EXPLORING Research in Algebra: Tackling algebra in middle school & high school

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Chair: Elizabeth Howell

Speaker: Dr. Candace Walkington

Speaker: Dr. Nick Wasserman

Algebra in the 21st Century

- Important issues with student motivation face schools today (Hidi & Harockwicz, 2000), especially in secondary mathematics (Mitchell, 1993)
- Interest in math declines over adolescence generally, and algebra classes specifically (Fredicks & Eccles, 2002; Frenzel et al., 2010; McCoy, 2005)
- Algebra I gatekeeper to higher-level mathematics, significant implications for equity & access (Cogan, Schmidt, & Wiley, 2001; Moses & Cobb, 2001)

Algebra in the 21st Century

- Failure rates in Algebra I continue to be high, especially among low-income students and students of color (Allensworth, Nomi, Montgomery, & Lee, 2009; McCoy, 2005)

"I think we're growing serfs in our cities, young people who graduate with eighth grade education that can't access economic arrangements to support families. Kids are falling wholesale through the cracks – or chasms – dropping out of sight… people say they do not want to learn. The only ones who can dispel that notion are the kids themselves." ~Bob Moses, Algebra Project

Algebra in the 21st Century

- Algebra I considered such a devastating subject, that some people suggest omitting the subject from high school altogether.

"A TYPICAL American school day finds some six million high school students and two million college freshmen struggling with algebra. In both high school and college, all too many students are expected to fail. Why do we subject American students to this ordeal? To our nation's shame, one in four ninth graders fail to finish high school. Most of the educators I've talked with cite algebra as the major academic reason."

Algebra in the 21st Century

- Rather than scrapping Algebra, an alternative view is that we could learn to **teach algebra better**
- But what's going wrong? Why is Algebra so difficult for students to learn?

Algebra Classrooms in the U.S.

- Let's look at a clip of what might be considered a typical Algebra classroom in the United States
- Consider:
  - How are students **engaging** with Algebra concepts?
  - How are **symbolic representations** and **visuals** used?
  - What do you think students **understand** about Algebra?

Video 1

- Eighth grade Algebra class reviewing concepts related to graphing linear functions on a coordinate plane.

Using a pencil and the large piece of graph paper, graph the following linear equations:

1) $y = \frac{2}{3}x + 8$
2) $y = \frac{3}{5}x - 10$
3) $y = 3x + 7$
4) $y = \frac{1}{4}x - 4$
5) $y = x - 5$

After these five equations are graphed, check with me before proceeding.

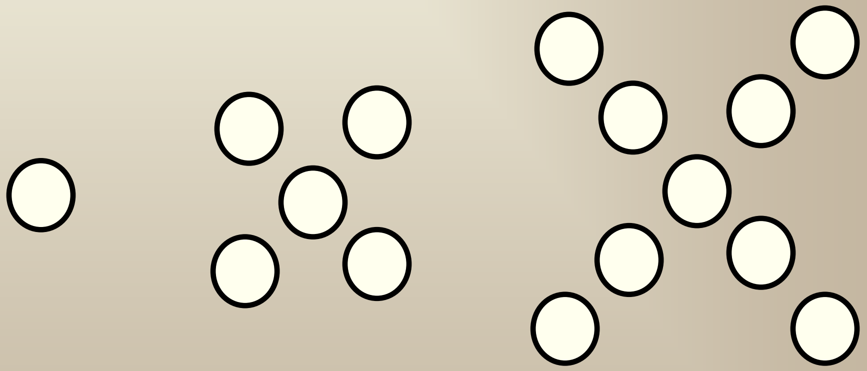
Algebra Classrooms in the U.S.

- Now let's look at a clip of a perhaps less typical Algebra class
- Consider:
  - How are students **engaging** with Algebra concepts?
  - How are **symbolic representations** and **visuals** used?
  - What do you think students **understand** about Algebra?

Video 2

- Algebra class learning about linear functions
- Students have been working on problem, now presenting to class

Describe the pattern.
Assuming the sequence continues in the same way, how many dots are there at 100 minutes?



number of dots = 4m+1

Algebra Classrooms in the U.S.

- **Video 1**: Procedural engagement with the task and the symbols, manipulating expressions with little understanding, poor connection to the visual
- **Video 2**: Deeper engagement with the symbols (meaningful), multiple viewpoints, visual representation well-connected to symbols

