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Research in Mathematics Education

Imagination Station (Istation): Algebra Readiness Progress Monitoring System Development for Grades 6-8

Technical Report 16-05

Imagination Station (Istation): Algebra Readiness Progress Monitoring System Development for Grades 6-8

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Summer 2016

Published by
Southern Methodist University
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This research was supported by Imagination Station Inc. Opinions expressed herein do not necessarily reflect those of Imagination Station Inc. or individuals within.

Abstract

In this technical report, we describe the development of an Algebra Readiness Progress Monitoring (ARPM) system for students in Grades 6-8 for Imagination Station (Istation). The ARPM system, delivered through the ARPM probes, or parallel assessment forms, support teachers' instructional decision-making and support students in tracking their personal growth. State and national mathematics content standards related to algebra readiness inform the mathematics topics underlying the ARPM probes. In this technical report we describe: (a) the process used to identify the mathematics content assessed in the ARPM system and specifically, the format and content of the ARPM probes, (b) the ARPM system blueprinting process and the development of the exemplar items, and (c) the processes and reviews conducted for the creation of the ARPM probes.

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Introduction

The purpose of the Algebra Readiness Progress Monitoring (ARPM) system for Grades 6-8 for Imagination Station is to provide teachers with data that can be used to monitor students' development of algebra readiness knowledge and skills throughout the academic year. The ARPM system is delivered through the ARPM probes(i.e., parallel forms) focused on the following content areas: Quantity Discrimination, Number Properties, and Proportional Reasoning. By administering the ARPM probes at frequent and consistent intervals, teachers and administrators can use the results to help determine if the student is progressing at an adequate rate to reach his or her instructional goals for algebra readiness. Additionally, students can compare their scores over time to see their own personal growth. Based on the Response to Intervention (RTI) model for curriculum intervention, the ARPM system is designed for struggling students, identified by a universal screener assessment, who are receiving content intervention based on the results of a diagnostic assessment or skills inventory.

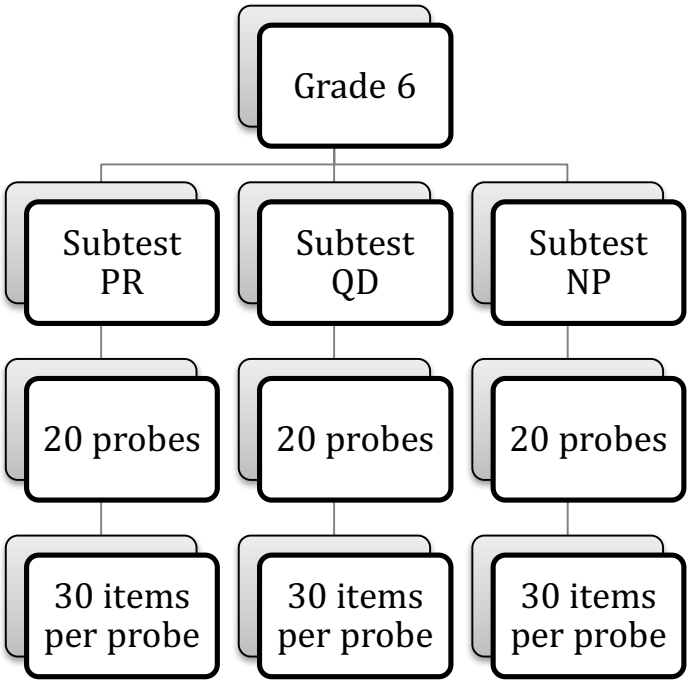
The purpose of this technical report is to describe the development of the ARPM system. The development processes used to create the items for the ARPM probes represent best practice in test development and align with the *Test Standards* published by the American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement in Education (NCME) (2014).

Assessment System Structure

The ARPM system is designed to address algebra readiness content for Grades 6-8. Each grade level is assessed by a subtest for each of the three identified content areas known as subtests: Quantity Discrimination (QD), Number Properties (NP), and Proportional Reasoning (PR). For each subtest, 20 ARPM probes were developed. Each ARPM probe has 30 items related to the specific subtest content. The ARPM system framework for Grade 6 is shown in *Figure 1*.

Figure 1

ARPM system framework for Grade 6.



ARPM Probes. At each grade level, there are 20 probes for each subtest. Each subtest probe is described as a parallel form because the probes are designed with different, yet comparable questions that assess the same construct (skill) at a similar level of difficulty. While the items on each probe are unique, items were written to target both the same content and difficulty level across all probes. An example of an item for each subtest probe is shown in *Table 1*. These subtest probes support ongoing progress monitoring (Kelley, B., Hosp, J. L., & Howell, K. W., 2008). The ARPM probes are designed to be administered frequently, at least once per month. If necessary, a student can take the subtests weekly (up to 20 weeks) in order to monitor progress through instructional intervention.

Item Format. Each subtest probe consists of 30 unique items**.** The format of each item is a comparison of two numerical expressions or values. Students are asked to determine which symbol (<, =, or >) will make the statement true. The response options are provided to students in a selected-response format. The items were developed to discourage lengthy numerical calculations. Instead, numerical reasoning strategies can be used to quickly determine which

symbol makes the statement true. Possible numerical reasoning strategies are explained for sample items in *Table 1*. Additional information about how the items were developed can be found in Ketterlin-Geller, Gifford, and Perry (2015).

Construct Definition

The mathematics content for the assessed construct of the ARPM system is based on the algebra readiness knowledge and skills identified by Ketterlin-Geller, et al. (2015) and on focused discussions with mathematics educators and mathematics researchers. The identified knowledge and skills were partitioned into three content specific subtests:

- Quantity Discrimination (QD): The knowledge and skills associated with the QD subtest are related to relationships among and between number systems (e.g., whole numbers, integers, and rational numbers). For students to use algebraic reasoning flexibly with number systems, it is necessary for them to have a solid conceptual understanding (National Mathematics Advisory Panel, 2008).

- Number Properties (NP): The knowledge and skills associated with the NP subtest are related to students' application of number properties (i.e., distributive property, commutative, associative, identity and inverse properties of addition and multiplication, and equality). The correct application of these properties enables students to efficiently solve mathematics problems, understand mathematical algorithms, and build a foundation for symbol manipulation in algebra (Geary et al., 2008).

- Proportional Reasoning (PR): Fluency with number systems and properties enable students to develop fluency with algorithms. Fluency with algorithms allows students to solve more complex problems conceptually instead of relying on procedural calculations. Thus, the knowledge and skills associated with the PR subtest focus on the generalization of algorithmic fluency through application.

Administration Consistent with the RTI model for intervention, a student identified in Tier II or Tier III on a universal screener should be further assessed (e.g., diagnostic assessment or skills inventory) to identify specific areas of growth that can be targeted for intervention. During the intervention period, progress should be monitored. If a teacher determines the ARPM system is the appropriate tool for progress monitoring, the student should be given all three subtests for his/her grade level at each administration. Individually, the subtests are not predictive of algebra readiness; rather, when combined the knowledge and skills represented in the three subtests support teachers in making valid decisions when providing instruction for algebra readiness.

In total, a student is allocated nine minutes to complete the assessment (three subtests). For each subtest probe, students are given three minutes to answer as many questions as possible. Students are given directions to specifically focus on the use of content knowledge and numerical reasoning skills as opposed to simply performing calculations before beginning each probe. Examples of a numerical reasoning strategy that could be used by students and the associated items are shown in *Table 1*.

Table 1

ARPM System Sample Items and Numerical Reasoning Strategies.

Subtest	Quantity Discrimination	Number Properties	Proportional Reasoning
Problem	3.35 ☐ $3\frac{1}{4}$	$3\frac{5}{9}+1\frac{3}{4}$ □ $1\frac{3}{4}+8\frac{5}{9}$	40% of 40 ☐ 40% of 60
Possible Numerical Reasoning Strategy	The common fraction $\frac{1}{4}$ is routinely converted to 0.25, making this comparison of magnitude about recognizing and evaluating values between number systems. Since the whole numbers are the same and 0.35 > 0.25, then 3.35 > 3.25.	Because of the commutative property of addition, the order of the addends does not affect the sum. Therefore, the comparison $a + b$ □ $b + c$ can be determined by comparing a and c. Since $3\frac{5}{9} < 8\frac{5}{9}$, then $3\frac{5}{9}+1\frac{3}{4} < 1\frac{3}{4}+8\frac{5}{9}$.	The same percent, 40%, is specified for both quantities represented in the comparison. Since 40 is less than 60, then 40% of 40 < 40% of 60.

Content Blueprint for the ARPM System

The blueprint for each of the subtests was developed based upon the content standards and underlying knowledge and skills related to algebra readiness. More specifically, the blueprint identified the number systems (e.g., whole numbers, fractions, decimals,) and mathematical operations (e.g., addition, subtraction, multiplication, or division) to be assessed at each grade level through an alignment of the content standards specified in the Common Core State Standards in Mathematics (CCSS-M; National Governors Association & Council of Chief State School Officers, 2010), the Texas Essential Knowledge and Skills (TEKS, 2012), and the Virginia Mathematics Standards of Learning (SOL, 2009).

To facilitate this process, the identified content standards and related knowledge and skills (for each subtest) were placed into categories specific to each subtest. For example, two categories for the Number Properties subtest were identified as: Commutative Property of Multiplication and Commutative Property of Addition. These categories were then broken into subcategories based on the number types to be assessed. For example, two subcategories for the Commutative Property of Multiplication were identified as: Whole Numbers and Fractions.

