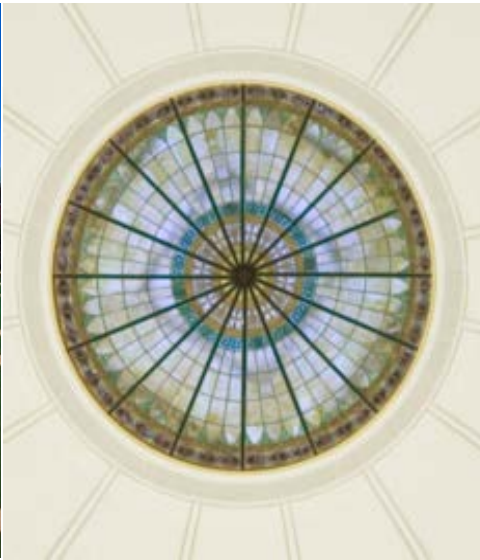


Developing Pedagogical Content Knowledge for Teaching Mathematics: Focus on Assessment

Research in Mathematics Education

Research to Practice Conference

February 15, 2013



Welcome and Introductions

Deni Basaraba, Ph.D.

Beth Richardson

Dawn Woods

Sharri Zachary

Response to Intervention Model

Tier III: **Intensive**
Instructional Support



B: 1-5th Percentile Rank

A: 6-14th Percentile Rank

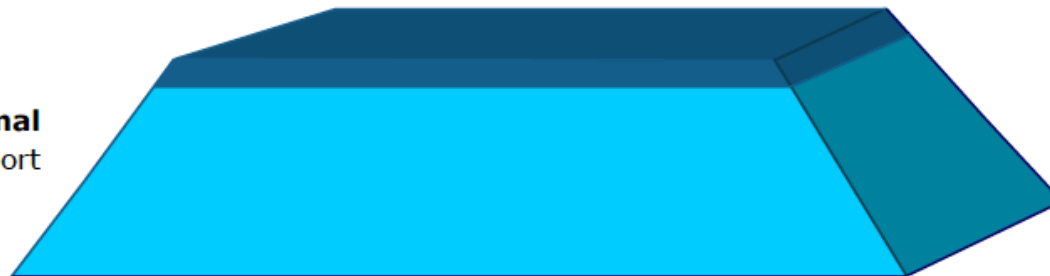
Tier II: **Strategic**
Instructional Support



B: 15-24th Percentile Rank

A: 25-39th Percentile Rank

Tier I: **Minimal**
Instructional Support



B: 40-49th Percentile Rank

A: 50-99th Percentile Rank



Performance Level	Instructional Need	Level Label	Range of Performance	Level of Additional Instructional Support
Tier III	Intensive Instructional Support	B	1-5 th Percentile Rank	Student needs urgent and intensive interventions that are highly specified to his/her individual needs. Additional instructional time is needed. Progress should be frequently and consistently monitored.
		A	6-14 th Percentile Rank	Student needs intensive interventions that are highly specified to his/her individual needs. Diagnostic assessments are needed to determine areas in need of improvement. Additional instructional time is needed. Progress should be frequently and consistently monitored.
Tier II	Strategic Instructional Support	B	15-24 th Percentile Rank	Student needs supplemental interventions that are targeted to his/her individual needs. Diagnostic assessments are needed to determine areas in need of improvement. Additional instructional time is needed. Progress should be consistently monitored.
		A	25-39 th Percentile Rank	Student needs targeted support including differentiated and scaffolded instruction, additional practice, corrective feedback. Additional instructional time may be warranted. Progress should be closely monitored to evaluate growth.
Tier I	Minimal to No Instructional Support	B	40-49 th Percentile Rank	Student needs minimal to no additional instructional support beyond the core instructional program. Student may benefit from differentiated instruction and strategic review to reinforce proficiency. Progress should be closely monitored to evaluate growth.
		A	50-99 th Percentile Rank	Student does not need additional instructional support beyond the core instructional program. Student may benefit from differentiated instruction and periodic review to reinforce proficiency.

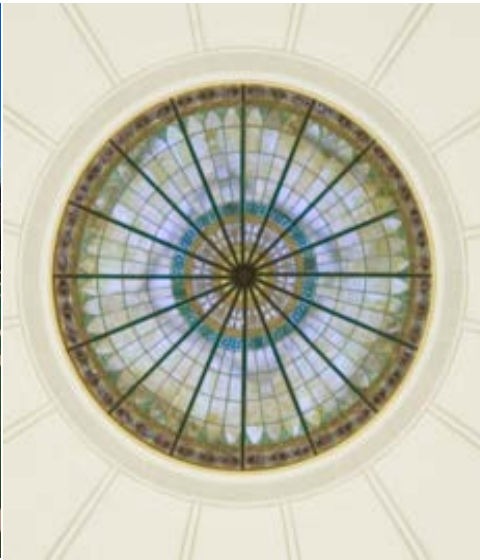
No child
left behind

IS THIS THE
TEST TO TEST
US FOR THE TEST
TO SEE IF WE
ARE READY FOR
THE TEST?



Focus on Assessment

Discussion Points	Outcome
<ul style="list-style-type: none">• Cognitive Complexity of a Test Item: Knowledge Representations• Levels of Mathematics Proficiency• Appropriate Question Stems• Answer Choices: Including Student Misconceptions	<ul style="list-style-type: none">• Understand the anatomy of a test item• Write test items at different proficiency levels
<ul style="list-style-type: none">• Importance of technically adequate assessments• Examining student performance: Moving beyond the overall score	<ul style="list-style-type: none">• Reliable data decision making



Introduction to Developing Pedagogical Content Knowledge for Teaching Mathematics