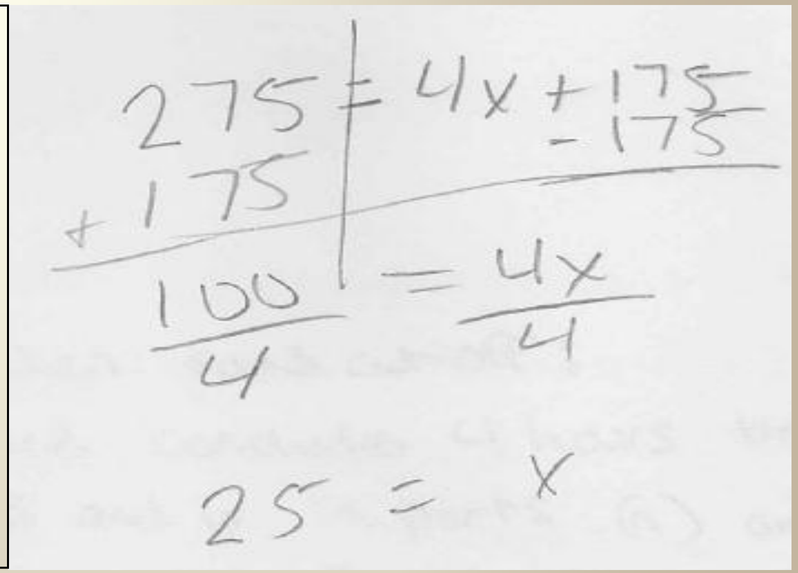
TEKS – Algebra I

In Algebra I, students must learn to work with **symbolic representations**:

- *Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.*
- Students should be able to "describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations." (111.32b)
- Coordinating symbols with story problem situations is a particularly difficult skill (Bardini, Pierce, & Stacey, 2004; Koedginer & McLaughlin, 2010)

Symbolization in Algebra

The distance in feet that a machine called the Crawler has traveled from its hangar is given by the equation y = 4x+175, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler be a total of 275 feet from the hangar?

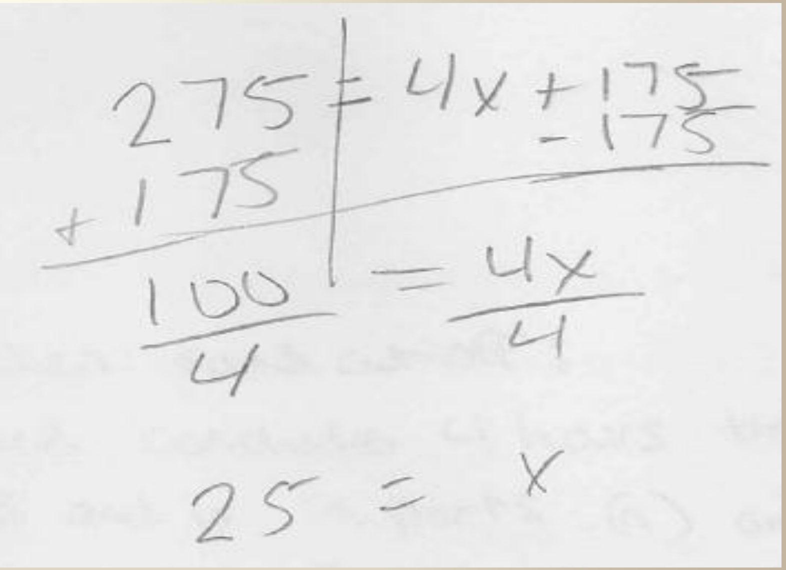


What *probing questions* might you ask this student, to assess if they understand key concepts related to using symbolic expressions?

Walkington, Sherman, & Petrosino (2012)

Symbolization in Algebra

The distance in feet that a machine called the Crawler has traveled from its hangar is given by the equation y = 4x+175, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler be a total of 275 feet from the hangar?

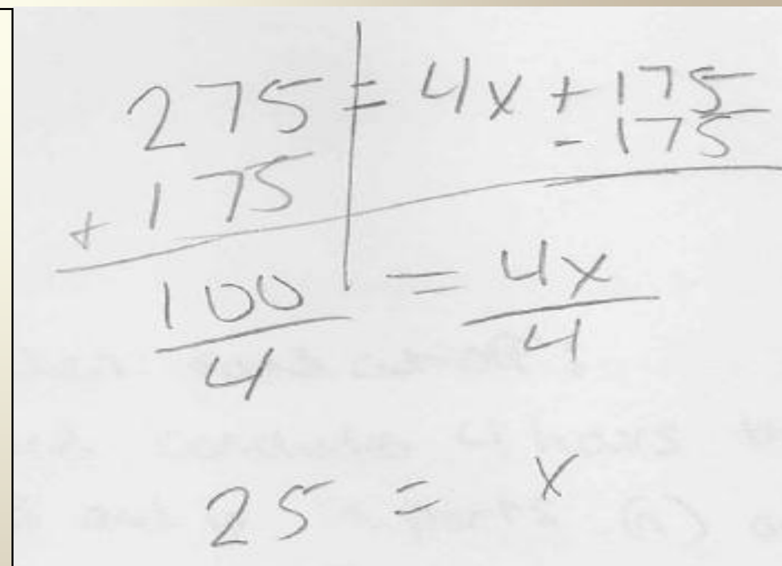


What could the "4" represent in the given situation? What could the "175" represent?

Walkington, Sherman, & Petrosino (2012)

Symbolization in Algebra

The distance in feet that a machine called the Crawler has traveled from its hangar is given by the equation $y = 4x+175$, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler be a total of 275 feet from the hangar?



"It could have already started at 4 feet... and 175 could equal the time it took for it to go 4 feet."

