Thermal Conductivity Measurement and Prediction from Geophysical Well Log Parameters with Borehole Application

RONALD GOSS

JIM COMBS

Institute for Geosciences, The University of Texas at Dallas, P.O. Box 688, Richardson, Texas 75080, USA

ABSTRACT

Problems related to thermal conductivity measurements of rocks were examined using the divided-bar method. Thermal contact resistance, measurement pressure, and sample thickness were found to have significant effects on the measured thermal conductivity. For accurate determinations, the effect of pressure on reference standards must be known. Tentative values of K_{fs} (mcal/cm·sec·°C) = $3.30 - 0.18 \times 10^{-3} P$ (bars) for fused silica, and $K_{\|q} = 25.3 + 1.2 \times 10^{-3} P$ for Z-cut natural quartz crystals were obtained. Thermal contact resistance obscured these relationships below a uniaxial pressure of 150 bars.

Measurements on each of 50 samples were made for thermal conductivity, bulk density, porosity, permeability, electrical resistivity and conductivity, compressional and shear velocity, and free fluid index. These physical properties were analyzed to derive predictive equations for thermal conductivity. An empirical relationship developed for the Imperial Valley of southern California is $K_{prd} = 2.01 - 0.095 \, \Phi \, (\%) + 1.66 \, V_P \, (\text{km/sec})$. A standard deviation of ± 0.7 mcal/cm·sec·°C implies a reliability of approximately 10% for the predicted thermal conductivity. Application of this relationship to thoroughly investigated geothermal borehole sections indicates that prediction from standard geophysical well logs may be more reliable than cell measurements in determining the thermal conductivity of unconsolidated sedimentary sequences.

INTRODUCTION

Knowledge of thermal conductivity is an absolute necessity in heat flow studies. It is an important parameter in the detection and development of geothermal fields. Similarly, thermal properties have importance for secondary and tertiary recovery techniques in the petroleum industry. Presently, the most economical and efficient approach to determine thermal conductivity is to collect samples from a borehole for laboratory analysis.

There are many problems in obtaining an accurate thermal conductivity value. The sample collection process is complex. For example, questions arise as to what interval of a borehole should be collected and how the in situ conditions, that is, saturation, pressure, temperature, and so on, should be reproduced. If properly prepared samples are obtained, laboratory measurements are reasonably accurate, although relatively time consuming and expensive. The most common method for the determination of the thermal conductivity of earth materials is the divided-bar apparatus (Birch, 1950). Bar materials and designs are not standard and therefore each apparatus is subject to its own subtle problems.

Though values of thermal conductivity may be necessary in a particular study, the rock samples and requisite data are often not economically feasible to collect. Alternative approaches to obtaining thermal conductivities of rocks are needed.

In many circumstances, downhole and laboratory methods for measuring thermal conductivity are unsatisfactory. A conductivity logging tool would be ideal, but none exists. Many of the properties which are regularly measured during geophysical logging of boreholes relate to the same physical phenomena that control thermal conductivity; therefore it should be possible to derive thermal conductivity from a correlation with other physical properties.

Theoretical relationships between properties like thermal conductivity and velocity have been derived for specific media (Debye, 1914; Kittel, 1971). These theoretical relationships apply reasonably well only to ideal materials, not to rocks. Thus theoretical work is not likely to predict thermal conductivities in the earth.

Empirical studies have had some success (Karl, 1965; Tikhomirov, 1968; Anand, Somerton, and Gomaa, 1973). Most empirical studies have been concerned with either correlation of only one physical property at a time, or with suites of rocks from many different environments. Since thermal conductivity can seldom be related closely to rock type, the results of these studies have not been generally applicable.

In this study an attempt was made to find an empirical relationship between thermal conductivity and several physical properties in combination. Each set of physical properties including thermal conductivity was measured with the same sample. Details pertaining to the samples, experimental equipment, and measurement techniques are presented elsewhere (Goss, 1974).

