 and 1:00 - 2:00 Omni Stockyards 3
- **ESTAR: Understanding the Value of an Assessment Plan** - Tuesday, July 22, 1:00 - 2:00, Omni FW 5
- **Implementing the NEW TEKS with Best Practices** - Tuesday July 22, 1:00 - 2:00, CC 114
- **Money Management: Developing Appreciation Through Mathematics** - Tuesday, July 22, 1:00 - 2:00, CC 204AB
- **Spaghetti & Meatballs and Algebraic Reasoning** - Wednesday, July 23, 10:00 - 11:00, Omni Sundance 2
- **ESTAR: Understanding the Value of an Assessment Plan** - Wednesday, July 23, 2:30 - 3:30, Omni FW 5

RME Information

www.smu.edu/RME

Email:
RME@smu.edu

@RME_SMU

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Works Cited

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Suh, J.M. (2007). Tying it all together. *Teaching Children Mathematics, 14*(3),164-169

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