The purpose of the subcategories was to define the content eligible to be assessed at each grade level. Subcategories that were eligible at a particular grade level were labeled *E*. An example of subcategories and grade levels assessed for the Commutative Property of Multiplication is shown in *Table 2*. Subcategories that were not eligible were labeled *NE*.

Table 2

Blueprint Subcategories for NP Commutative Property of Multiplication

Category: Commutative Property of Multiplication	**Grade level eligibility[a]**		
Subcategory	***G6***	***G7***	***G8***
Whole Numbers	E	E	E
Fractions	NE	E	E
Improper Fractions	NE	E	E
Mixed Numbers	NE	E	E
Decimals that Terminate	E	E	E
Integers	NE	E	E
Negative Fractions	NE	NE	E
Negative Mixed Numbers	NE	NE	E
Negative Improper Fractions	NE	NE	E
Negative Decimals that Terminate	NE	NE	E

[a] Eligible (E); not eligible (NE)

Due to the specific content focus of each subtest, the items that represent the categories and subcategories for each subtest were also unique.

Blueprint Review and Finalization

An external review of the blueprint was conducted before proceeding with the item writing process. The goal of the external review was to: (a) confirm the categories, subcategories, and eligible grade levels; (b) prioritize the subcategories within a category; and (c) engage in discussion about the weighting of each category on a form of the subtest.

The external review was conducted with a mathematics education expert (see Appendix B for the qualifications for Expert Reviewer 1). The expert was provided the blueprint one week prior to a scheduled meeting with the research team. The expert was asked to carefully review the blueprint and make note of any initial observations and questions about the blueprint prior to the meeting. Each subtest was considered separately for the review, but the same guiding questions were used. The guiding questions were:

- What feedback do you have about the categories for this subtest? Are there any categories that are not necessary to be assessed when considering algebra readiness and the format of this assessment? Why?

- Looking at each subcategory within a category, how would you rank order the subcategories when considering algebra readiness fluency?

    - Are there any subcategories that you would eliminate?

    - Are there any subcategories that should receive more weight in a certain grade level?

The discussion with the mathematics education expert resulted in modifications around three themes. These themes included:

- Determinations of whether or not specific comparisons were important to assess for struggling middle schoolers and necessary or useful for algebra readiness, for example fractions (single representation) was identified as a very important skill,

- Determinations of the relative weight of each type of comparison, (i.e., should each type be assessed multiple times or simply one or two times?), and

- Determinations of the relative weight of each comparison at each grade level (6, 7, and 8.)

After revisions were made to the blueprint based on the external review, the research team analyzed each subcategory to determine which answer choices were possible. Our initial goal was to develop each subtest with an equal distribution of the <, >, and = symbols. However, after analyzing the types of items that could be written for each subcategory, an equal distribution was not possible. For example, on the PR subtest, the subcategory of *Equivalent Fraction* only had a possible answer of "equal to". Similarly, comparing integers on the QD subtest would never produce a result of "equal to" while preserving a reasonable level of difficulty. Despite these nuances, every effort was made to stay as close to an equal distribution as possible on each subtest.

Development of Exemplar Items

After finalizing the blueprint and identifying the categories and subcategories to be assessed the development of the 20 probes (parallel forms) began. The items for the first probe were determined to be the *exemplar items*, with a total of 35 developed for each subtest. This development began with the creation of an item template. Then, item designers were hired to develop of the remaining 24 *exemplar items* per subtest. After the *exemplar items* were developed, mathematical education experts conducted a review of the *exemplar items*. Finally, the exemplar items were pilot tested.

Item Template Design

Item designing began with the development of an item template. Each item template served two purposes: (a) creation of the exemplar items and (b) construction of item constraints to be used on the remaining items for the probes. By using a structured item template for the exemplar items, the research team was able to verify that the subtest as a whole had various representations, misconceptions, and strategies represented that aligned with the eligible categories and subcategories for each subtest. The item constraints were used to ensure that the items on the probes were representative of and the same format as the exemplar item. These item constraints also ensured that the items would be theoretically similar in difficulty level across the parallel forms.

To initiate the development process, the research team reviewed the previously developed items by Ketterlin-Geller et al. (2015). These items were previously pilot tested and data from the study was available for further analysis. Approximately five items per subtest that had acceptable item statistics (e.g., difficulty level, mean-square infit and outfit, point biserial) and matched a category and subcategory from the blueprint were selected. These items were used as exemplar items and supported the development and refinement of the item template. The final version of the item template (*Figure 2*) includes the following: the exemplar item, parallel item (additional probe) constraints, and any additional notes necessary to solidify the expectation of the constraints.

Figure 2

Item Template

Exemplar Item: -35 ☐ -72
Parallel Item (additional probe) Constraints: $-ab$ ☐ $-cd$ Additional Notes: $-ab > -cd$ $-100 < ab < -10$ $-100 < cd < -10$

Through this process, critical components were identified that needed to be included in the additional notes on each item template. These critical components were often identified because the perceived difficulty of the parallel items varied. For example, comparing six-tenths (6/10) to three-fourths (3/4) could be considered easier than comparing six-tenths (6/10) to seven-elevenths (7/11) because the denominators 10 and 4, can easily be used to write an equivalent fraction with a denominator of 100. These denominators also lend themselves to converting the fraction to a decimal. However, the denominator of 11 cannot be used to write a fraction with a denominator of 100 or be used to easily convert the fraction to a decimal. Thus, it was determined that an additional note should be added to explicitly state that the denominator would be a factor of 100.

After finalizing the design of the item template, the research team created six additional exemplar items per subtest, per grade level. By creating these, the team was able to facilitate the creation of items that would test for some common misconceptions. These items were reviewed by multiple team members. In addition, this process informed the item designer training, in particular, the refinement of necessary processes and procedures for the remaining item

templates yet to be created. Following this process, item designers were hired to create the remaining 24 exemplar items per subtest, per grade level.

Exemplar Item Development

Item Designers

Applicants were invited to apply to be a designer via an email and website job posting. All applicants were screened based on experience and expertise in teaching mathematics at the sixth through eighth grade level. Priority consideration was given to those with curriculum and assessment design experience. A total of 18 item designers (6 per subtest) were hired, but one item designer was not able to fulfill the requirements of the position. The design work for this contractor was shared among the other designers for that subtest. Item writer biographies can be found in Appendix C.

Item Designer Pre-Training

Before beginning the design process, item designers engaged in a one-hour training webinar. The purpose of the webinar was to: (a) provide item designers with background information on Response to Intervention (RTI) and the importance of progress monitoring systems as a part of the RTI process, (b) provide item designers with an overview of the ARPM project goals, c) share the proposed timeline for development, and (d) outline the roles and responsibilities of each item designer.

Item Template/Exemplar Item Design and Training Workshop

Following the pre-training webinar, the item designers came to the SMU campus for a training and item design workshop. Three workshops were scheduled, one for each of the subtest content areas (QD, NP, and PR). During the selection process, the item designers were assigned to one of the three subtest content areas and attended the appropriate workshop. Each workshop was composed of two sessions: (a) project training, and (b) item design.

The project training session provided the item designers with a more in-depth look at the evidence of the knowledge and skills predictive of algebra readiness and the rationale for the format of the test design. Item designers engaged in an iterative process, working collaboratively to understand how to utilize the item template to create an exemplar item and identify constraints to support the creation of the additional probes. Since twenty probes with items of a similar level of difficulty and numerical reasoning strategy were needed for each exemplar item, the majority of the workshop focused on ensuring that designers understood the goals and expectations for creating an item template.

During the item design session, item designers were given twelve item templates to create. Item templates were provided for the item designers with a specified category, subcategory, and the answer type (<, =, >). After writing the exemplar item and constraints, designers collaborated with another designer as a partner to review their work. During this collaborative review process, the partner utilized the constraints on each item template to write two additional items to the intended constraints. There were three purposes in writing these additional items: (a) to verify

that additional items could be written of similar difficulty; (b) to verify that a similar numerical reasoning strategy could be used to find the answer; and (c) to identify language in the constraints that needed refinement. This collaborative review process also allowed for the item designers to engage in discussion about common strategies that students would use to solve the problems.

After the collaborative review, item designers submitted the item templates to the research team for final approval. In reviewing each item template, the research team first verified that the exemplar item was written for the intended category and subcategory. Then, the two additional items written by the partner were compared to the exemplar item to verify that a similar numerical reasoning strategy could be used and that the level of difficulty across the three items was maintained. A member of the research team then wrote a third additional item that tested the limits of the constraints to ensure that the numerical reasoning strategy and difficulty were still maintained. The research team documented the numerical reasoning strategy that was used to verify that a variety of strategies were represented on the subtest. After this process, items were either approved, or returned to the item designer with feedback and suggestions for revisions. Item designers were encouraged to review the feedback with their collaborative partner to revise and resubmit the template for a final approval. This final review process continued until all item templates were approved by the research team.

Exemplar Item Review by Mathematics Education Experts

A research team member not present at the workshop verified the item templates created by the item designers one additional time. During this review, the research team member verified the following: (a) category, (b) subcategory, and (c) the numerical reasoning strategies used to solve the problems varied. During the review by the research team member, an overlap in strategies and content across the grade levels was determined. In order to address this overlap, it was determined that a subset of the item templates would be reviewed externally by mathematics education experts. This subset of item templates represented approximately 25% of the item templates. The exemplar items that were selected for external review were representative of the categories and subcategories at each grade level for each subtest.