A suite of samples, petrologically similar and obtained from the same geological environment, was chosen. An investigation was then made of possible extensions and limitations over a wider range of rock types. Finally, some use of, and application to, borehole logging data was included.
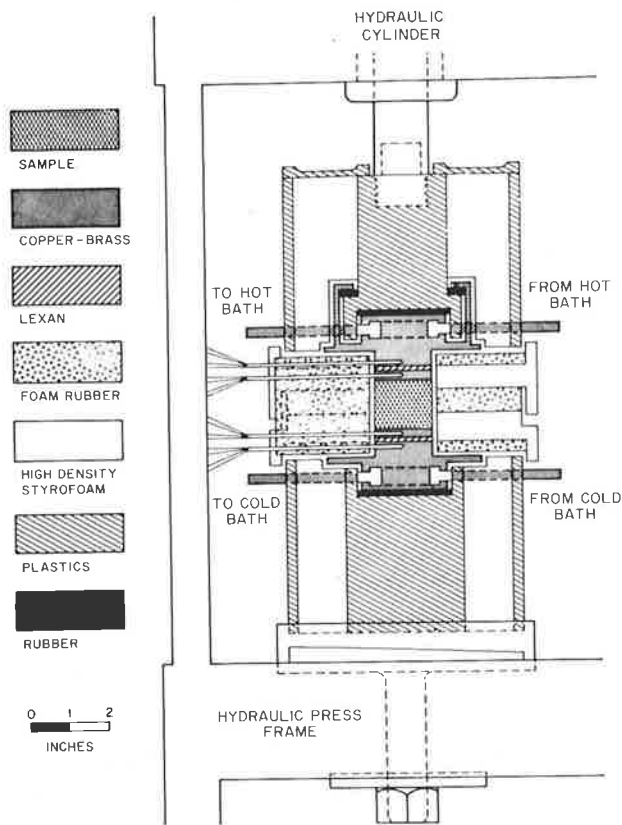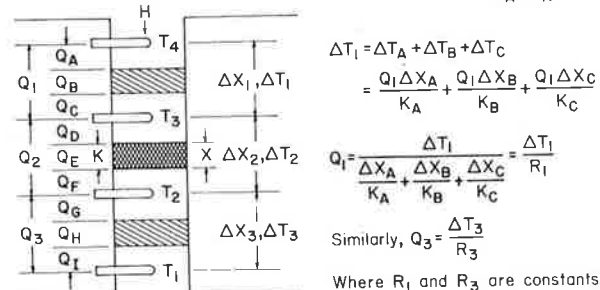
Figure 1.   An idealized section for the divided-bar apparatus.

DIVIDED-BAR APPARATUS

An idealized section of the divided-bar used in the present study is shown in Figure 1. The design is an attempt to incorporate a short bar, having "constant" thermal resistance with a good insulation system capable of accepting a variable length sample. The theory for this design is presented in Figure 2. From Figure 2, it can be seen that any reference thermal conductivity standard gives a comparison factor

$$C^* = K_R^* \Delta T_{2R} / X_R (\Delta T_{1R} + \Delta T_{3R}) \qquad (1)$$

In order to determine the thermal conductivity for an unknown sample, the comparison factor C is substituted into the following relationship

$$K = CX(\Delta T_1 + \Delta T_3)/\Delta T_2 \qquad (2)$$

The calculation of the comparison factor C is based on the assumption that the thermal conductivity of the reference standard K_R^* is known and constant. However, C^* is found to be a function of pressure. Details of the effect of pressure on the thermal conductivity of reference materials will be published elsewhere (Goss and J. Combs, in preparation, 1975). Nevertheless, a few comments are appropriate.

Since the apparent pressure effect can be in opposite directions for different reference materials, it cannot be

DERIVATION OF DIVIDED-BAR EQUATIONS

For one-dimensional steady state heat flow $Q = \dfrac{q}{A} = \dfrac{H}{tA} = -K\dfrac{\Delta T}{\Delta X}$

Reversing directions of increasing T and X $Q = K\Delta T/\Delta X$

In a "stack" of materials A, B, C, ... I, $Q_1 = Q_2 = Q_3 = Q_A \cdots = Q_I$

To add, write as thermal resistance, $Q_1 = Q_A = \dfrac{\Delta T_A}{\Delta X_A/K_A}$, etc.