Walkington, Sherman, & Petrosino (2012)

Symbolization in Algebra

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"In general, if students engage extensively in symbolic manipulation before they develop a ***solid conceptual foundation*** *for their work, they will be unable to do more than mechanical manipulations." (NCTM, 2000)*

NCTM

Making Algebra Meaningful

- My research explores ways to make algebraic representations more **meaningful** to students, by connecting their learning to things they know, understand, and are interested in

"Well the one [video game] that I play a lot is probably… Commander… There's stuff like, this unit has 1000 health and does 100 damage per attack. And then the other units they might have 10,000 health and they might to 20 damage per attack. If I have them attack each other, who will win?"

Walkington, Sherman, & Howell (in preparation)

Making Algebra Meaningful

"In a day, I would maybe estimate 400 [texts], 'cuz I've gotten over 14,000 in one month before… I was pretty surprised about that one… We were comparing who was the worst texter, me or my cousin, and I was the worst one. Hers was like 10,000 and mine was like *14 [thousand]. She was like "oh my gosh…"*

"I work at Barney's, a restaurant around *here… it's* just minimum wage [7.25], plus tips. At the end of the night a *bunch of my friends… we all* like sit around and like we count *each other's [tips] up to make sure… like see whether they* were good or not. To see who has better tables we add them all up and divide by how many *tables…"*

Walkington, Sherman, & Howell (in preparation)

Making Algebra Meaningful

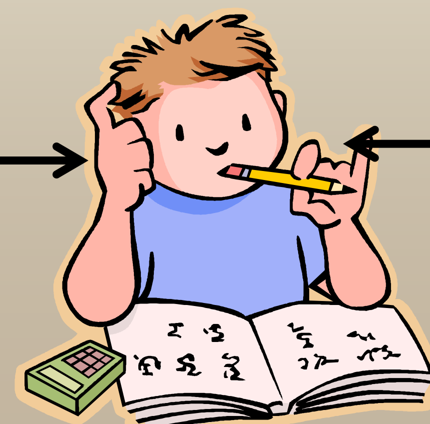
- Many students have experiences from their everyday lives where concepts like rate of change could be utilized
- By connecting instruction in Algebra to these students' interests and experiences, we can ***personalize*** each students' learning

Personalization principle: Matching Algebra problems to students' out-of-school interests improves learning

The Theory: Why is Personalization Effective?

- Connections to **prior knowledge** and experience support inference-making
  - "Grounding"

$y = x + 3$



r^2

The Theory: Why is Personalization Effective?

- Connections to **prior knowledge** and experience support inference-making
  - "Grounding" (Goldstone & Son, 2005)
- Elicit students' **interest** in the concepts they are learning (Hidi & Renninger, 2006)
  - Focus of attention, use of learning strategies, persistence, confidence, goal-setting

Personalization Research

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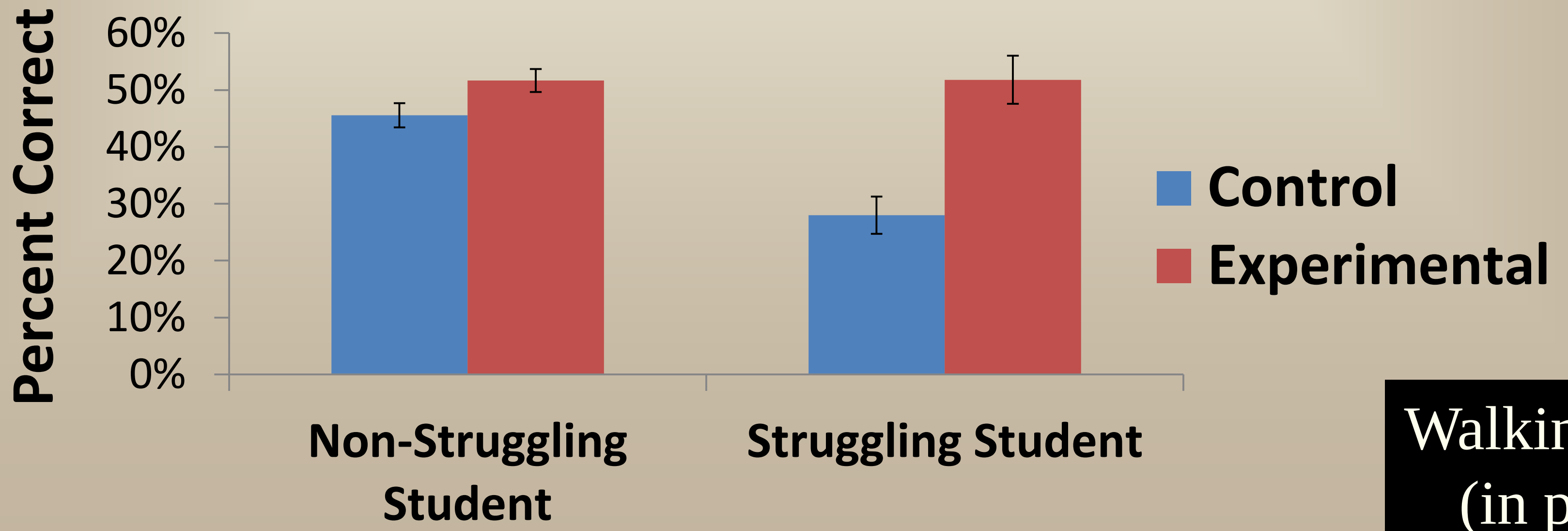
- **145** Algebra I students assigned to 2 groups:
  - Receive personalized problems for one unit (Experimental)
  - Receive normal problems for one unit (Control)
- Within **Cognitive Tutor Algebra** (CTA) environment
- CTA presents interests survey & selects variations