Cognitive Engagement

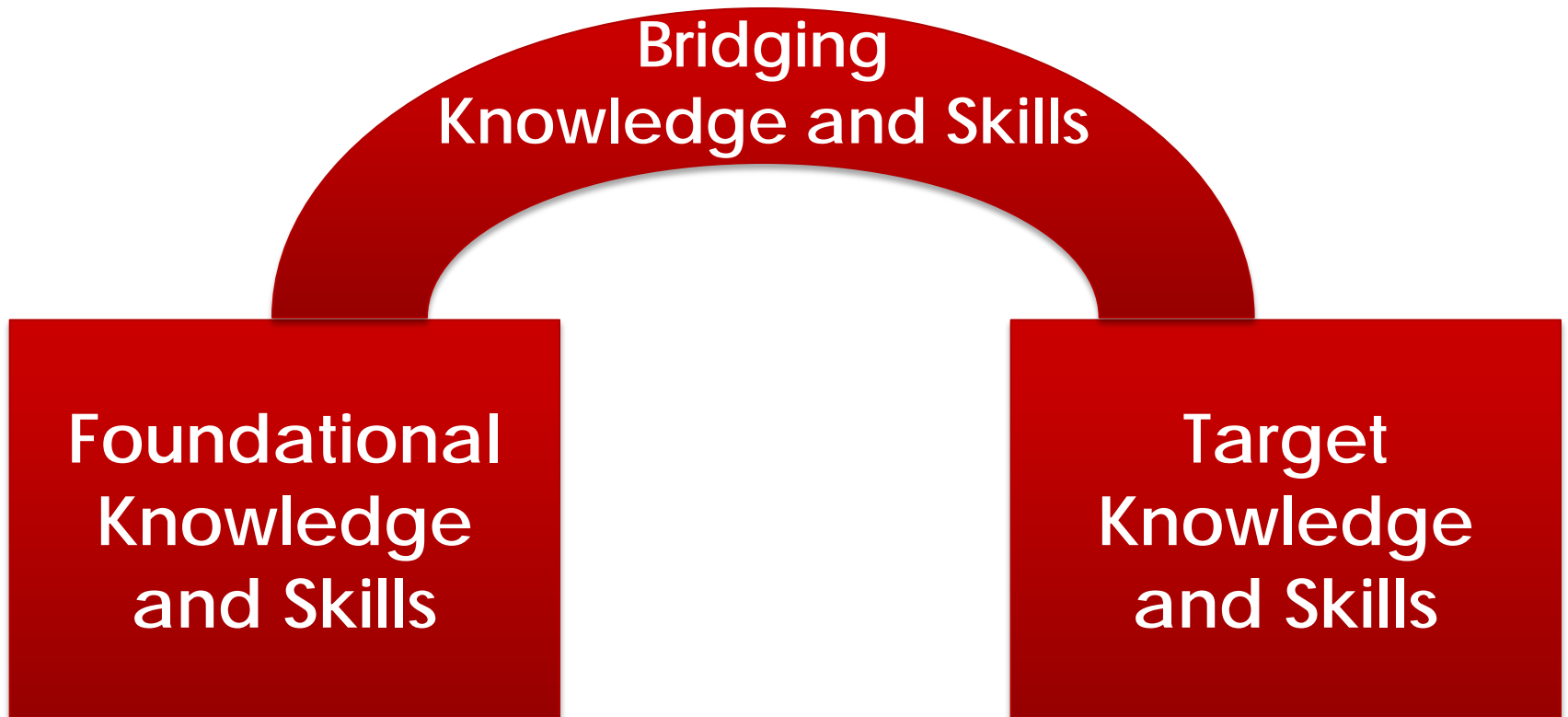
Item Writing Template

Item Writing Template		
Course/Grade Level:		
TEKS:		
Knowledge Representation (Circle One): Foundational Bridging Target		
Cognitive Engagement (Circle One): Procedural Conceptual Strategic Competence Adaptive Reasoning		
Relative Difficulty (Circle One): Easy Medium Difficult		
Question Stem	Response Choices	
	Content/Graphic	Student Misconception(s)
	Answer	
	Distractor 1	
Stem Graphic	Distractor 2	
	Distractor 3	

Cognitive Engagement of a Test Item

- **Knowledge Representations**
 - Target knowledge and skills
 - Bridging knowledge and skills
 - Foundational knowledge and skills
- **Levels of Mathematical Proficiency**
 - Conceptual understanding
 - Procedural fluency
 - Strategic competence
 - Adaptive reasoning

Knowledge Representations



Target Knowledge and Skills

- Grade level knowledge and skills
- Supports **future success in mathematics**
- Often are abstract representations of formal mathematical knowledge, but not always

Foundational Knowledge and Skills

- Knowledge and skills that support the target content and are accumulated from previous learning
- **Prerequisite knowledge and skills** needed to be successful at the target knowledge and skills

Bridging Knowledge and Skills

- Knowledge and skills needed to **connect or support students' learning** from the foundational knowledge to the target knowledge and skills
 - Often represents an integration of knowledge and skills (may be conceptual or model-based)
- The knowledge and skills that students learn from the teacher or instructional materials

3rd grade Target Skills From TX-RCFP

Curriculum Focal Points are:

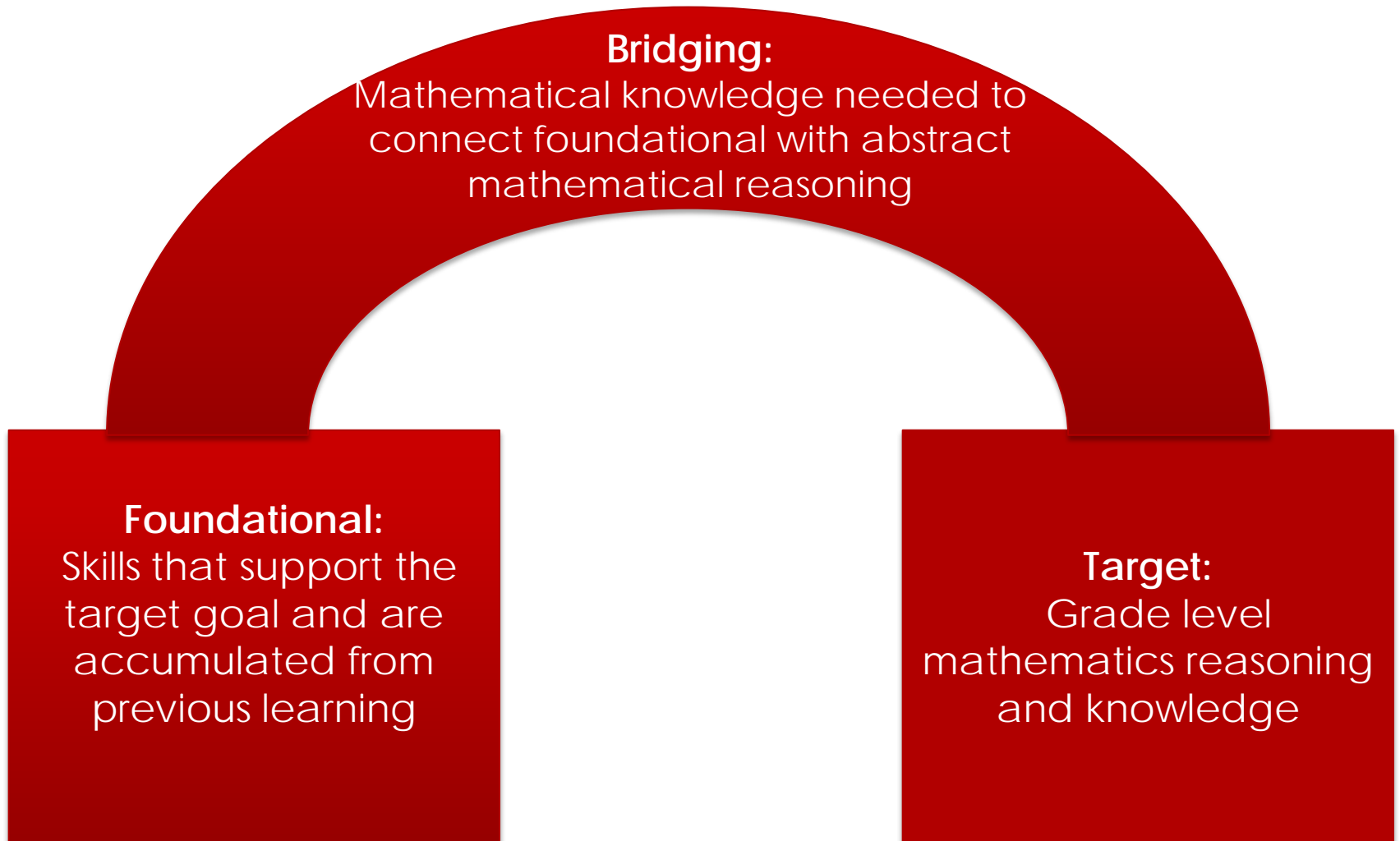
- Framework of critical areas of mathematics instruction
- A mathematical theme, not a single TEKS statement