$\Delta T_1 = \Delta T_A + \Delta T_B + \Delta T_C$

$= \dfrac{Q_1\Delta X_A}{K_A} + \dfrac{Q_1\Delta X_B}{K_B} + \dfrac{Q_1\Delta X_C}{K_C}$

$Q_1 = \dfrac{\Delta T_1}{\dfrac{\Delta X_A}{K_A} + \dfrac{\Delta X_B}{K_B} + \dfrac{\Delta X_C}{K_C}} = \dfrac{\Delta T_1}{R_1}$

Similarly, $Q_3 = \dfrac{\Delta T_3}{R_3}$

Where R_1 and R_3 are constants.

By design $R_1 \approx R_3$, and since $Q_E = Q_1 = Q_3$

$Q_E = \dfrac{Q_1 + Q_3}{2} = \dfrac{\Delta T_1/R_1 + \Delta T_3/R_3}{2} \approx \dfrac{\Delta T_1 + \Delta T_3}{2R_1} \approx \dfrac{\Delta T_1 + \Delta T_3}{2R_3}$

By design $\Delta X_D = \Delta X_F \ll \Delta X_E$ and $K_D = K_F \gg K_E$

$\Delta T_E \to \Delta T_2$ as $R_2 = \dfrac{\Delta X_D}{K_D} + \dfrac{\Delta X_E}{K_E} + \dfrac{\Delta X_F}{K_F} \to \dfrac{\Delta X_E}{K_E} = \dfrac{X}{K}$

$2R_1 = 2R_3 = \dfrac{\Delta X_E(\Delta T_1 + \Delta T_3)}{K_E \Delta T_E} \longrightarrow 2R_1 = 2R_3 = \dfrac{X(\Delta T_1 + \Delta T_3)}{K\Delta T_2}$

Or as material E is changed to i, j, k

$\dfrac{X_i(\Delta T_{1i} + \Delta T_{3i})}{K_i\Delta T_{2i}} = \dfrac{X_j(\Delta T_{1j} + \Delta T_{3j})}{K_j\Delta T_{2j}} = \dfrac{X_k(\Delta T_{1k} + \Delta T_{3k})}{K_k\Delta T_{2k}}$

Figure 2.   Theoretical derivation with basic assumptions for the steady-state comparative thermal conductivity technique using the divided-bar apparatus.
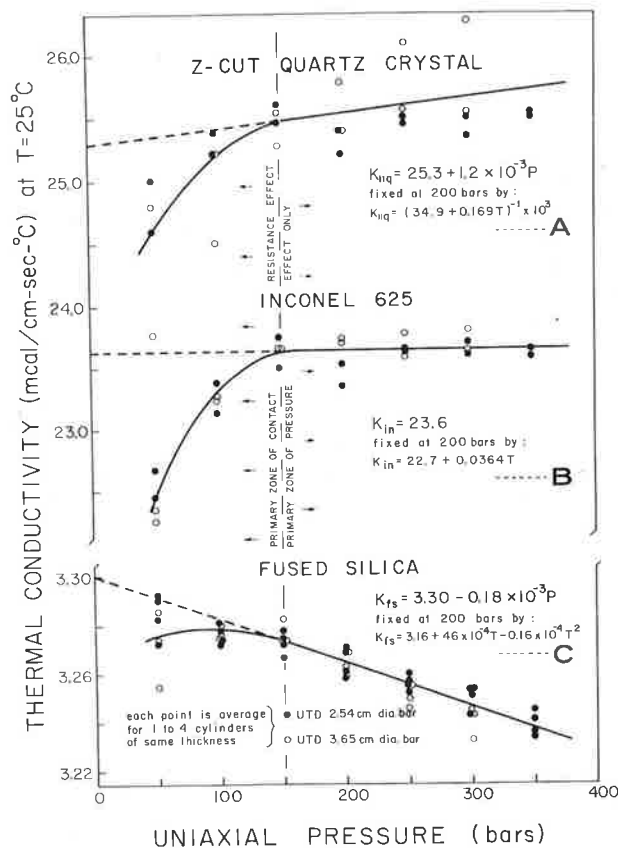

Figure 3.   Thermal conductivity of three reference standards as a function of pressure at constant temperature. The curves are fixed to agree at 200 bars with the starting $K = f(T)$ as given by: (A) Mean of two equations given by Horai and Simmons (1969), (B) Linear fit to data between −80 to 80°C as published in Huntington Alloys (1970), (C) Ratcliffe (1959).

primarily an instrument effect. If it is assumed that all of the change is due to the effect of pressure on the reference thermal conductivity K_R^*, then use of the "true" reference conductivity which is a function of pressure, $K_R = f(P)$, would result in a constant comparison factor C.