The subset of item templates was reviewed by two mathematics education experts (see Appendix B for the qualifications for both experts). For each item template, the mathematics education experts were each asked to select *agree* or *disagree* for the following statements:

- The targeted category and subcategory is assessed.
- The four items assess the skills at a similar level of difficulty.
- A similar numerical reasoning strategy can be used on all four items.
- The intended correct answer is true for all four items.

If the mathematics education experts *disagreed* with any of the statements, they were asked to provide a rationale and recommend possible revisions. The results of the review are summarized below by subtest.

QD Subtest

The mathematics education experts' ratings for the QD subtest can be seen in *Table 3*. Overall, the ratings indicate that the items on each template for the QD subtest assess the targeted category and subcategory, assess the skills at a similar level of difficulty, use a similar numerical reasoning strategy, and have the same correct answer.

Table 3

Mathematics Education Experts' Ratings for Quantity Discrimination Subtest

	Expert Reviewer 1		Expert Reviewer 2	
Review Criteria	Agree	Disagree	Agree	Disagree
The targeted category and subcategory is assessed.	100%	0%	100%	0%
The four items assess the skills at a similar level of difficulty.	92%	8%	85%	15%
A similar numerical reasoning strategy can be used on all four items.	92%	8%	92%	8%
The intended correct answer is true for all four items.	96%	4%	100%	0%

While the mathematics education experts *agreed* with most statements, the written feedback provided by the expert reviewers was used to modify and improve the items. Out of the items reviewed by the mathematics education experts, 15% of the Grade 6 items, 12% of the Grade 7 items, and 15% of the Grade 8 were modified in some way. The primary reason for modification was to ensure that the constraints were written so that the difficulty level of the items for each template remained similar across the parallel forms of the subtest. More specifically, the mathematics education experts noted instances when some items may be easier, such as when fractions with a more recognizable denominator of 2, 5, or 10 were compared to fractions with denominators of 7, or 9. The reviewers also commented when quantities that could be simplified in one step were compared with quantities requiring two steps to simplify, citing differing levels of difficulty to complete. Additionally, some items were adjusted for increased clarity and one item needed to be reconceptualized.

Since only a subset of the items were reviewed by the experts, their comments were also used to modify the constraints of related items not reviewed to better ensure the items written for each template would remain similar in difficulty level across the subtest probes. For example, because the experts noted that some denominators are more recognizable than others, the items not directly reviewed by the experts were internally examined and modified, if needed, to ensure that the denominators were similar in difficulty within each item across the probes.

NP Subtest

The mathematics education experts' ratings for the NP subtest can be seen in *Table 4*. Overall, the ratings indicate that the items on each template for the NP subtest assess the targeted category and subcategory, assess the skills at a similar level of difficulty, use a similar numerical reasoning strategy, and have the same correct answer.

Table 4

Mathematics Education Experts' Ratings for Number Properties Subtest

	Expert Reviewer 1		Expert Reviewer 2	
Review Criteria	Agree	Disagree	Agree	Disagree
The targeted category and subcategory is assessed.	87%	13%	90%	10%
The four items assess the skills at a similar level of difficulty.	100%	0%	84%	16%
A similar numerical reasoning strategy can be used on all four items.	100%	0%	87%	13%
The intended correct answer is true for all four items.	97%	3%	100%	0%

While the mathematics education experts *agreed* with most statements, the written feedback provided by the expert reviewers was used to modify and improve the items. Out of the items reviewed by the experts, 50% of the Grade 6 items, 27% of the Grade 7 items, and 29% of the Grade 8 were modified in some way. The primary reason for modification was to ensure that the constraints were written so that the difficulty level of the items for each template remained similar across the parallel forms of the subtest. More specifically, the mathematics education experts' noted instances when some items may be easier, such as when simplified fractions were used instead of non-simplified fractions or when more familiar multiplication facts were utilized, such as doubles. The reviewers also commented when the number of digits may affect the difficulty level of an item (e.g., fractions with two-digit denominators versus fractions with one-digit denominators).

Since only a subset of the items were reviewed by the experts, their comments were also used to modify the constraints of related items not reviewed to better ensure the items written for each template would remain similar in difficulty level across the subtest probes. For example, all exemplar items were reviewed to ensure that the constraints specified whether fractions should be simplified or non-simplified, since the experts noted this may impact the difficulty level of an item.

PR Subtest

The mathematics education experts' ratings for the PR subtest can be seen in *Table 5*. Overall, the ratings indicate that the items on each template for the PR subtest assess the targeted category and subcategory, assess the skills at a similar level of difficulty, use a similar numerical reasoning strategy, and have the same correct answer.

Table 5

Mathematics Education Experts' Reviewer Ratings for Proportional Reasoning Subtest

	Expert Reviewer 1		Expert Reviewer 2	
Review Criteria	Agree	Disagree	Agree	Disagree
The targeted category and subcategory is assessed.	100%	0%	96.3%	3.7%
The four items assess the skills at a similar level of difficulty.	67%	33%	74.1%	25.9%
A similar numerical reasoning strategy can be used on all four items.	100%	0%	88.9%	11.1%
The intended correct answer is true for all four items.	100%	0%	100%	0%

While the mathematics education experts *agreed* with most statements, the written feedback provided by the expert reviewers was used to modify and improve the items. The primary reason for modification was to ensure that the constraints were written so that the difficulty level of the items for each template remained similar across the parallel forms of the subtest. Specifically, the experts noted instances when some fractions or percentages may be easier to compare by reasoning. For example, items with common fractions (e.g., 1/2, 1/3, 2/8) were considered easier than items with fractions involving multi-digit denominators. Reviewers also noted items that were more likely to prompt calculations than reasoning strategies. These comments were used to modify the constraints on 34% of all of the items to better ensure the items written for each template would remain similar in difficulty level across the parallel forms of the subtest.

Since only a subset of the items were reviewed by the experts, these comments were applied to the constraints of related items not reviewed to better ensure the items written for each template would remain similar in difficulty level across the subtest probes.

Exemplar Item Pilot Study

Before developing the items for the parallel forms, the exemplar items were pilot tested to examine their technical adequacy and to assist with final item selection.

Sample

Students in Grades 6-8 from one middle school in a high-achieving school district in Texas participated in the pilot study. Students were recruited at the classroom level, and participation was voluntary. The number of pilot test participants by grade level and subtest can be found in *Table 6*.

Table 6

Pilot Test Participants

	Number of Participants		
ARPM Subtest	Grade 6	Grade 7	Grade 8
Number Properties	42	52	39
Quantity Discrimination	34	56	38
Proportional Reasoning	43	59	39

Procedures

Each student was administered one subtest in an online testing environment using an iPad. Trained test administrators read standardized instructions to the students prior to beginning the assessment. The pilot test was not timed; however, timing data was collected to determine the average length of time per item (see *Table 7* in the *Results and Finalization of Form A Items* section for more information). While the test was untimed, the test was completed in one class period. Students were not allowed to revisit items or go backward in the online testing environment.

The pilot study was conducted at the end of the fall semester.

Analysis

The results from the pilot study were analyzed using the Rasch model. The Rasch model is a one-parameter logistic item response theory (IRT) model that estimates the probability of a correct response based on a person's ability level and an item's difficulty level. The item difficulty parameters were estimated using the Winsteps Rasch measurement computer program (Linacre, 2012). Other item statistics, including discrimination, point measure, and mean squares infit and outfit estimates, were also calculated in Winsteps and used to examine the technical adequacy of the items.

Results and Finalization of Exemplar Items

The results from the pilot test were used to select 30 of the 35 exemplar items on each of the subtests. Only the 30 selected items for each subtest had an additional 19 parallel forms created (for a total of 20 parallel forms). The 5 lowest functioning items were removed from each subtest. Ranges and cutoffs for acceptable item statistics were developed to determine which items to keep as exemplars. The data criteria used to select items can be found in *Table 7*. These criteria were modified from typical ranges to account for low sample sizes.

Table 7

Data Criteria for Item Selection

Item Statistic	Acceptable Range	
	Low	High
Item difficulty	-4	4
Mean-square infit	0	2
Mean-square outfit	0	2
Discrimination	0.5	2
Point measure	0.15	No maximum

A summary of the item statistics for the exemplars (selected items) is shown in *Table 8*.

Table 8

Summary of Item Statistics for Items Selected as Exemplars

	ARPM Subtest								
	Number Properties			Quantity Discrimination			Proportional Reasoning		
Grade	6	7	8	6	7	8	6	7	8
Mean item difficulty (SD)	0.03 (0.82)	-0.14 (1.45)	-0.11 (1.10)	-0.13 (1.54)	0.03 (1.00)	0.27 (1.32)	-0.08 (1.24)	0.03 (1.04)	-0.01 (1.30)
Min item difficulty	-1.36	-2.52	-1.95	-2.36	-1.77	-2.73	-2.21	-1.75	-1.85
Max item difficulty	2.76	3.35	1.9	3.74	2.06	2.85	2.46	1.6	3.33
Average mean-square infit (SD)	0.98 (0.10)	0.98 (0.09)	0.97 (0.11)	0.93 (0.19)	0.97 (0.09)	1.00 (0.12)	0.94 (0.15)	0.95 (0.12)	0.95 (0.11)
Average mean-square outfit (SD)	0.99 (0.28)	0.95 (0.17)	0.97 (0.23)	0.80 (0.44)	0.92 (0.23)	1.03 (0.32)	0.87 (0.32)	1.00 (0.26)	0.87 (0.31)
Average discrimination (SD)	1.02 (0.20)	1.00 (0.19)	1.08 (0.26)	1.07 (0.27)	1.05 (0.19)	1.01 (0.27)	1.10 (0.29)	1.09 (0.29)	1.07 (0.22)
Average point measure (SD)	0.28 (0.05)	0.28 (0.06)	0.34 (0.05)	0.38 (0.12)	0.33 (0.08)	0.34 (0.08)	0.33 (0.07)	0.37 (0.07)	0.34 (0.08)

The full version of the item statistics for the selected items can be found in Appendices D-L.