Walkington (in press)

Personalization Research

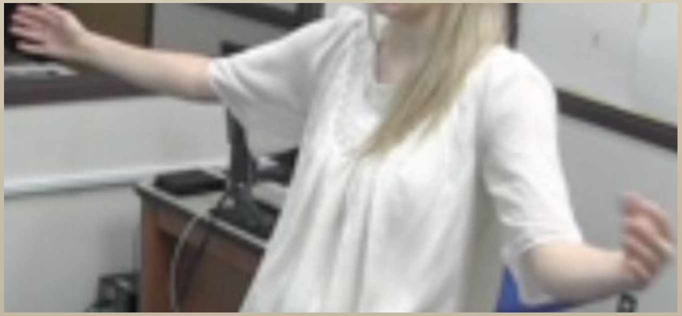
- Students receiving personalized problems:
  - Solved problems in the unit **more accurately** and **faster**
  - “**Gamed the system**” less often
  - One month later, **still** solved **more difficult** Algebra problems more accurately and faster



Walkington (in press)

Making Algebra More Meaningful

- Personalization is not the only way to ***ground*** mathematics instruction in students' **experiences** and enhance **motivation**
- Model concepts by moving the body – kinesthetic learning – better problem-solvers



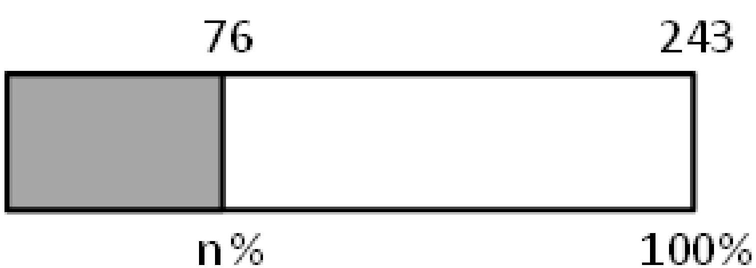
Walkington et al. (2012)

Making Algebra More Meaningful

- Using visuals to make math problems interesting and understandable (Study of 139 students)

BETTER

A Southwest airlines passenger plane can seat 243 passengers…



WORSE

A Southwest airlines passenger plane can seat 243 passengers…

(***effects strongest for *low interest* students)

Walkington, Cooper, & Howell (submitted)

Making Algebra More Meaningful

- In order to access algebra learning, students need to be able to *read* and *understand* problems (especially ELL)
- Problems with relevant, coherent, and readable language support problem-solving (Study of 3000 Algebra I students)
  - Problems about "you" are easier to solve
  - Less likely to attempt problems with more sentences, more words, or more words per sentence
  - Problems that have similar words or repetitive verbs across sentences, and that clearly define how events are related ("because of", "due to") are also easier

Walkington, Clinton, & Howell (submitted)

Making Algebra Meaningful

- ***Summary: What can teachers do to help make Algebra meaningful for students?***
  - Talk to your students about their interests
  - Craft problem scenarios situated in their experiences and typical language
  - Work to enhance motivation and grounding though visual and kinesthetic components
  - Be creative – Algebra is a rich and dynamic subject that students should enjoy exploring!