Careful examination of the mathematical relationships in Figure 2 indicates that $K_R(P) = C_{avg}^* K_R^*/C^*(P)$, where the average measured comparison factor, C_{avg}^*, is used as an approximation to the constant C, should be a good approximation to the curve $K_R = f(P)$. This reduction has been performed for three reference materials and the results are plotted in Figure 3. The result,

$$K_{fs} = 3.30 - 0.18 \times 10^{-3} P \qquad (3)$$

in which K_{fs} is in mcal/cm·sec·°C and P is in bars, has been obtained for fused silica using General Electric (GE) types GE 101, GE 102, and GE 125 fused silica. A more tentative result for Z-cut natural quartz is

$$K_{\|q} = 25.3 + 1.2 \times 10^{-3} P \qquad (4)$$

Inconel 625 was also investigated as one means of determining whether instrument effects were significant. Because of the elastic properties of this metal alloy, it would not be expected to have a discernible pressure effect over the considered range, 0 to 400 bar. From Figure 3, it can be seen that the assumption of insignificant pressure effects is correct for pressures above 150 bar. Below a pressure of 150 bar for all standards, thermal contact resistance at the bar-reference standard interface produces an apparent decrease in conductivity. Thermal contact resistance can be eliminated by using several thicknesses of a material, assuming the resistance of the bar material is well-known; or it may be successfully eliminated by calibration as

attempted here if all sample ends are similarly prepared and theoretical assumptions are verified.

The above method provides a reliable estimation of the slope for the pressure effects, but each curve (Fig. 3) must be tied to some known point for absolute values. K_R at $P = 200$ bar was chosen to agree with the K_R^* values obtained from the temperature functions of Figure 3 at $T = 25°C$. This point was chosen since it is well above the contact resistance zone and probably in the range of confining pressures in which studies of the temperature effect on thermal conductivity of the reference materials were conducted.

A question exists as to whether the effect of pressure on the sample length, $X = f(P)$, can be ignored, as has been done. From the respective values of Young's modulus for these materials, $E_{fs} \approx 0.7$, $E_{\|q} \approx 1$ and $E_{in} \approx 3.5$ Mb, the maximum changes for the lengths of the standards are $\Delta L_{fs} \approx 0.001$, $\Delta L_{\|q} \approx 0.0009$, and $\Delta L_{in} \approx 0.0002$ cm. These changes of length would produce an effect of less than 0.002 for a typical value of 1.200 mcal/cm²·sec·°C, and can thus be neglected.