As previously noted, timing data was also recorded to determine the average time spent on each item. These results are available on request.

Item-person maps were also created in Winsteps (Linacre, 2012) to examine the distribution of the student abilities and item difficulties. These diagrams area available upon request.

Development of Parallel Items

After the pilot test was conducted and the 30 exemplar items for each subtest was determined from the data, we then began the development of the parallel forms.

Parallel Item Designing

Training

In preparation for utilizing each item template to expand the exemplar items into 20 parallel forms, a Parallel Form Item Design Guide (PFIDG) was created. The PFIDG outlined the procedures for designing parallel forms for an exemplar item, procedures for electronic submission of each set of items and expectations for communication with the RME team throughout the design and review process. The PFIDG was provided to item designers one week before a webinar training session. The focus of the half-hour webinar was to refresh the designers on how to use the item templates they previously designed to write the additional parallel form items and to practice the design expectations using a sample item. Group discussion was facilitated on the webinar to ensure all item designers understood the expectations. Additionally, since this design work would be done remotely, designers were trained in the procedures and expectations for submitting their items.

Item Design and Internal Review Procedures

To create the parallel forms for each item template, item designers engaged in an eight-week process to submit and finalize items. Each designer was given three item templates to write the parallel form items to within a week. Each group of three item templates was called an *item set*. A detailed work flow chart can be seen below in *Table 9*. Set A was developed and submitted by the item designers in week one. Then, in week two, the RME Staff reviewed the item template and accompanying parallel form items using the following criteria:

- Does each parallel form item adhere to the constraints on the item template?
- Does each parallel form item have a similar level of difficulty in comparison to the exemplar item?
- Can a similar numerical reasoning strategy be applied to each parallel form item based on the intent of the exemplar item?
- Are all parallel form items unique (e.g., is any single item repeated within the 20 items)?

After reviewing set A, feedback was provided to item designers to ensure that the expectations for design were understood and additional training was provided on an individual basis. Item designers were given an opportunity to revise set A during week three and also began

development of set B. RME staff worked with each designer to ensure that they were confident and competent in the process and procedures before continuing.

Table 9

Parallel Form Development Work Flow

Item Set	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
A	Design items	Review Items	Revise Items	Finalize Items				
B			Design items	Review Items	Revise Items	Finalize Items		
C				Design items	Review Items	Revise Items	Finalize Items	
D					Design items	Review Items	Revise Items	Finalize Items

Parallel Item Finalization

At the end of the eight-week period, the parallel items were finalized. Because so many items were being created simultaneously and within each domain there was potential for some overlap, the researchers determined that it was important to verify that no item was inadvertently duplicated during the item writing process. Templates with possible overlap were identified and a member of the research team compared the paper copies to the electronic database within the electronic database to search for duplicate items. If a duplicate item was discovered, it was modified (adhering to constraints) and reentered into database.

Following the identification and changes made as a result of any duplicate items found, the team conducted one last item review. A subset of the item templates was selected and reviewed by the research team. This subset represented approximately 25% of the item templates. The exemplar items that were selected were representative of the categories and subcategories at each grade level for each subtest. The following criteria were used for the review:

- Does each parallel form item adhere to the constraints on the item template?
- Does each parallel form item have a similar level of difficulty in comparison to the exemplar item?
- Can a similar numerical reasoning strategy be applied to each parallel form item based on the intent of the exemplar item?
- Are all parallel form items unique (e.g., is any single item repeated within the 20 items)?

The results of the review can be seen in *Table 10*.

Table 10

Parallel Item Finalization Review

	Grade Level	Number of Item Templates Reviewed	Number of Items Reviewed	Number of Items Modified
Number Properties	6	8	160	8
	7	8	160	3
	8	9	180	5
Quantity Discrimination	6	9	180	1
	7	8	160	9
	8	8	160	2
Proportional Reasoning	6	9	180	4
	7	11	220	8
	8	10	200	10

As reflected in *Table 10*, only a few items were modified during this final round of review. Most modifications involved changing the numerical values within a template to ensure the items had a similar level of difficulty. A small number of items were modified to ensure that a similar numerical reasoning strategy could be used to solve each of the items on the template.

Conclusion

In this technical report, we described the development of an Algebra Readiness Progress Monitoring (ARPM) system for students in Grades 6-8 for Imagination Station (Istation). The ARPM system, delivered through the ARPM probes or parallel assessment forms, support teachers' instructional decision-making and support students in tracking their personal growth. State and national mathematics content standards related to algebra readiness inform the mathematics topics' underlying the ARPM probes. In this technical report we described (a) the process used to identify the mathematics content assessed in the ARPM system and specifically, the format and content of the ARPM probes, (b) the ARPM system blueprinting process and the development of the exemplar items, and (c) the processes and reviews conducted for the creation of the ARPM probes.

References

American Educational Research Association, American Psychological Association, & National Council on Measurement in Education. (2014). *Standards for educational and psychological testing*. Washington, DC: American Psychological Association.

Geary, D. C., Boykin, A. W., Embretson, S., Reyna, V., Siegler, R., Berch, D. B., & Graban, J. (2008). *Chapter 4: Report of the task group on learning processes*. Retrieved from http://www.ed.gov/about/bdscomm/list/mathpanel/report/learning- processes.pdf

Kelley, B., Hosp, J. L., & Howell, K. W. (2008). Curriculum-based evaluation and math: An overview. *Assessment for Effective Intervention*, *33*, 250-256.

Ketterlin-Geller, L. R., Gifford, D. B., & Perry, L. (2015). Measuring middle school students' algebra readiness: Examining validity evidence for three experimental measures. *Assessment for Effective Intervention*, *41*(1), 28-40.

Linacre, J. M. (2012). Winsteps Rasch measurement computer program. Beaverton, OR: Winsteps.com.

National Governors Association & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Research in Mathematics Education. (2016). *Research in Mathematics Education Cousin Design Style Guide*. Dallas, TX: Southern Methodist University.

Appendix A: State Content Standards Referent Sources

Texas
The Texas Essential Knowledge and Skills (adoption 2012) were retrieved from:
http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html

Common Core Standards
The Common Core Standards in Mathematics were retrieved on May 10, 2015 from www.corestandards.org/the-standards/mathematics. These standards were published in 2010. They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

Virginia
Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved from
http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics

Appendix B: Mathematics Education Expert Biographies

Mathematics Education Expert 1 holds a Doctoral degree in Educational Curriculum and Instruction, a Master's degree in Mathematics Education, and a Bachelor's degree in Elementary and Secondary Education. She has experience working in education teaching Grade 3, post-secondary mathematics, and mathematics methods for pre-service teachers. She is currently a university professor and Associate Dean in the Mathematics Department. Her curriculum design experience includes serving as the chair of the National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points writing committee and as a participant in Texas state curriculum design.

Mathematics Education Expert 2 holds a Doctoral degree in Curriculum & Instruction - Mathematics Education, a Master's degree in Mathematics, and a Bachelor's degree in Special Education. She has experience as a professor, mathematics specialist, and teacher. She has directed multiple mathematics research centers across the country, and has published a multitude of nationally known mathematics education books and articles. She is currently a research professor in mathematics education and a co-Principal Investigator of two algebra-research projects.

Appendix C: Item Designer Biographies

Item Designer 1 holds a Master's degree in Secondary Education with an emphasis in Mathematics Education and a Bachelor's degree in Computer Science with a minor in German. She has experience as a mathematics teacher, and as an instructional mathematics coach. She is currently an instructor at the University of North Texas where she teaches mathematics courses.

Item Designer 2 holds a Master's degree in Education and a Bachelor's degree in Elementary Education. She has experience as a mathematics educator and mathematics specialist. She is currently a mathematics instructional dean and instructional facilitator where she facilitates the design and delivery of summer school mathematics curriculum in her district.

Item Designer 3 holds a Master's degree in Curriculum & Instruction Cognate: Mathematics, a Bachelor's degree in Elementary Education with Specialization in Gifted and Talented Education, and certifications in Gifted and Talented Education and General Education (EC-6). As a graduate assistant, she worked on the GEAR UP Math Initiative project where she participated in educational research, wrote curriculum for math summer camps, and worked with high school students in multiple districts. She is currently a 6th grade mathematics teacher.

Item Designer 4 holds a Master's degree in Educational Administration, a Bachelor's degree of Interdisciplinary Studies with a concentration in Mathematics, an Associate's Degree in Science, and certifications in Principal Administration, Mathematics Education (4-8), General Education (4-8), and ESL. She has experience as a 6th and 7th grade math teacher and an Instructional Dean of Math. She is currently a middle school math specialist.