Shifting the focus

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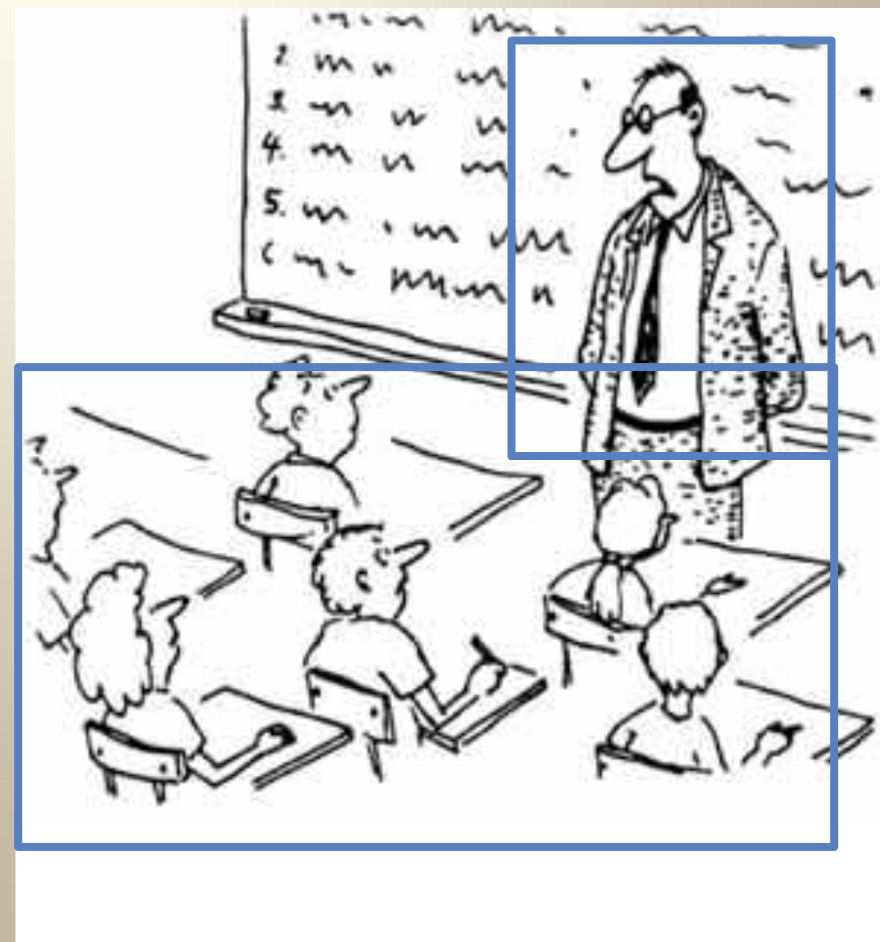
- In addition to finding ways to make the content of Algebra relevant to students, the teaching of Algebra is also impacted by the teacher
- While many things may impact teaching (e.g., beliefs, personality, etc.), we will focus on his/her content knowledge for teaching Algebra

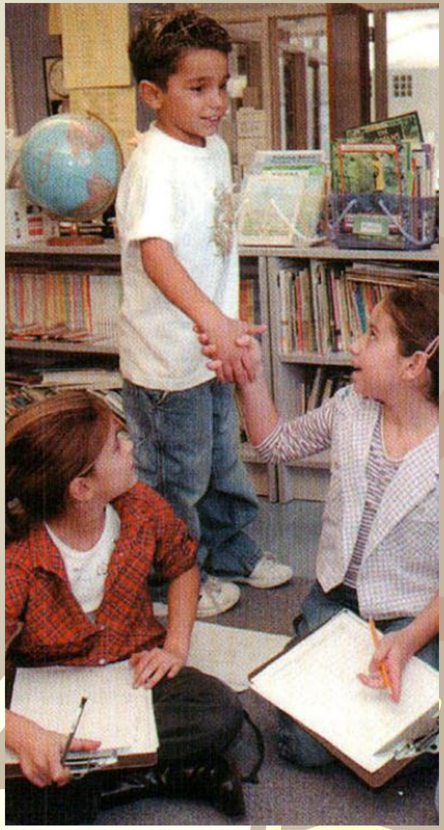


Knowledge for Teaching

- Some (e.g., Monk, 1994) have found that teachers' content knowledge is aligned with student gains. Others (e.g., Rowan, Correnti, & Miller, 2002) have found that too much content knowledge adds very little.
- In characterizing Mathematical Knowledge needed for Teaching (MKT), a distinction is generally made between Content Knowledge and Pedagogical Content Knowledge. (Some work, e.g., Ball, et al., 2008, further delineates these into other categories.)
- Both aspects of MKT, however, are specific ways that *mathematical knowledge* impacts the work of teaching.

Expanded Meaning of Algebra

- In the past few decades, there has been an increasing push to further "Algebrafy" the K-12 curriculum.
- This means using tasks *throughout* K-12 that will help students build algebraic reasoning, with concepts such as equivalence, ordering, pattern, etc. (There are a number of great practitioner articles that exemplify these ideas; e.g., Blanton & Kaput, 2003)



Blanton & Kaput (2003). NCTM

Knowledge for Algebra Teaching

- What kinds of knowledge of Algebra should teachers have to teach an “Algebrafied” curriculum?
- We look at one framework, Knowledge for Algebra Teaching (Ferrini-Mundy, et al, 2012), that attempts to characterize how various Categories of Algebra Knowledge overlap with their Use in Algebra Teaching.

Knowledge for Algebra Teaching

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Categories of Teachers Knowledge / Use in Teaching	Knowledge of School Algebra	Knowledge of Advanced Mathematics	Mathematics-for-Teaching Knowledge
Trimming Removing complexity while maintaining integrity			
Bridging Making connections across topics, assignments, representations, and domains			
Decompressing Unpacking complexity in ways that make it comprehensible			

Knowledge for Algebra Teaching

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- Trimming, Bridging, and Decompressing are ways that teachers may use their knowledge in the classroom.
- The impact on teaching spans across various tasks:
  - Creating mathematical tasks
  - Probing students' mathematical ideas
  - Accessing the mathematical thinking/knowledge of learners
  - Knowing and using the curriculum
  - Explaining mathematics
  - Supporting productive mathematics discourse

Research Question

- Within the perspective of knowledge, I am particularly interested in how “advanced” algebra ideas can impact teaching (Knowledge of Advanced Mathematics)
- In what ways and to what degree can knowledge of the algebraic structure of Group Theory (advanced knowledge) impact teachers’ Knowledge for Algebra Teaching and their practices?