Before writing assessments, it is important to:

- Analyze the focal point description to identify key target skills
- Analyze related TEKS statements
- Synthesize the overlaying skills

TEXAS RESPONSE TO CURRICULUM FOCAL POINTS FOR GRADE 3 MATHEMATICS	
Developing an understanding of multiplication and division	
Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). Students use properties of addition and multiplication to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems. Students relate multiplication and division as inverse operations. [1, p. 15]	
Related Grade 3 TEKS:	
3.4 (A)	The student is expected to learn and apply multiplication facts through 12 x 12 using concrete models and objects.
3.4 (B)	The student is expected to solve and record multiplication problems (up to two digits times one digit).
3.4 (C)	The student is expected to use models to solve division problems and use number sentences to record the solutions.
3.6 (B)	The student is expected to identify patterns in multiplication facts using concrete objects, pictorial models, or technology.
3.6 (C)	The student is expected to identify patterns in related multiplication and division sentences (fact families) such as $2 \times 3 = 6$, $3 \times 2 = 6$, $6 \div 2 = 3$, $6 \div 3 = 2$.
3.7 (A)	The student is expected to generate a table of paired numbers based on a real-life situation such as insects and legs.
3.13 (A)	The student is expected to collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data.
3.14 (all)	The student applies Grade 3 mathematics to solve problems connected to everyday experiences and activities in and outside of school.
3.15 (all)	The student communicates about Grade 3 mathematics using informal language.
3.16 (all)	The student uses logical reasoning.

Knowledge Representations





SMU Sample Target Item

3rd grade

- Jake goes to lunch with Ed and Bella. The total bill was \$36. If they decided to equally split the bill, how much would each person pay?
- A. \$6
 - B. \$9
 - C. \$12
 - D. \$18

Correct answer: **C**



SMU Sample **Foundational** Item
Grade 3

Gracie has 12 books and an empty bookshelf with 3 shelves.

If she puts the same number of books on each shelf, how many books will be on each shelf?

A. 3 books

A. 4 books

B. 6 books

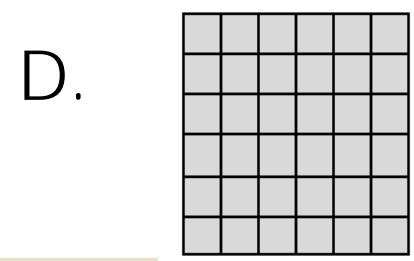
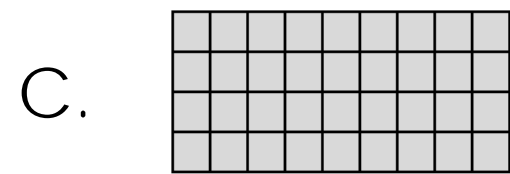
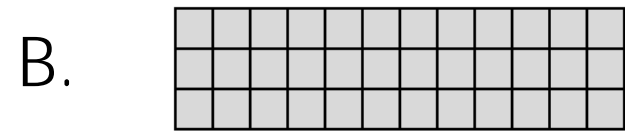
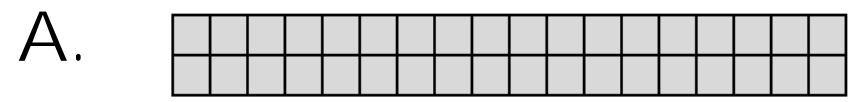
A. 12 books