Item Designer 5 holds Master's degrees in Educational Administration, and Curriculum and Instruction, a Bachelor's degree in Middle School Mathematics Education, and certifications in Principal Administration, in Mathematics Education, and TEI. She has experience as a master teacher in mathematics. She is currently an assistant principal where she works closely with her principal and leads weekly professional learning communities.

Item Designer 6 holds a Master's degree in Education Administration, a Bachelor's degree in Mathematics with a minor in Education and certificates in Secondary Mathematics, Principal Administration, and Superintendent Administration. She has experience teaching mathematics including Algebra I, Algebra II, and Geometry. She is currently a principal where her duties include coordinating with instructional facilitators, working closely with special education, and evaluating effectiveness of classroom instruction.

Item Designer 7 holds a Masters in Teaching with certifications in Mathematics Education (4-8) and Physical Education, and a Bachelor's degree in Sociology. She has experience teaching 6th and 8th grade math honors classes and 7th grade support math courses. She is currently a math teacher and sports coach at the middle school level.

Item Designer 8 holds a Master's degree in Math Education, Bachelor's degrees in Political Science and Accounting, and certifications in Generalist Subjects (1-6), and Mathematics Education (4-8). She has experience as a tutoring initiative math trainer, math school teacher,

math instructional facilitator, and math instructional specialist. She is currently a math instructional coach at the elementary school level.

Item Designer 9 holds a Master's degree in Educational Administration, a Bachelor's degree in Bioenvironmental Sciences, and certifications in Mathematics for grade levels 4-8 and 8-12. She has experience teaching Algebra 1, Geometry, and Algebra 2, has served as a math instructional specialist, and as a curriculum writer for STAAR exams. She is currently a STEM instructional specialist.

Item Designer 10 holds a Master's degree in Educational Administration and a Bachelor's degree in Chemical Engineering. She has experience as a math teacher, instructional coach, and a math methods teacher. She currently works as a graduate research assistant, an instructional specialist, and an adjunct instructor at the University of North Texas and Dallas County Community College District.

Item Designer 11 holds a Bachelor's degree in Finance, and a Certification in Education Grades 4-8. He has experience in education as a teacher, mathematics educator, instructional technology specialist, and in his current role as a mathematics department chair. He has experience developing and implementing professional development lessons for his mathematics department.

Item Designer 12 holds a Bachelor's degree in Speech Communications and holds certifications in General Education (4-8), Mathematics Education (4-12) and a ESL (4-8). She has experience in education as a teacher, mathematics coach, and in her current role as a mathematics specialist. In her work as a mathematics specialist, she has developed assessments and worked to create programs to support struggling learners.

Item Designer 13 holds a Bachelor's degree in Interdisciplinary Studies with an emphasis on Reading, and certifications in Mathematics (4-8), English as a Second Language (PK-12), Elementary Reading (1-8), and Generic Special Education (PK-12). He has experience teaching special education, English as a second language, and mathematics (6-8). He is currently an 8th grade math teacher department chair.

Item Designer 14 holds a Bachelor's degree in Kinesiology and is certified in Mathematics (4-8) and Physical Education (EC-12). She has been an educator for K-8 in mathematics and physical education. She is currently a math department head, an 8th grade pre-algebra teacher, and a 7th grade pre-AP math teacher.

Item Designer 15 holds Bachelor's degrees in Sociology and Biology and is certified in Mathematics Education (4-8). She has experience as an AVID tutor where she worked with students to facilitate group interaction and problem solving. She currently teaches 7th grade pre-AP and 8th grade math.

Item Designer 16 holds a Bachelor's degree in Business Administration, Finance with a minor in Marketing, and has certifications in Mathematics Education (4-8), General Education (4-8), and English as a Second Language. She has experience as an 8th grade math teacher. She is currently a 6th grade math teacher.

Item Designer 17 holds a Bachelor's degree in Business Administration with an emphasis in Organizational Behavior and Human Resource Management, and credentials in General Education (4-8 and EC-6). He is currently a mathematics teacher at the middle school level.

Appendix D: Item Statistics for Selected Grade 6 Number Properties Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	-0.77	1.0193	0.8147	1.0175	0.2407	Distributive	Decimals
2	-0.53	0.8765	0.633	1.1634	0.2438	Commutative: Addition	Fractions
4	0.16	0.8891	0.7886	1.2008	0.2834	Inverse: Multiplication	Whole numbers
6	-0.63	1.0221	1.0795	0.9677	0.2393	Distributive	Whole numbers
8	0.02	0.8722	0.7035	1.2401	0.2807	Inverse: Multiplication	Whole numbers
9	-1.32	1.0776	1.7256	0.894	0.1844	Associative: Addition	Whole numbers
10	0.81	1.1221	1.1226	0.6162	0.3423	Distributive	Whole numbers
11	-0.2	0.9451	0.7434	1.1249	0.2688	Commutative: Addition	Mixed numbers
12	0.35	0.9827	0.9784	1.0358	0.3061	Commutative: Multiplication	Decimals
14	0.49	1.2014	1.2589	0.5152	0.3058	Associative: Multiplication	Decimals
15	-0.77	1.0309	0.9965	0.976	0.2163	Distributive	Whole numbers
17	0.57	1.0701	1.1173	0.7955	0.3276	Distributive	Decimals
18	0.42	0.8453	0.7344	1.3825	0.308	Associative: Addition	Decimals
19	-1.36	1.0711	1.4919	0.9108	0.1785	Inverse: Multiplication	Whole numbers
20	-0.52	1.1244	1.5919	0.8079	0.2403	Associative: Multiplication	Whole numbers
20	-0.06	0.9323	0.8287	1.1231	0.2756	Distributive	Decimals
21	-0.12	0.882	0.7834	1.1533	0.2597	Distributive	Decimals
22	0.58	0.9837	0.9218	1.0757	0.3175	Inverse: Multiplication	Whole numbers
23	-0.24	0.8226	0.6696	1.2423	0.2624	Identity: Addition	Fractions
24	0.47	0.8589	0.8176	1.3064	0.3037	Associative: Addition	Mixed numbers
25	1.25	1.0383	1.0679	0.7989	0.3546	Distributive	Whole numbers
26	0.59	0.8805	0.9128	1.2644	0.3273	Identity: Multiplication	Decimals
27	-0.89	1.0524	1.2918	0.9231	0.205	Commutative: Addition	Improper fractions
28	0.05	0.8793	0.917	1.1553	0.2806	Commutative: Multiplication	Decimals
29	0.44	0.9963	1.0623	0.9722	0.3183	Associative: Multiplication	Decimals
31	2.76	1.0348	1.2024	0.9176	0.3855	Distributive	Whole numbers
32	-0.58	0.8636	0.6193	1.1694	0.2378	Commutative: Addition	Decimals
33	-0.74	0.8777	0.7128	1.1227	0.2268	Inverse: Multiplication	Whole numbers
34	0.59	1.0039	0.9517	1.0229	0.3273	Associative: Multiplication	Decimals
35	0.08	1.116	1.2145	0.8147	0.2796	Distributive	Decimals

Appendix E: Item Statistics for Selected Grade 7 Number Properties Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	0.77	0.9494	0.9383	1.2421	0.3345	Associative: Multiplication	Improper
2	-2.52	0.9788	0.9306	1.0111	0.1334	Distributive	Integers
3	1.42	1.0737	1.093	0.7598	0.3519	Associative: Addition	Fractions
4	-1.75	1.0455	0.8312	0.9924	0.2026	Commutative: Addition	Whole numbers
5	2.9	0.9871	1.517	0.9253	0.2805	Distributive	Mixed numbers
6	-1.81	1.0284	0.9853	0.9804	0.1995	Identity: Multiplication	Fractions
7	-0.61	1.0944	0.9555	0.915	0.2902	Inverse: Multiplication	Improper
9	-0.68	0.9469	0.9214	1.0619	0.2751	Associative: Addition	Whole numbers
10	0.77	1.0416	1.0399	0.8272	0.3526	Associative: Multiplication	Mixed numbers
11	-1.12	0.9487	0.7525	1.0781	0.2567	Inverse: Addition	Integers
12	-0.49	0.7834	0.6415	1.3508	0.2951	Associative: Addition	Mixed numbers
14	-0.62	0.9951	0.9127	1.0215	0.2879	Commutative: Addition	Improper
15	-1.62	0.9132	0.8209	1.0735	0.2117	Identity: Multiplication	integers
16	0.09	0.9672	0.9672	1.0891	0.3342	Inverse: Multiplication	Improper
17	1.31	0.893	0.8908	1.3439	0.3552	Commutative: Multiplication	Integers
19	0.47	1.1725	1.1322	0.405	0.3381	Associative: Multiplication	Decimals
21	0.56	0.9986	0.975	1.0226	0.3529	Identity: Addition	Improper
22	-1.78	0.8505	0.8297	1.0956	0.2057	Commutative: Multiplication	Mixed numbers
23	-1.86	0.8808	0.5999	1.1025	0.1951	Inverse: Multiplication	Fractions
24	2.4	0.9432	0.8395	1.0844	0.3154	Distributive	Decimals
25	-1.03	0.8294	0.7559	1.1677	0.2601	Commutative: Addition	Integers
26	3.35	0.945	0.9653	1.0244	0.2435	Distributive	Fractions
27	1.01	0.9685	1.0324	1.0658	0.3401	Inverse: Multiplication	Improper
28	0.38	1.1406	1.0797	0.5703	0.337	Associative: Addition	Integers
29	-0.86	0.9557	0.8249	1.069	0.273	Associative: Multiplication	Whole numbers
30	-1.56	1.0716	1.0717	0.9498	0.2092	Distributive	Fractions
31	-0.06	1.0724	1.1157	0.8261	0.3259	Commutative: Multiplication	Decimals
32	-0.13	1.0825	1.0715	0.8229	0.318	Associative: Addition	Integers
34	-1.17	1.038	0.8996	0.9863	0.2486	Inverse: Multiplication	Improper
35	-0.06	0.9408	1.0059	1.0992	0.3268	Distributive	Fractions