Decompressing: A glimpse into Algebraic Structure

- What would you do to solve this equation?

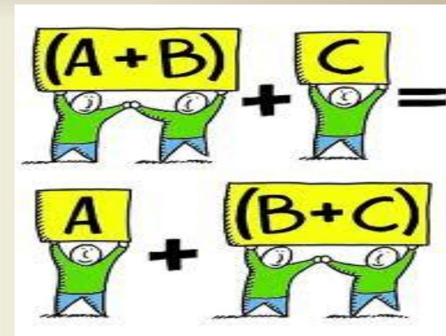
$$\begin{array}{ccc} x+5 & = & 12 \\ -5 & & -5 \end{array}$$

- The equals sign is a symbol for *equivalence*, not a sign to *compute*. The notion of equivalence justifies "what we do to one side do to the other" in order to remain equivalent.
- But my primary question is: What's next?

$$x=7\ ???$$

- There are actually four distinct ideas/steps required in this process… Can you find them?

Associative



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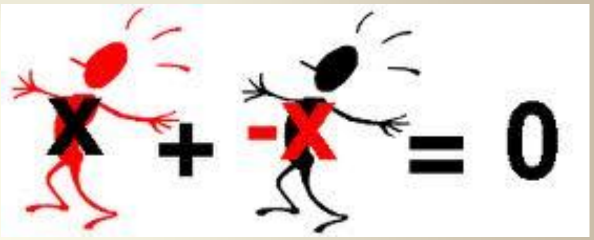
- In fact, addition is a *binary* operation, which means that we add *two* numbers at a time. So the first step actually leaves us with:

$$(x + 5) + -5 = 12 + -5$$

- To make any progress in solving, addition *must be* associative

$$x + (5 + -5) = 12 + -5$$

Inverse Element



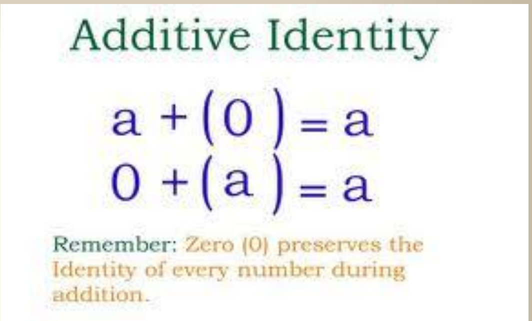
- It now becomes clear why we chose to add -5 to both sides in the first place:

$$x + (5 + -5) = 12 + -5$$

$$x + 0 = 12 + -5$$

- We added -5 because it gave was a "zero-pair," the *additive inverse* to 5.

Identity Element



- Unfortunately, the left side looks remarkably similar to our original expression ($x + 5$ – e.g., *sum with unknown*).

$$x + 0 = 12 + -5$$

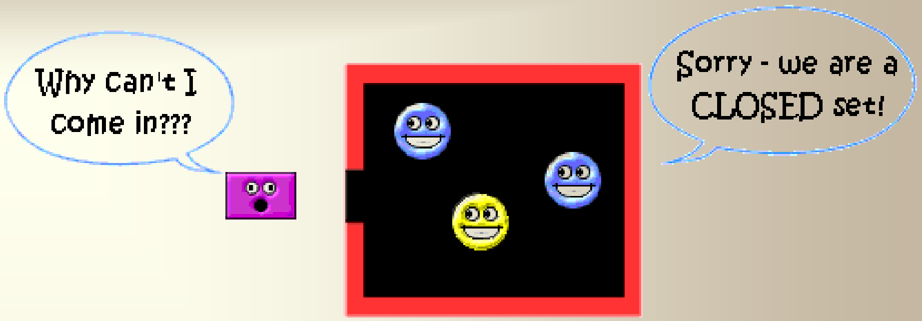
- Fortunately, having 0 instead of 5 makes the *sum with an unknown* remarkably dissimilar – it is a *known sum*.

$$x + 0 = 12 + -5$$

$$x = 12 + -5$$

- To make any progress in solving, an *identity element* under addition is absolutely necessary.
- Also, to note, is the intimate relationship between an *inverse* and the *identity* (an inverse element produces the identity element)

Closure



- Finally, for our solution to the unknown, x, to make any sense, it must be that the sum on the right side produces *a number*.

$$x = 12 + -5$$

$$x = 7$$

- *Closure* on an operation (that the sum of any two numbers results in a number) is important to the solving process, as it guarantees that our answer will be sensible.

Algebraic structure of a Group

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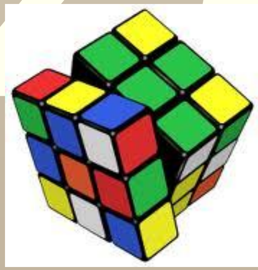
- These four properties – Closure, Associative, Identity element, existence of an Inverse element (that produces the Identity element) – form the axiomatic structure of a Group. (Note lack of *commutative* property.)
- Any binary operation (*) on a specified set of objects (S) that has these 4 properties can be treated in an algebraically similar way.
- Famous example: Rubik's cube



Research Study

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- Participants were current teachers that were introduced to Group theory in a graduate course (this included a number of other activities besides the previous example, with applications to more abstract situations)
- During interviews, written lesson plans, (and video recordings of their teaching), they identified ways that their teaching of algebra would (did) change. These were recorded with SmartPens – various excerpts are used in the following slides.