Correct answer: **B**

 **SMU** Sample **Bridging** Item
Grade 3

Which model could be used to represent

$$36 \div 9 = \square ?$$



Correct answer: **C**



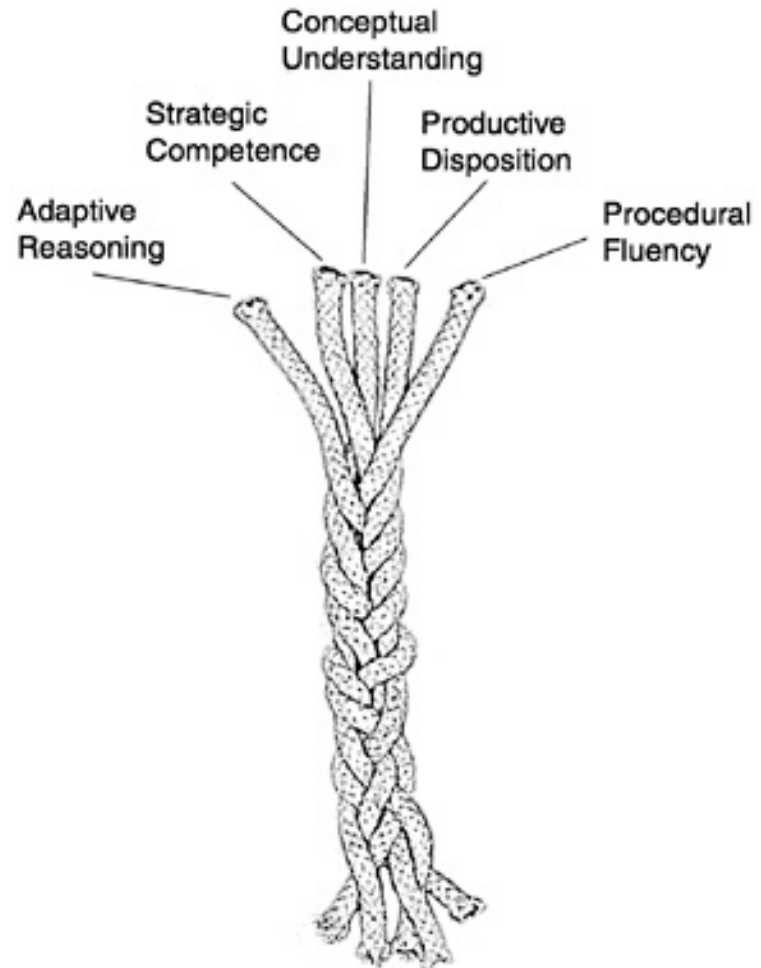
SMU™ Process for Articulating the Content of an Item

- Become familiar with the TEKS standard (content standard) for which you are writing a test item
 - Articulate the **TARGET SKILLS**
 - Articulate the **FOUNDATIONAL SKILLS**
 - Articulate the **BRIDGING SKILLS**
-



SMU Strands of Mathematical Proficiency

- Intertwined Strands of Proficiency



Activity

- Find someone with whom you have not talked this afternoon.
- With your partner, **in 2 minutes**, introduce yourselves and share one interesting fact about yourselves.
- **Based on your current level of understanding, determine the strand of mathematical proficiency associated with each item: CONCEPTUAL, PROCEDURAL, STRATEGIC, ADAPTIVE.**

Conceptual Understanding

- Demonstrate an integrated and functional grasp of mathematical ideas
- Understand specific task as it relates to a whole concept
- Find relationships between pieces of information
- Make connections to similar representations
- Use models and multiple representations (e.g. pictures, numbers, real-life situations, words)

Conceptual Understanding

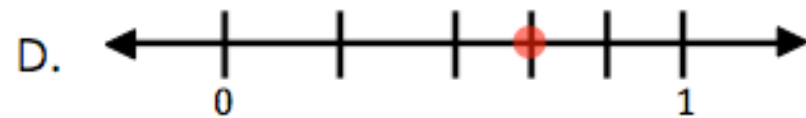
TEKS 3.3D

The student applies mathematical process standards to represent and explain fractional units.

The student is expected to:

(D) Compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts $1/b$.

Which number line shows $\frac{3}{5}$?



Correct answer: **B**

Procedural Fluency

- Use formal language or symbolic representations
- Carry out accurate computations
- Follow multiple steps sequentially
- Make proper use of algorithm and properties

Procedural Fluency

TEKS 7.11A

The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:

(A) **Model and solve one-variable, two-step equations and inequalities.**

Solve for w :

$$\frac{w + 12}{3} = 20$$

A. 72

B. 58

C. 48

D. 16

Correct answer: C

Strategic Competence

- Ability to formulate a problem in mathematical terms
- Represent problem solving strategically (verbally, symbolically, graphically, or numerically)
- Identify and use strategy necessary to solve problems effectively (e.g. use the distributive property to solve)

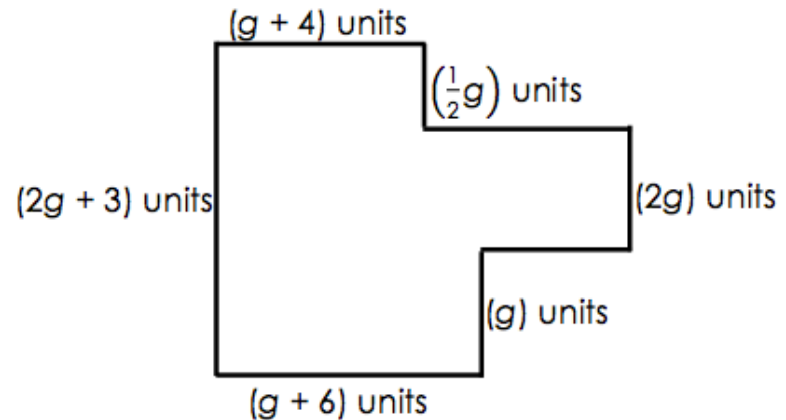
Strategic Competence

TEKS 7.11C

The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:

(C) Write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationship.

The perimeter of the figure below is 43 units.



Which equation can be used to solve for the variable, g ?

- A. $7.5g + 13 = 43$
- B. $7g + 13 = 43$
- C. $5.5g + 13 = 43$
- D. $5g + 13 = 43$

Correct answer: **A**

Adaptive Reasoning

- Think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach
- Rationalize and justify strategies
- Appropriately explain a procedure or concept

Adaptive Reasoning

TEKS 3.3H

The student applies mathematical process standards to represent and explain fractional units. The student is expected to:

(H) Compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models.

Why is $\frac{2}{3} > \frac{2}{7}$?

- A. 2 is equal to 2 and thirds are larger than sevenths.
- B. 2 is equal to 2 and thirds are smaller than sevenths.
- C. 2 is equal to 2 and 3 is smaller than 7.
- D. 2 is equal to 2 and 3 is greater than 7.

Correct answer: **A**

Level of Difficulty

◆ Easy

- Basic Knowledge
- Skills that are familiar to students
- Sometimes conceptually based

◆ Medium

◆ Difficult

- Skills that are peripheral to curriculum
- Not all students will have mastered these

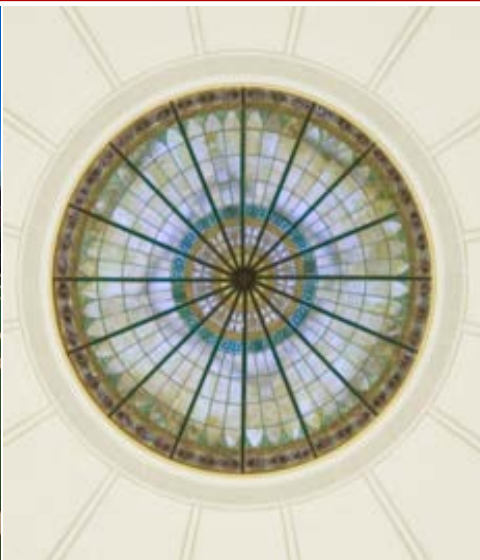
(Leong, 2006)

Gallery Walk

- In your packet of materials is a 4 x 3 matrix with the 4 strands of mathematical proficiency along the top and 3 levels of difficulty along the left side.
- Around the room are 12 problems written to align with the 6th grade TEKS.
- As you read each item, determine the strand of mathematical proficiency and relative level of difficulty.
- Write the number associated with the item in the appropriate cell.