Appendix F: Item Statistics for Selected Grade 8 Number Properties Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	0.49	0.9006	0.8761	1.3538	0.393	Distributive	Whole numbers
3	-1.18	1.0457	0.9074	0.9751	0.2968	Inverse: Multiplication	Improper fractions
4	1.09	1.0143	0.9927	0.9756	0.3963	Associative: Addition	Negative fraction
5	0.55	0.7611	0.7126	1.8546	0.3888	Distributive	Decimals that terminate
6	-1.95	1.015	1.2637	0.9503	0.2286	Commutative: Addition	Decimals that terminate
7	-1.42	0.798	0.5248	1.258	0.2703	Identity: Multiplication	Fractions
8	1.9	0.8133	0.6623	1.2869	0.3537	Associative: Multiplication	Negative mixed numbers
9	-0.52	0.7253	0.6039	1.5971	0.3526	Identity: Addition	Improper fractions
10	0.82	0.9635	0.9845	1.0989	0.3807	Inverse: Multiplication	Negative fractions
12	-1.68	0.9288	0.7497	1.0834	0.2517	Commutative: Multiplication	Whole numbers
13	-0.43	0.8732	0.7669	1.332	0.3487	Commutative: Addition	Integers
14	-0.05	0.9825	0.9684	1.0487	0.3726	Inverse: Addition	Negative improper
15	-1.41	1.0212	1.4932	0.9183	0.2742	Inverse: Multiplication	Improper
16	0.21	0.981	1.0093	1.044	0.3739	Associative: Multiplication	Integers
17	0.66	1.0039	1.0367	0.9469	0.3795	Identity: Multiplication	Negative decimals
18	-0.85	1.15	1.4548	0.6787	0.3207	Distributive	Negative fractions
20	0.31	1.0343	1.098	0.823	0.384	Associative: Addition	Mixed numbers
21	0.33	0.8939	0.8544	1.3805	0.3909	Inverse: Multiplication	Improper fractions
22	-0.57	0.9628	0.84	1.1247	0.3393	Identity: Addition	negative decimals
23	1.44	0.824	0.8969	1.2931	0.3767	Distributive	integers
25	1.31	1.1441	1.1719	0.6819	0.3816	Associative: Addition	Negative improper
26	-1.02	1.0521	1.3095	0.854	0.3115	Distributive	Fractions
27	0.27	0.9663	0.9202	1.159	0.3774	Commutative: Addition	Negative mixed numbers
28	-1.93	0.9154	0.772	1.078	0.2259	Commutative: Multiplication	Negative decimals
29	-0.79	1.0285	1.0376	0.9429	0.3379	Inverse: Addition	Negative mixed numbers
30	1.15	1.0694	1.0978	0.8065	0.3866	Distributive	Negative decimals
31	-0.7	0.9339	0.8016	1.1673	0.3313	Inverse: Multiplication	Improper
32	1.32	1.125	1.1212	0.7416	0.3736	Associative: Multiplication	Negative fraction
33	-1.38	1.0212	1.0235	0.9822	0.2851	Distributive	Improper fractions
35	0.67	1.0366	1.0021	0.9089	0.3899	Distributive	Negative decimals

Appendix G: Item Statistics for Selected Grade 6 Quantity Discrimination Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	1.3	0.4767	0.3749	1.8571	0.5336	Fractions and decimals (+)	Decimal that terminates: Mixed number
2	-1.44	0.9355	0.6109	1.0458	0.2649	Integers and fractions/decimals (-)	Integer (-): Integer (-)
3[a]						Types of fractions: Single representation (+)	Mixed number: Improper fraction
4	3.74	0.9141	1.6474	0.9429	0.5436	Fraction/decimals and percents: Mixed representation (+)	Decimal that terminates: Percent > 100
5	-0.15	0.7397	0.499	1.2799	0.397	Types of fractions: Single representation (+)	Fraction: Fraction
7	-1.51	0.9594	0.6739	1.0243	0.2663	Integers and fractions/decimals (-)	Integer (-): Integer (-)
8	0.31	0.8604	0.6989	1.1898	0.4406	Types of fractions: Single representation (+)	Mixed number: Improper fraction
10	-0.86	0.7093	0.2878	1.2678	0.3257	Percents (+)	Percent containing a decimal: Percent containing a decimal
11[a]						Types of fractions: Single Representation (+)	Fraction: Fraction
12	-0.71	1.1246	1.0403	0.9002	0.3431	Types of Fractions: Single representation (+)	Improper fraction: Mixed number
13	-0.4	1.1572	1.4568	0.8035	0.3805	Fraction/decimals and percents: Mixed representation (+)	Decimal that terminates: Percent containing a decimal
14	2.05	1.2007	1.4979	0.5438	0.5594	Fraction/decimals and percents: Mixed representation (+)	Percent < or = 100: Decimal that terminates
15	1.01	0.6403	0.5521	1.5591	0.5135	Fractions and decimals (+)	Fraction: Decimal that terminates
16	-2.19	0.8622	0.2225	1.117	0.1767	Positive to negative	Whole number: Integer (-)
17	-0.65	1.0804	1.7066	0.7851	0.3599	Percents (+)	Percent < or = 100: Percent containing a decimal

18	-0.27	1.2405	1.1907	0.7686	0.4038	Types of fractions: Single representation (+)	Improper fraction: Improper fraction
19	-2.35	0.9096	0.2582	1.0938	0.1835	Positive to negative	Integer (-): Whole number
20	1.14	1.2007	1.0729	0.7087	0.5144	Types of fractions: Single representation (+)	Fraction: fraction
21	-1.29	0.6908	0.207	1.2436	0.295	Integers and fractions/decimals (-)	Integer (-): Integer (-)
22	-0.4	1.0387	0.8357	0.9764	0.3805	Types of fractions: Single representation (+)	Mixed number: Mixed number
24	1.14	1.0706	0.9906	0.8984	0.5144	Fraction/decimals and percents: Mixed representation (+)	Fraction: Percent < or = 100
25	0.5	0.8681	0.5883	1.2503	0.4622	Fractions and decimals (+)	Improper fraction: Decimal that terminates
26	-0.71	0.8391	0.6835	1.1304	0.3462	Percents (+)	Percent containing a decimal: Percent containing a decimal
27	0.09	0.8404	0.633	1.2133	0.4138	Types of fractions: Single representation (+)	Mixed number: Mixed number
28	-0.03	0.785	0.5473	1.2749	0.4214	Fractions and decimals (+)	Mixed number: Decimal that terminates
29	-1.44	1.2076	0.9667	0.8618	0.2553	Fraction/decimals and percents: Mixed representation (+)	Fraction: Percent < or = 100
30	3.17	0.8647	0.9449	1.1259	0.5717	Fractions and decimals (+)	Decimal that terminates: Fraction
32	-2.36	0.909	0.2533	1.0939	0.1811	Integers and fractions/decimals (-)	Integer (-): Integer (-)
33[a]						Types of fractions: Single representation (+)	Mixed number: Mixed number
34	-1.42	0.9836	1.1326	0.9605	0.2706	Percents (+)	Percent containing a decimal: Percent < or = to 100

[a]Item statistics for original item were unsatisfactory. An item from the existing pilot testing conducted by Ketterlin-Geller, et al. (2015) was substituted for this item.

Appendix H: Item Statistics for Selected Grade 7 Quantity Discrimination Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	0.76	0.7639	0.6689	1.6308	0.3947	Fractions and decimals (+)	Decimal that terminates: Mixed number
2	0.26	1.0369	0.9061	0.975	0.3625	Types of fractions: Single representation (+)	Mixed number: Mixed number
3	-0.38	0.9825	1.1285	0.9798	0.3027	Decimals or fractions: Single representation (-)	Negative fraction: Negative fraction
4	0.08	0.8984	0.7876	1.1769	0.3409	Integers and fractions/decimals (-)	Negative integer: Negative fraction
5	-1.75	0.9368	0.5484	1.0659	0.174	Decimals or fractions: Single representation (-)	Negative decimal that terminates: Negative decimal that terminates
6	-0.02	1.0341	0.9107	0.9827	0.33	Fractions and percents (+)	Percent > 100: Mixed number
8	0.65	1.0864	1.2452	0.7489	0.3946	Decimals or fractions: Single representation (-)	Negative improper fraction: Negative improper fraction
11	0.67	0.9957	0.8855	1.0612	0.3892	Types of Fractions: Single representation (+)	Mixed number: Improper fraction
12	0.19	1.1682	1.1848	0.7253	0.3467	Types of Fractions: Single representation (+)	Improper fraction: Improper fraction
14	-0.41	0.8738	0.7404	1.1511	0.2997	Types of Fractions: Single representation (+)	Mixed number: Mixed number
15	-0.63	0.9662	0.9131	1.0275	0.2712	Positive to negative	Decimal that terminates: Integer (-)
16	0.81	1.0488	0.9947	0.8936	0.3986	Fractions and decimals (+)	Decimal that terminates: Fraction
17	-0.33	0.88	0.6425	1.1819	0.2993	Integers and fractions/decimals (-)	Integer (-): Negative mixed number
18	0.1	1.1684	1.5161	0.695	0.3229	Fractions and percents (+)	Fraction: Percent containing a decimal