Findings

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- Participants’ responses will be discussed according to some of the ways that the Advanced Mathematics Knowledge impacted different aspects of teaching:
  - Creating mathematical tasks
  - Probing students’ mathematical ideas
  - Accessing the mathematical thinking/knowledge of learners
  - Knowing and using the curriculum (trajectories)
  - Explaining mathematics
  - Supporting productive mathematics discourse

Findings

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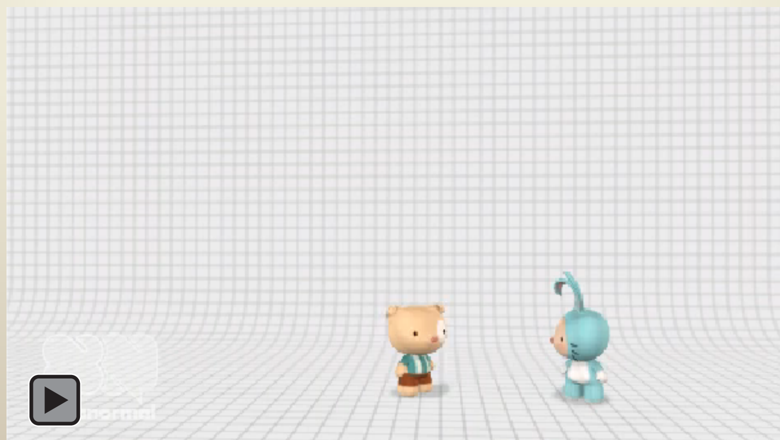
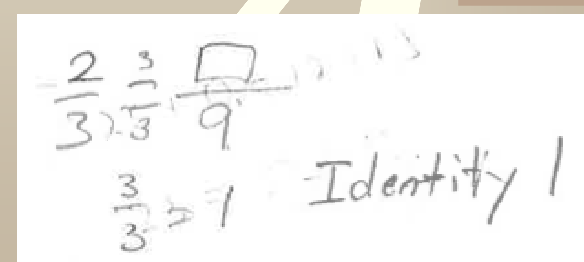
<table>
<tr><th>Categories of Teachers Knowledge
Tasks of Teaching</th><th>Knowledge of School Algebra
Arithmetic Properties</th></tr>
<tr><td>Creating mathematical tasks</td><td rowspan="2">Commutative
$\square + 3 = 3 + 5$ (Balancing equation)

Identity Element → Multiplication Facts
$4 + \square = 4$ (0)
$4 - \square = 4$ (0)
$4 \times \square = 4$ (1)</td></tr>
<tr><td>Knowing and using the curriculum</td></tr>
</table>

Findings

<table>
<tr><td>Categories of Teachers Knowledge
Tasks of Teaching</td><td>Knowledge of Advanced Mathematics
Mathematical reasoning and proof</td></tr>
<tr><td>Probing students' mathematical ideas</td><td rowspan="2">Pre-Video Teaching Episode

$$2\frac{1}{3} = \frac{2}{6}$$

</td></tr>
<tr><td>Explaining mathematics</td></tr>
</table>

Findings

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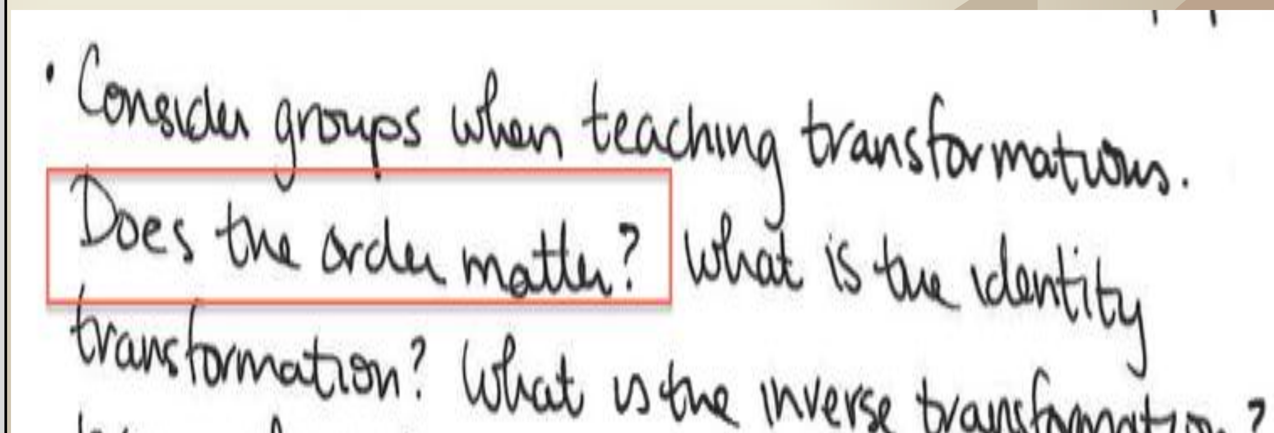
Categories of Teachers Knowledge / Tasks of Teaching	**Knowledge of Advanced Mathematics** Mathematical language
Creating mathematical tasks	 • Consider groups when teaching transformations. Does the order matter? What is the identity transformation? What is the inverse transformation? What is the identity function? Inverse function? Does order matter?