Gallery Walk – Answer Key

	Procedural	Conceptual	Strategic	Adaptive
Easy	5	3	1	12
Medium	10	6	4	7
Difficult	8	11	9	2



Introduction to Developing Pedagogical Content Knowledge for Teaching Mathematics

Assessment Item Development

Guidelines for Item Development

- Item writing requires careful consideration not only to general item-writing procedures and the overall content of the items but also, in the case of **multiple-choice item writing**, careful consideration of the **stem and response options** as well.

General Item-Writing (Procedures)

- Avoid the complex multiple-choice format. (i.e., A and D, B and C).
- Use plain language. Avoid conditional phrases (if..., then...).
- Keep the language of the stem and response options at the appropriate grade level.
- Minimize examinee reading time in phrasing each item.

Stem Development

- State the stem in question form. Minimize use of completion form. When using the completion format, do not leave a blank for completion in the beginning or middle of the stem.
- Include only the material needed to make the problem clear. Avoid extraneous information.
- Word the stem positively; avoid negative phrasing. If an item must be stated negatively, underline or capitalize the negative word.
- Keep all essential information in the stem. Items that require students to read and evaluate each response option prior to selecting an answer increase the cognitive load required.

General Item-Writing (Content)

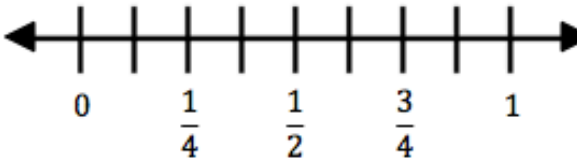
- Base each item on important content to learn; avoid trivial content.
- Keep the content of each item independent from content of other items on the test.
- Avoid cuing one item with another; keep items independent of one another.
- Avoid items based on opinions.
- Develop items that measure higher-level thinking.
- Avoid potentially insensitive content or language.

Response Development

- Make all distractors plausible.
 - If you're interested in obtaining more information about students' understanding, create distractors that represent common misconceptions they may have about the content being assessed
- Keep all options in an item homogenous in content and grammatical structure.
- Keep the length of options brief and fairly consistent.
- Phrase options positively, not negatively.



SMU Example of a Well-Written Item

Item Writing Template			
Course/Grade Level: 3 rd Grade			
TEKS: The student is expected to determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line.			
Knowledge Representation (Circle One): Foundational Bridging Target			
Cognitive Engagement (Circle One): Procedural Conceptual Strategic Competence Adaptive Reasoning			
Relative Difficulty (Circle One): Easy Medium Difficult			
Question Stem	Response Choices		
<p>Which fraction is greater than zero but less than $\frac{1}{2}$?</p>	Content/Graphic	Student Misconception(s)	
	Answer	$\frac{1}{4}$	
	Distractor 1	$\frac{1}{2}$	May not understand that $\frac{1}{2}$ is equal to $\frac{1}{2}$.
Stem Graphic	Distractor 2	$\frac{3}{4}$	Found fraction greater than zero but greater $\frac{1}{2}$.
	Distractor 3	$\frac{1}{1}$	May not understand that $\frac{1}{1}$ names the whole as a fraction.



SMU™ Uh Oh ☹️

Examples of Poorly Written Math Items

Dan enjoys going to the park. The park has lots of trees and a pond. Dan goes to the park at 10:15 a.m., plays on the swings, slides down the slide, and runs around the pond. He is hungry when he finally makes it home 1 hour 55 minutes later. What time did he arrive at home?

- A. 12:00 p.m.
- B. 12:10 p.m.
- C. 8:20 a.m.
- D. 6:00 p.m.



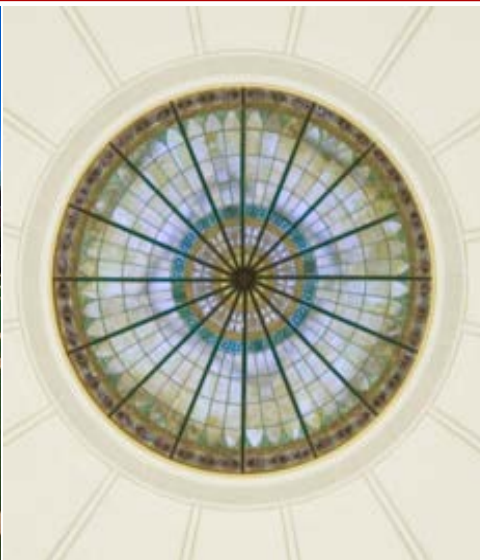
SMU™ Uh Oh ☹️

Examples of Poorly Written Math Items

Sue has a box of 2 stars and 2 circles. She wants to make Michelle's box of 6 stars and 3 circles proportional to her box. How many stars does she need to add to her box to make Michelle's box proportional to hers?

- A. 6
- B. 0
- C. 2
- D. 20





Introduction to Developing Pedagogical Content Knowledge for Teaching Mathematics