19	-0.29	0.9644	0.9371	1.0384	0.3048	Decimals or fractions: Single representation (-)	Negative fraction: Negative fraction
20	-1.74	0.9074	0.9531	1.0451	0.1729	Positive to negative	Integer (-): Fraction
21	-1.13	0.885	0.5618	1.1127	0.2241	Integers and fractions/decimals (-)	Negative integer: Negative decimal that terminates
22	0.35	0.9285	0.7668	1.1951	0.3701	Fractions and decimals (+)	Mixed number: Decimal that terminates
24	2.06	0.9763	1.0593	1.0161	0.4271	Fractions and percents (+)	Mixed number: Percent < or = 100
25	1.88	1.0196	1.1319	0.8882	0.4564	Decimals or fractions: Single representation (-)	Negative mixed number: Negative mixed number
26	0.57	1.0601	1.0049	0.8774	0.3813	Integers and fractions/decimals (-)	Negative integer: Negative improper fraction
27	-0.8	0.9045	0.623	1.1181	0.2553	Fractions and percents (+)	Fraction: Percent < or = 100
28	2.04	0.9046	0.8206	1.2444	0.4651	Fractions and decimals (+)	Fraction: Decimal that terminates
29	-0.92	0.9148	0.9731	1.0607	0.2477	Positive to negative	Negative fraction: Fraction
30	-0.68	1.0053	1.1989	0.9734	0.2589	Types of fractions: Single representation (+)	Fraction: Fraction
31	0.1	1.0293	1.1727	0.8949	0.3357	Integers and fractions/decimals (-)	Negative fraction: Integer (-)
32	0.61	0.886	0.7484	1.3266	0.3746	Fractions and decimals (+)	Decimal that terminates: Improper fraction
33	0.89	0.9236	0.8138	1.2679	0.409	Decimals or fractions: Single representation (-)	Negative improper fraction: Negative improper fraction
34	-0.16	0.8686	0.6074	1.2278	0.3012	Types of fractions: Single representation (+)	Fraction: Fraction
35	-1.77	1.0816	1.0711	0.9415	0.1745	Fractions and percents (+)	Percent > 100: Mixed number

Appendix I: Item Statistics for Selected Grade 8 Quantity Discrimination Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
3	0.21	0.9449	1.0104	1.0626	0.3639	Fractions and decimals (+)	Improper fraction: Decimal that terminates
4	-2.73	1.0467	0.8704	0.9772	0.1261	Positive to negative	Decimal that terminates: Integer (-)
5	0.8	0.8647	0.7908	1.4036	0.3945	Decimals or fractions: Single representation (-)	Negative fraction: Negative fraction
6	-0.01	0.9351	1.0517	1.0501	0.3339	Fractions and decimals (+)	Mixed number: Decimal that terminates
7	0.06	1.1107	0.9604	0.8719	0.3525	Decimals or fractions: Single representation (-)	Negative decimal that terminates: Negative decimal that terminates
10	0.44	0.9987	0.9257	1.0315	0.3755	Squares and square root relationships	Exponent: Square root of a perfect square
11	1.92	1.2261	1.2055	0.4946	0.4099	Exponents	Integer (+ or -): Exponent (-)
12	1	0.9864	0.9514	1.0606	0.4027	Fractions and percents (+)	Mixed number: Percent > 100
13	-0.07	1.1119	1.1077	0.826	0.3457	Decimals or fractions: Single representation (-)	Negative improper fraction: Negative improper fraction
14	1.19	0.9152	0.8674	1.2841	0.4158	Squares and square root relationships	Square root not of a perfect square: Whole number
15	-0.09	1.0528	1.1507	0.8703	0.34	Types of fractions: Single representation (+)	Improper fraction: Improper fraction
16	-1.93	1.0342	0.7062	1.003	0.1824	Positive to negative	Fraction: Negative fraction
17	-0.67	0.8563	0.7604	1.1475	0.2897	Types of fractions: Single representation (+)	Mixed number: Mixed number
18	-0.37	1.1631	1.0844	0.8202	0.3222	Fractions and decimals (+)	Decimal that terminates: Improper fraction

19	0.58	0.8032	0.7049	1.5482	0.3791	Fractions and percents (+)	Improper fraction: Percent > 100
20	1.66	1.1037	1.0849	0.7365	0.4144	Exponents	Expanded notation: Exponent
21	2.85	1.2632	1.8606	0.5629	0.3461	Exponents	Integer (+): Exponent (+)
22	1.33	1.1866	1.2605	0.3992	0.4182	Squares and square root relationships	Square root not of a perfect square: Whole number
23	0.57	0.9254	1.1605	1.0731	0.3877	Types of fractions: Single representation (+)	Mixed number: Improper fraction
24	-0.1	0.8948	0.9698	1.1294	0.3395	Decimals or fractions: Single representation (-)	Negative mixed number: Negative mixed number
25	1.15	0.8686	0.8036	1.4459	0.4105	Fractions and percents (+)	Percent < or = 100: Mixed number
26	-1.98	1.0778	1.3399	0.928	0.1764	Exponents	Exponent: Expanded notation
27	0.05	0.9691	0.7651	1.1234	0.3445	Squares and square root relationships	Square root: Exponent
28	-1.98	0.9539	0.4667	1.0694	0.1764	Exponents	Exponent (+): Integer (+)
29	0.85	0.8616	0.8144	1.4127	0.3966	Decimals or fractions: Single representation (-)	Negative mixed number: Negative mixed number
30	-0.29	0.8664	0.7064	1.1996	0.3131	Fractions and decimals (+)	Fraction: Decimal that terminates
31	1.66	0.9684	0.9732	1.0787	0.4144	Fractions and decimals (+)	Fraction: Decimal that terminates
32	-0.9	0.9613	1.9736	0.9573	0.2655	Types of fractions: Single Representation (+)	Mixed number: Mixed number
34	2.67	0.9862	1.3568	0.9479	0.3757	Fractions and percents (+)	Percent containing a decimal: Improper fraction
35	0.21	1.1004	1.3544	0.7377	0.3639	Decimals or fractions: Single representation (-)	Negative fraction: Negative fraction

Appendix J: Item Statistics for Selected Grade 6 Proportional Reasoning Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	-0.96	1.0155	0.8053	1.018	0.275	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
2	-0.01	0.6882	0.6152	1.533	0.3512	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
3	0.82	1.0902	1.014	0.7889	0.4085	Fraction comparison	Two fractions
4	-1.16	0.9457	0.6889	1.0801	0.2584	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
6	-0.07	0.6884	0.587	1.5083	0.3502	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
7	-1.18	0.917	0.5608	1.1214	0.2551	Unit fraction of a whole number	Unit fraction to multiple of unit fraction, same whole 1/b of N □ a/b of N
8	0.7	0.7709	0.6972	1.663	0.3882	Fraction comparison	Equivalent fractions a/b □ c/d, where a/b = c/d
9	-0.84	0.8302	0.6792	1.1803	0.2903	Unit fraction of a whole number	Unit fraction to multiple of unit fraction, same whole 1/b of N □ a/b of N
11	-0.14	0.9081	1.3023	1.07	0.3385	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
13	-0.57	0.8213	0.6822	1.2353	0.3092	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
14	-0.07	1.2396	1.113	0.6483	0.3529	Fraction comparison	Two fractions
15	-2.17	0.9834	0.5525	1.0409	0.1667	Fraction comparison	Two fractions, same denominator a/b □ c/b
16	-0.93	0.8141	0.4773	1.2377	0.2817	Unit fraction of a whole number	Unit fraction to equivalent, same whole c/d of N □ 1/b of N, where 1/b = c/d
17	-0.76	0.9635	0.7368	1.081	0.2903	Fraction of a whole number	Fraction to fraction, same whole a/b of N □ c/d of N
19	-2.21	1.0236	0.6973	1.0045	0.1665	Fraction comparison	Two fractions, same numerator a/b □ a/d

20	-0.72	1.1147	1.1328	0.8723	0.2995	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
21	1.21	0.8729	0.8421	1.4006	0.4176	Fraction of a whole number	Fraction to fraction, same whole a/b of N □ c/d of N
22	0.53	0.7175	0.6519	1.7186	0.3881	Fraction comparison	Equivalent fractions a/b □ c/d, where a/b = c/d
23	-0.99	0.8611	0.6881	1.1454	0.2686	Unit fraction of a whole number	Unit fraction to equivalent, same whole 1/b of N □ c/d of N, where 1/b = c/d
24	2.42	1.2647	1.7865	0.4697	0.422	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
25	-0.51	0.9747	0.9641	1.0256	0.3117	Fraction comparison	Two fractions, same numerator a/b □ a/d
27	2.26	1.1883	1.5215	0.5812	0.4248	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
28	-0.83	1.0733	1.0324	0.9288	0.2748	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
29	-0.33	0.7792	0.5979	1.3498	0.3266	Fraction of a whole number	Fraction to fraction, same whole a/b of N □ c/d of N
30	-0.5	0.8753	0.6916	1.1913	0.3087	Unit fraction of a whole number	Unit fraction to equivalent, same whole 1/b of N □ c/d of N, where 1/b = c/d
31	0.21	0.9431	0.8055	1.1707	0.374	Fraction comparison	Two fractions
32	-0.97	0.8586	0.5276	1.1854	0.2741	Unit fraction of a whole number	Unit fraction to multiple of unit fraction, same whole 1/b of N □ a/b of N
33	2.15	1.0679	1.1987	0.8305	0.4215	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
34	0.87	0.9749	0.9197	1.104	0.3961	Fraction comparison	Equivalent fractions a/b □ c/d, where a/b = c/d
35	2.46	1.0551	1.4029	0.8102	0.4162	Unit fraction of a whole number	Same unit fraction, different whole 1/b of M □ 1/b of N