Findings

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Categories of Teachers Knowledge / Tasks of Teaching	**Knowledge of Advanced Mathematics** Mathematical discourse
Supporting productive mathematics discourse	Use terms that are vertically aligned with older grade levels

Findings

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Categories of Teachers Knowledge / Tasks of Teaching	**Knowledge of School Algebra** Conceptual depth
Explaining mathematics	*In addition to the interviews, lesson plans, and videotaped lessons, participants took an identical pre- and post assessment.* Participants demonstrated statistically significant increases in their knowledge of the vocabulary for familiar arithmetic properties, as well as increased **conceptual understanding** and explanations of their meaning in more **abstract situations**

Using Advanced Knowledge

DECOMPRESSING: Unpacking complexity in ways that make it comprehensible

Inverse

$$\frac{5x}{5} = \frac{25}{5}$$

$$\frac{5}{5} \cdot x = \frac{25}{5}$$

$$1x = 5$$

$$\begin{array}{r} -3 + x = 8 \\ +3 \qquad +3 \\ \hline 0 + x = 11 \end{array}$$

$$\log_4 x = 6$$

$$f(x) = \log_4 x; f^{-1}(x) = 4^x$$

$$4^{\log_4 x} = 4^6$$

$$f \circ f^{-1}(x) = i(x) = x$$

$$x = 4^6$$

Many things (commutative, cancelling, etc.) that we take for granted are not always obvious to students, particularly ones that struggle. Students need to be reminded and made aware of these things.

Using Advanced Knowledge

001

TRIMMING: Removing complexity while maintaining integrity

Inverse

How to respond to a 9th grade student question:

After learning that cos x = A/H and sec x = H/A in a right triangle, a student asks:

Is sec x the inverse of cos x?

Using Advanced Knowledge

001

BRIDGING: Connections across topics, assignments, representations, & domains

Inverse

Understanding an inverse requires knowing:

- The operation
- The set on which the operation is defined
- The identity element for that operation and set
- Awareness of associativity and closure of operation on that set

"I am able to connect these [arithmetic] properties to a bigger picture in mathematics."

Teachers Algebra Knowledge

Conclusions

- Teachers' knowledge of "advanced" algebraic structures, can impact algebra teaching practices (KAT)
- The mathematical activities of Trimming, Bridging, and Decompressing comprise many ways teachers use content knowledge
- The interaction of advanced content with K-12 content is not the only way to improve teachers' KAT.
  - Deeper study of students' learning (as opposed to content) trajectories
  - Developing pedagogical content knowledge for teaching different types of learners
  - Identifying how Algebra can be (is) Decompressed, Trimmed, and Bridged with integrity at each phase K-12

Teachers Algebra Knowledge

Concluding thoughts:

- Teachers play an irreplaceable role in the learning process
- Districts and Schools need to continue to invest in and provide opportunities for teachers' professional development – across many domains, including:
  - content knowledge
  - pedagogical content knowledge
  - available resources & technology
  - students' developmental learning
  - etc.

Algebra in the 21st Century

- A national survey of Algebra I teachers found:
  - "Working with unmotivated students" as most challenging aspect of teaching algebra
  - Second place: "Making mathematics accessible and comprehensible to all students."

  (Loveless, Fennel, Williams, Ball, & Banfield, 2008)
- Algebra is a difficult subject, and novel approaches to teaching & curriculum needed
- Algebra teachers need to account for the enormous diversity in the backgrounds and needs of their students

Algebra in the 21st Century

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- Teaching algebra for understanding involves…
  - Building **interest**, **motivation, and prior knowledge** for doing mathematics for students who come into the classroom without these resources
  - Making the new & challenging representations introduced in algebra meaningful
  - Finding ways to decompress, trim, and bridge algebra content that maintain integrity in different grade levels and with different students
  - Creating tasks that foster and reinforce fundamental algebraic concepts throughout K-12

Discussion Question

- How you could practically incorporate students’ interests into your teaching?

Discussion Question

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- How could the instructional team at your school support teachers with trimming, decompressing, and bridging?

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General Q&A

Research Participation

- We are looking for **mathematics teachers** interested in partnering on research studies, like the ones we described today!
- A fun and rewarding experience – you have the opportunity to contribute to issues of **national significance** in math education
- Please contact **cwalkington@smu.edu** if you think you might be interested

MMT Program at SMU

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- SMU offers the “Master Mathematics Teacher” certification, either on its own, or as part of a Masters in Education
- Learn the content and pedagogy of how teachers can facilitate math learning
- Application Deadline: Rolling Admissions

SMU. Annette Caldwell Simmons School of Education & Human Development

CHANGING MINDS