Data-Driven Decision Making



SMU Trustworthy & Reliable Decision Making

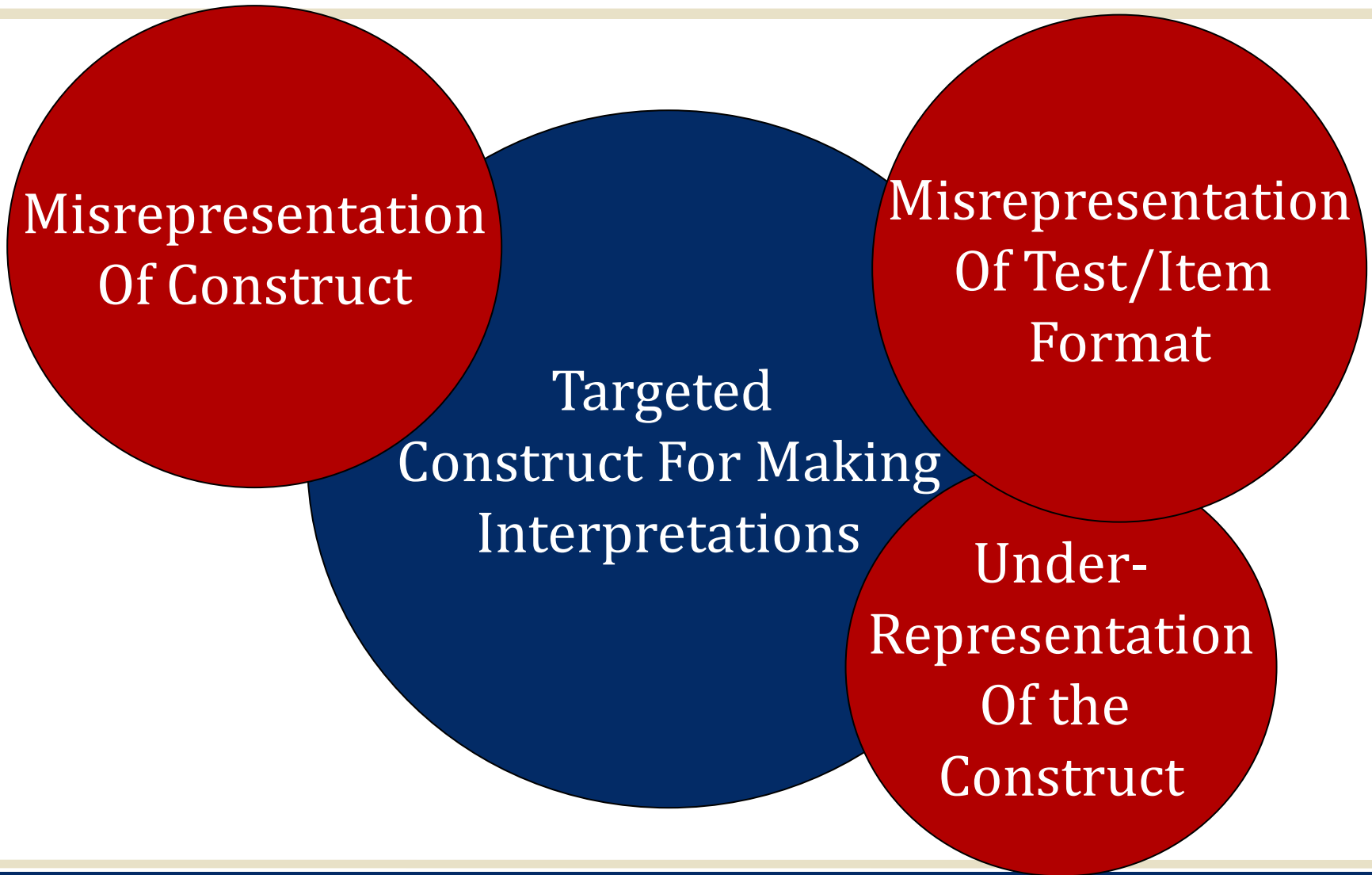
- Validity –Trustworthiness and meaningfulness of the uses and interpretations of the test results
- Reliability –Consistency of the results across items, setting, time, and raters
- Fairness –Free from sources of bias, equitable treatment of test takers

Decision Making Checklist

Teachers should agree to analyze the assessments around the same set of criteria. The decisions should be directed toward:

- ✓ Validation about the appropriateness of the assessment
- ✓ Ensuring the assessment is congruent to the stated mastery objective and/or state or district standards
- ✓ Consistency of opinion about the assessment and evaluation of the work
- ✓ Adjustments in teacher directions and support for all students

Barriers to Decision Making





SMU Using Data to Inform Instruction: Overall Student Performance

- One of the most common ways to examine student performance data is by making normative comparisons of their **overall** performance:

Grade	Teacher	Student	Number Correct	Percent Correct
5	Richardson	Swan, B.	20/40	50%
5	Richardson	Black, J.	22/40	55%
5	Richardson	Cullen, E.	21/40	52.5%
5	Richardson	Newton, M.	20/40	50%
5	Hatfield	Everdeen, K.	37/40	92.5%
5	Hatfield	Hawthorne, G.	36/40	90%
5	Hatfield	Meelark, P.	37/40	92.5%
5	Hatfield	Abernathy, H.	35/40	87.5%



SMU Using Data to Inform Instruction: Digging a Little Bit Deeper



- Although students' overall performance may be similar, this does not necessarily mean that they have similar levels of mastery on the assessed content:

Student	Number of Items Correct				Total Correct
	Number Line Structure	Magnitude as Distance	Part-to-Whole Relationships	Unit Fractions	
Swan, B.	7/10	6/10	5/10	2/10	20/40
Black, J.	5/10	7/10	4/10	6/10	22/40
Cullen, E.	8/10	5/10	3/10	5/10	21/40
Hale, J.	5/10	7/10	3/10	5/10	20/40



SMU Using Data to Inform Instruction: Digging Even Deeper



- Similarly, even though students may have responded correctly to the same number of items within a given sub-level, again this does not necessarily mean they have mastered the same skills or have the same level of proficiency with the targeted skill

Student	Part-to-Whole Relationships (Items 1-10)										Number Correct
	1	2	3	4	5	6	7	8	9	10	
Swan, B.	1	0	1	1	0	1	0	0	0	1	5/10
Black, J.	1	1	0	0	1	1	0	0	0	0	4/10
Cullen, E.	1	1	0	0	0	1	0	0	0	0	3/10
Hale, J.	1	0	1	0	0	1	0	0	0	0	3/10

0 = Incorrect

1 = Correct

SMU Using Data to Inform Instruction: How Far Down the Rabbit Hole Should I Go?