Appendix K: Item Statistics for Selected Grade 7 Proportional Reasoning Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	-0.82	0.9843	1.3697	0.9715	0.2968	Fraction Comparison	Two fractions, same numerator a/b □ a/d
2	0.14	0.9559	0.9026	1.1158	0.3909	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
3	0.81	0.8712	0.8022	1.418	0.4314	Percent of a whole number	Same percent, different wholes A% of M □ A% 0f N
4	-0.56	0.8895	0.7755	1.1649	0.3366	Fraction of a whole number	Fraction to fraction, same whole a/b of N □ c/d of N
5	-0.25	1.1978	1.2853	0.6214	0.3629	Unit fraction of a whole number	Unit fraction to equivalent, same whole 1/b of N □ c/d of N, where 1/b = c/d
6	-0.71	0.9031	0.9249	1.0992	0.3077	Unit fraction of a whole number	Unit fraction to multiple of unit fraction, same whole 1/b of N □ a/b of N
8	1.08	0.8265	0.7741	1.5329	0.4347	Percent of a whole number	Same percent, different wholes A% of M □ A% 0f N
9	1.6	0.9239	0.8963	1.176	0.4542	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
10	1.1	0.9734	1.0274	1.0383	0.4525	Unit fraction of a whole number	Same unit fraction, different whole 1/b of M □ 1/b of N
11	1.52	1.0884	1.1999	0.7282	0.452	Unit fraction of a whole number	Same unit fraction, different whole 1/b of M □ 1/b of N
13	-0.93	0.8843	0.9665	1.092	0.304	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
14	-0.8	0.889	1.3148	1.0702	0.3154	Fraction Comparison	Two fractions, same denominator a/b □ c/b
15	1.34	0.7901	0.7326	1.5582	0.454	Percent of a whole number	Same percent, different wholes A% of M □ A% 0f N
16	1.31	0.8315	0.8002	1.4387	0.4527	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N

17	-1.16	0.9131	0.8703	1.0647	0.2882	Unit fraction of a whole number	Unit fraction to multiple of unit fraction, same whole 1/b of N □ a/b of N
18	-0.92	0.8004	0.7909	1.2078	0.3089	Percent of a whole number	different percents, same whole A% of N □ B% 0f N
19	-0.64	1.0357	1.2495	0.8816	0.3316	Fraction Comparison	Two fractions
20	-0.23	1.1637	1.1829	0.6797	0.3646	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
22[a]						Fraction Comparison	Improper fractions b/a □ d/c, where b > a and d > c
23	-1.75	1.076	1.6683	0.8802	0.2378	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
24	0.93	0.7424	0.6896	1.8051	0.4298	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
25	-1.07	1.0471	0.8425	0.9854	0.295	Fraction of a whole number	Fraction to fraction, same whole a/b of N □ c/d of N
26	0.84	1.147	1.1232	0.5728	0.4322	Fraction Comparison	Improper fractions b/a □ d/c, where b > a and d > c
27	1.26	1.0129	1.1169	0.9102	0.4499	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
28	-0.41	1.0607	1.0676	0.89	0.3506	Unit fraction of a whole number	Unit fraction to equivalent, same whole 1/b of N □ c/d of N, where 1/b = c/d
29	-1.41	0.836	0.4967	1.1812	0.2616	Percent of a whole number	Different percents, same whole A% of N □ B% 0f N
31	-0.84	0.9824	1.0521	0.9982	0.2861	Fraction Comparison	Two fractions, same numerator a/b □ a/d
33	1.48	0.77	0.7186	1.5399	0.4603	Percent of a whole number	Same percent, different wholes A% of M □ A% 0f N
34	0.55	0.9956	0.9391	1.0425	0.4175	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
35	-0.45	1.0312	1.3887	0.8652	0.3458	Unit fraction of a whole number	Unit fraction to equivalent, same whole 1/b of N □ c/d of N, where 1/b = c/d

[a]Original item was not tested.

Appendix L: Item Statistics for Selected Grade 8 Proportional Reasoning Items

Item	Item Difficulty	Mean-square Infit	Mean-square Outfit	Discrimination	Point Measure	Category	Sub Category
1	0.87	1.0053	0.9244	1.0368	0.4273	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
2	-0.45	1.0757	1.7705	0.7582	0.303	Unit fraction of a whole number	Unit fraction to equivalent, same whole c/d of N □ 1/b of N, where 1/b = c/d
3	0.78	1.0183	1.0029	0.9557	0.4277	Fraction comparison	Two fractions
4	-0.84	0.9669	0.8053	1.0627	0.2914	Fraction of a whole number	Improper fraction to improper fraction, same whole b/a of N □ d/c of N, where b > a and d > c
5	0.04	0.8913	0.8819	1.2317	0.3681	Unit fraction of a whole number	Unit fraction to improper fraction, same whole 1/b of N □ d/c of N, where d > c
6	1.4	0.9524	0.8935	1.1403	0.4366	Percent of a whole number	Same percent, different wholes A% of M □ A% 0f N
7	1.93	1.0797	1.1995	0.8234	0.4535	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
8	-1.55	0.8583	0.5135	1.1433	0.2279	Fraction comparison	Equivalent fractions a/b □ c/d, where a/b = c/d
10	1.39	0.9293	0.9503	1.1467	0.4485	Unit fraction of a whole number	Same unit fraction, different whole 1/b of M □ 1/b of N
11	-1.85	1.0171	0.6564	1.0258	0.206	Unit fraction of a whole number	Unit fractions, same whole 1/b of N □ 1/c of N
13	0.2	0.8812	0.7326	1.3797	0.3975	Fraction of a whole number	fraction to fraction, same whole a/b of N □ c/d of N
14	1.36	0.831	0.7879	1.4131	0.4461	Percent of a whole number	Same percent, different wholes A% of M □ A% 0f N
15	-0.45	0.8612	0.7159	1.2448	0.3316	Fraction comparison	Two fractions
16	-0.33	1.2186	1.3483	0.5609	0.3379	Unit fraction of a whole number	Unit fractions, different whole 1/b of M □ 1/c of N

17	-1.28	0.8137	0.6022	1.1809	0.2478	Unit fraction of a whole number	Unit fraction to improper fraction, same whole 1/b of N □ d/c of N, where d > c
18	-0.98	0.8702	0.6766	1.1562	0.2738	Percent of a whole number	Different percents, same whole A% of N □ B% 0f N
19	0.96	1.0077	0.9677	0.9976	0.4341	Fraction of a whole number	Same fraction, different whole a/b of M □ a/b of N
20	1.89	0.8349	0.7204	1.3065	0.4596	Fraction of a whole number	Improper fraction to improper fraction, same whole b/a of N □ d/c of N, where b > a and d > c
21	-0.27	1.0721	0.9716	0.8929	0.3464	Fraction of a whole number	Improper fraction to fraction, same whole d/c of N □ a/b of N, where d > c
22	-1.46	0.7907	0.4725	1.1958	0.2358	Percent of a whole number	Different percents, same whole A% of N □ B% 0f N
23	-0.66	1.0786	1.2215	0.8541	0.3083	Fraction comparison	Two fractions, same numerator a/b □ a/d
24	0.87	0.9009	0.8587	1.3137	0.4273	Unit fraction of a whole number	Same unit fraction, different whole 1/b of M □ 1/b of N
25	3.33	0.7167	0.369	1.2698	0.3855	Fraction of a whole number	Equivalent fractions, same whole a/b of N □ c/d of N, where a/b = c/d
27	-0.44	1.0012	0.7928	1.065	0.3316	Unit fraction of a whole number	Unit fractions, different whole 1/b of M □ 1/c of N
28	-1.81	0.8908	0.457	1.121	0.2119	Fraction of a whole number	Improper fraction to fraction, same whole d/c of N □ a/b of N, where d > c
30	-0.79	0.8357	0.6576	1.2148	0.2995	Unit fraction of a whole number	Unit fractions, different whole 1/b of M □ 1/c of N
31	0.01	1.131	1.5371	0.588	0.3721	Fraction of a whole number	Improper fraction to improper fraction, same whole b/a of N □ d/c of N, where b > a and d > c
33	-1.55	0.9266	0.7394	1.066	0.2246	Fraction of a whole number	Fraction to fraction, same whole a/b of N □ c/d of N

34	-1.49	0.9451	0.6978	1.0704	0.2301	Unit fraction of a whole number	Unit fraction to improper fraction, same whole 1/b of N □ d/c of N, where d > c
35	0.87	1.0593	1.1477	0.7607	0.4273	Fraction comparison	Equivalent fractions a/b □ c/d, where a/b = c/d