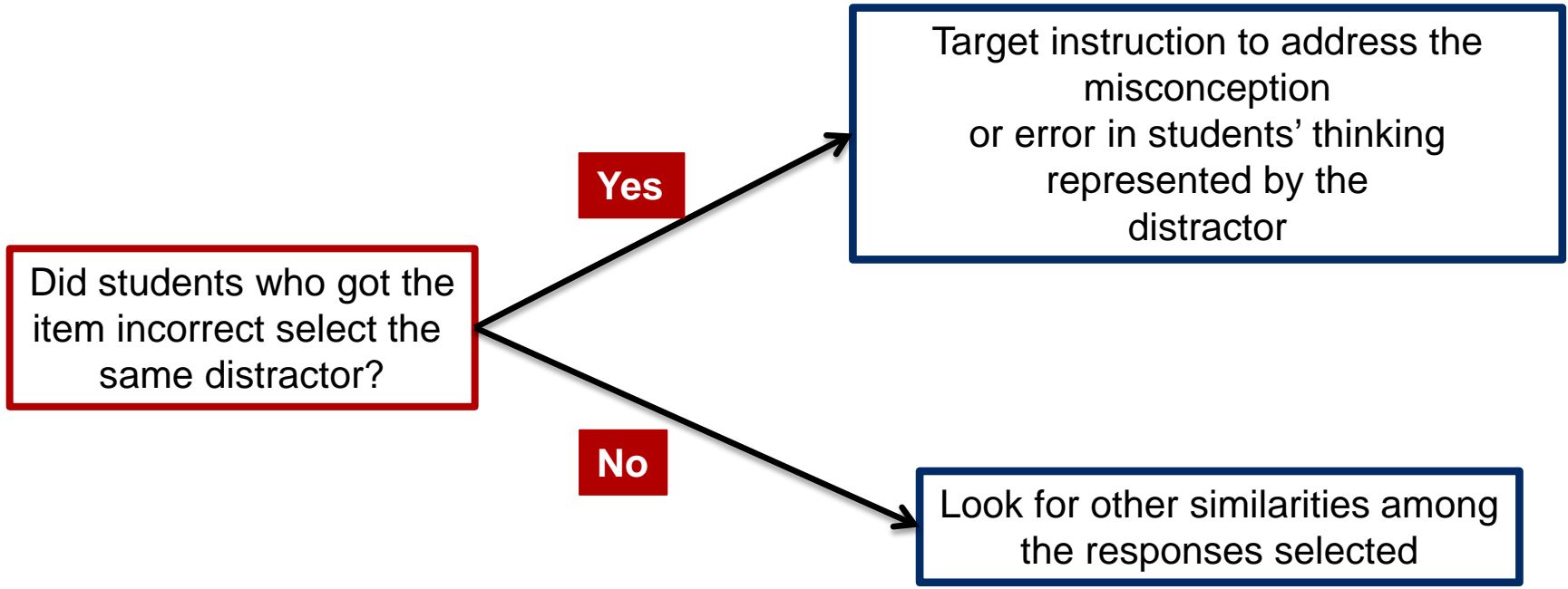
Clearly, the level at which you can examine student performance can become increasingly fine-grained. For example, once you start looking at item-level performance you can examine any of the following attributes:

- Whether students selected one distractor more frequently than the others (e.g., **Did all students who got the item incorrect select the first distractor?**)
 - Whether students responded correctly to items targeting a specific level of proficiency (e.g., **Did students get all of the items targeting procedural fluency correct?**)
 - Whether students consistently selected the incorrect response for items targeting a specific level of proficiency (e.g., **Did students consistently get items targeting strategic competence or adaptive reasoning incorrect?**)

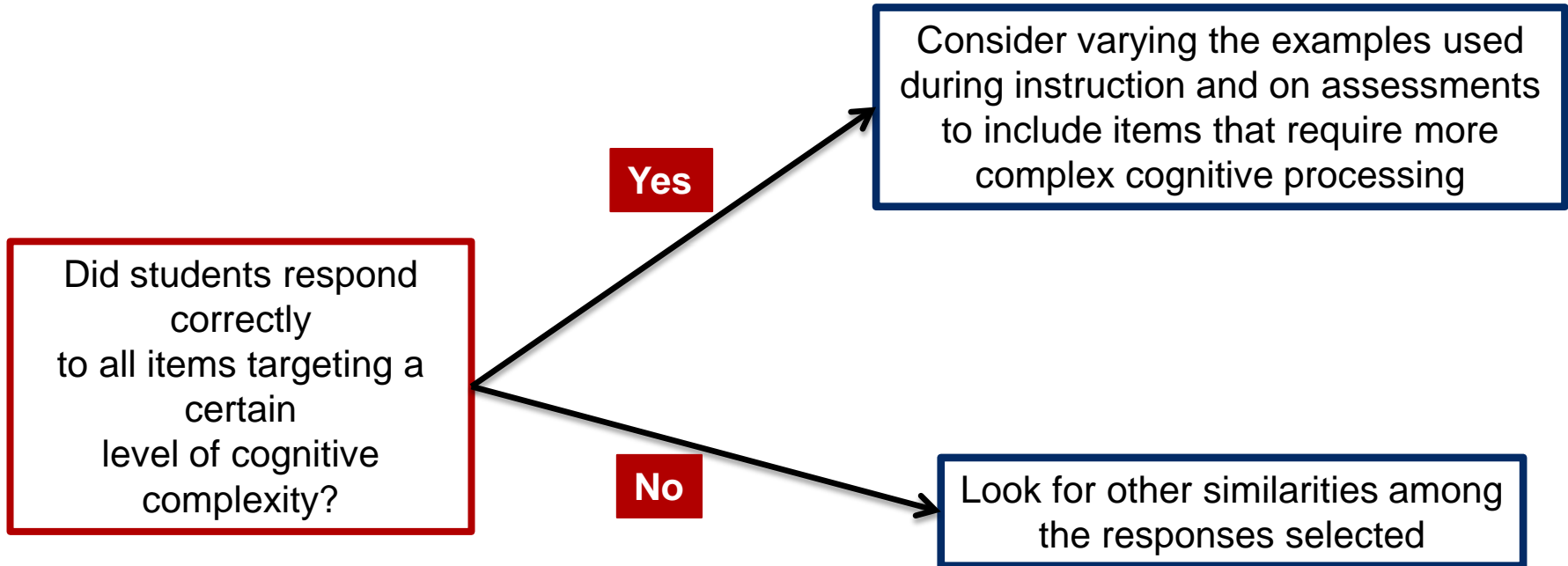
 **SMU** Using Data to Inform Instruction:
How Far Down the Rabbit Hole
Should I Go?



Responses to these questions can then be used to further guide instructional planning. For example:



 **SMU** Using Data to Inform Instruction:
How Far Down the Rabbit Hole
Should I Go?



 SMU Using Data to Inform Instruction:
How Far Down the Rabbit Hole
Should I Go?



Now it's time to consider the **BIG** question....

Do I need to look at the data this closely
for **every** student in my classroom?

Our response....

Probably not. For students who are on track, monitoring their progress using the overall score is probably enough. For students who are struggling, however, digging a bit deeper into the data to try and figure out **why** they are struggling may be very useful.

Understanding **why** students are struggling can be used to help us target our instructional efforts to meet students' needs.



SMU Some Final Thoughts & Take-Aways

- **Assessment and instruction should be considered together** – performance on assessments can inform instruction and assessments can be specifically designed to provide students with opportunities to demonstrate what they've learned during instruction.
- When considering the level of knowledge represented while designing a test item, start with the outcome (**target knowledge**) in mind. Then consider what **foundational knowledge** the student needs and the **bridging knowledge** that will help the student acquire the **target knowledge** or skill.



SMU Some Final Thoughts & Take-Aways

- When designing tests or assessments for use in your classroom, be sure to include items that target **multiple levels of cognitive engagement**.
- Revisit the **guidelines for item development** as often as needed to ensure that the items you write provide students with the best opportunity possible to demonstrate their knowledge and understanding of the content.
- Don't hesitate to **look beyond the overall test score** to students' performance on groups of items or individual items to try and understand what instructional supports you can provide to support students' learning

- Haladyna, T. M. (2004). *Developing and validating multiple-choice items*. New York, NY: Routledge.
- Leong, S.C. (2006). *On varying the difficulty of test items*. Paper presented at the 32nd Annual Conference of the International Association for Educational Assessment, Singapore.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics* . Washington, DC: National Academies Press.
- Rutherford, P. (2008) *Instruction for all students* (2nd ed.). Alexandria, VA: Just Ask Publications & Professional Development

ACTIVITY

Procedural Easy: testing simplifying expressions (easy because only 2 terms to combine, one variable, and terms are given in order)

Simplify the expression: $7x - 3x + 3$	$4x + 3$
	$7x$
	$-4x + 3$
	$13x$

Procedural Medium: testing simplifying expressions (medium because combining more than 2 terms and more than one variable)

Simplify the expression: $11x + 5y - 2y + 4x$	$15x + 3y$
	$16x + 2y$
	$18xy$
	$11x + 7y$

Procedural Difficult: testing simplifying expressions (difficult because distributing and combining like terms, and order is not necessarily easy to deal with)

Simplify the expression: $4(3r + 2) + 5r$	$17r + 8$
	$32r + 8$
	$12r + 6$
	$17r + 2$

Conceptual Easy: tests understanding of the distributive property (easy because whole numbers, split up...students only choosing which operations are correct)

<p>Which expression is equivalent?</p> <p>$(5 + 3) \times 7$</p>	$(5 \times 7) + (3 \times 7)$
	$(5 + 7) + (3 + 7)$
	$(5 \times 3) + (5 \times 7)$
	$(5 \times 3) + (3 \times 7)$

Conceptual Medium: tests understanding of the distributive property (medium because students must decide how to break up numbers and which operations to use)

<p>Which expression is equivalent?</p> <p>2.1×3.5</p>	$(2 \times 3.5) + (0.1 \times 3.5)$
	$(2 + 3.5) + (0.1 + 3.5)$
	$(2 \times 3) + (0.1 \times 0.5)$
	$(2 \times 0.5) + (0.1 \times 3)$

Conceptual Difficult: tests understanding of the distributive property (difficult because variables included, operations not as obvious, distribution required twice, and can't actually compute answer to check)

<p>Which expression is equivalent?</p> <p>$(x + 2)(x + 7)$</p>	$(x^2 + 7x) + (2x + 14)$
	$(x + x + 7) + (2 + x + 7)$
	$x^2 + 14$
	$7x + 2x$

Strategic Easy: tests process of converting measurement (easy because the only thing the student needs to distinguish is the operation)

The length of John's backyard is 50 feet. Which expression can be used to find the length of John's backyard in inches?	50×12
	$50 \div 12$
	$50 + 12$
	$50 - 12$

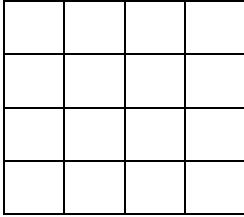
Strategic Medium: tests process of converting rate (medium because the student must look at units carefully...something students struggle with)

Jake reads 3 pages in 1 minute. At this rate, which expression can be used to find how many pages Jake can read in 1 hour?	$\frac{3 \text{ pages}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$
	$\frac{3 \text{ pages}}{1 \text{ minute}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$
	$\frac{1 \text{ minute}}{3 \text{ pages}} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$
	$\frac{3 \text{ minutes}}{1 \text{ page}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$

Strategic Difficult: tests process of converting from fraction to percentage (difficult because student must distinguish between correct operation and correct units)

A class has 12 girls and 16 boys. Which expression can be used to find what percentage of the students in the class are boys?	$\frac{16}{28} \times 100\%$
	$\frac{16}{12} \times 100\%$
	$\frac{12}{16} \div 100\%$
	$\frac{16}{28} \div 100\%$

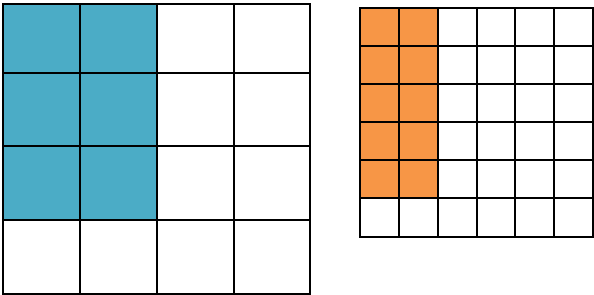
Adaptive Easy: tests justification of how to use equivalent fractions (I think it's easy because the visual model provides some support)

<p>Jane wants to shade $\frac{3}{4}$ of the model below. Which explanation describes why she multiplies $\frac{3}{4} \times \frac{4}{4}$?</p> 	She is finding an equivalent fraction.
	She is simplifying the fraction.
	She is finding a common denominator.
	She is finding the greatest common multiple.

Adaptive Medium: tests justification of how ratios change when num/den is increased (medium because they must first take the context and understand the underlying math...they'll hopefully write down the initial and new ratios)

<p>Doug has 4 fish and 2 dogs. He buys another fish. How does the additional fish change the ratio of dogs to fish?</p>	The ratio gets smaller because only the denominator increases.
	The ratio gets larger because the total number of pets increases.
	The ratio gets smaller because only the numerator increases.
	The ratio gets larger because the number of fish increases.

Adaptive Difficult: tests justification of fraction comparison with different wholes (difficult because they cannot compare models directly and must really think about the fraction each model represents)

<p>Which explanation best describes why Model A represents a larger fraction?</p> <p style="text-align: center;">Model A Model B</p> 	The shaded portion of Model A covers more of the total area than Model B.
	The total area of Model A is larger than the total area of Model B.
	The squares are larger in Model A than the squares in Model B.
	There are fewer un-shaded squares in Model A than in Model B.