The present bar design (Fig. 1), in conjunction with an exceedingly stable $5\frac{1}{2}$-digit multimeter manufactured by Systron-Donner, provided repeatability for C of ±0.005 or about 0.5% for different runs with the same piece of standard material. C has been found to be a function of standard thickness X_R, differing slightly depending on the particular standard (Fig. 4). However, the overall C is essentially the same for either standard, implying that the theoretical assumptions were justified. Until further work with other standards is finished, no meaningful correction for thickness exists, and therefore reductions have been made to the mean curve of Figure 4. Since results may deviate by as much as $\pm1.0\%$ from the mean curve, allowing for the same magnitude of effects on the unknown rock samples gives
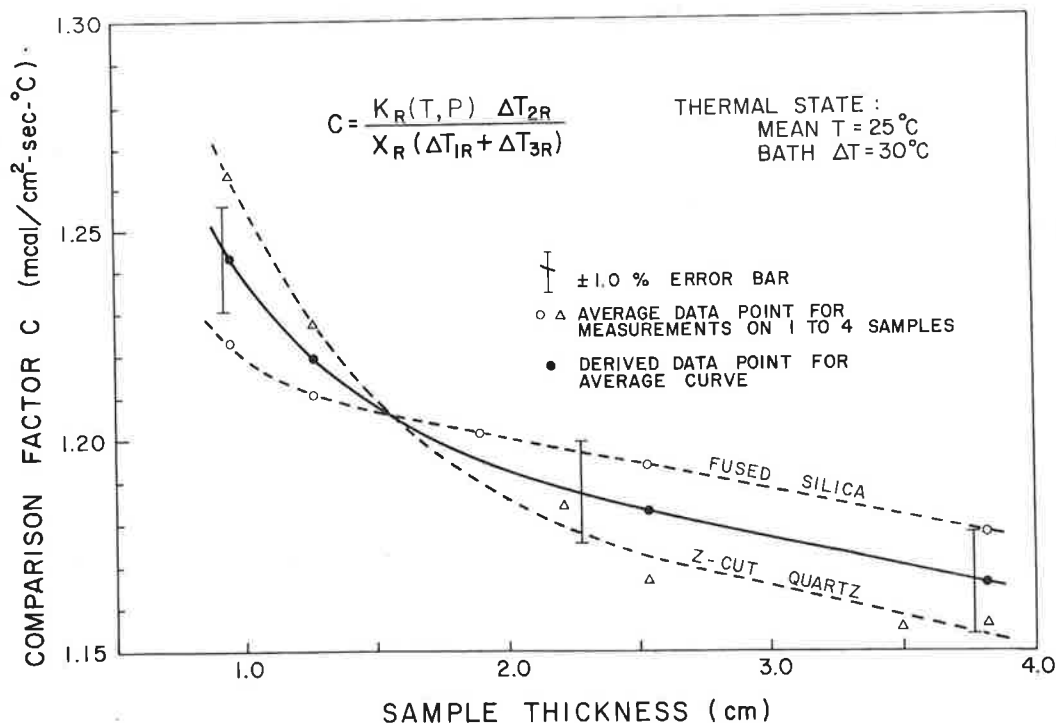

Figure 4. Comparison factor C as a function of sample thickness for two reference standards for the UCR 2.54-cm-diameter-bar.

an expected accuracy of 2% for a measured thermal conductivity value.

For measurements on borehole grab samples of drill chips and unconsolidated sediments, a cell arrangement similar to that described by Sass, Lachenbruch, and Munroe (1971) was used. The cell device essentially constitutes a new divided-bar design. The copper caps of the cell function as an extension of the bar. The cell device, therefore, would be expected to have a somewhat higher comparison factor C_c (that is, equation (1) where ΔT_2 is greater, ΔT_1 and ΔT_3 are the same or less) for its equivalent sample thickness (in the present study, 1.77 cm). This assumption was tested using four materials of known solid conductivity which were crushed and mixed with water. Each material gave a very similar C_c value after reversing the cell reduction calculations, that is, C_c was equal to 1.241 ± 0.006 mcal/cm$^2 \cdot$sec$\cdot$°C. The average C_c value was used for all cell reductions. As predicted, this C_c was higher than the equivalent for a fused silica standard, by about 3.5%.

All calculations involving thermal conductivity in aggregates, whether for the solid-liquid in a cell mixture or to convert the solid K_S to an in situ aggregate K_A based on the best estimate of saturated porosity, used the geometric mean equation for liquid volume fraction V

$$K_A = K_S^{1-V} K_L^V, \tag{5}$$

where the thermal conductivity of water is $K_L = 1.45$ mcal/cm$\cdot$sec$\cdot$°C. With the reduction estimates implied, cell measurement accuracy is 5 to 6%. However, with the common sampling problems for grab samples, for example, depth determination, drilling mud contamination, and in situ physical state, thermal conductivity values of 10% reliability are the best attainable for the cell technique.

EXPERIMENTAL DATA

Successively more general groups of samples have been collected and analyzed in this investigation. The first 25 core samples are from two exploratory holes at the Dunes geothermal anomaly in the Imperial Valley (Fig. 5). These Dunes samples vary from siltstones through graywackes to pebble sandstones, all having been hydrothermally altered and cemented (Bird, 1975).

An expanded group adds a Mesa core sample, and drill cuttings from boreholes drilled in the Mesa geothermal field. A set of these values represents averages over zones where

the logs and measurements changed conformably with each other. Five sets are from the deep zone (1670 to 1740 m) of the U.S. Bureau of Reclamation (USBR) Mesa No. 5-1 geothermal well, and eight sets of values are from the shallow zone (200 to 800 m) of the USBR Mesa No. 6-1 well. These Mesa samples are representative of the unconsolidated sediments of the Imperial Valley. This group of Dunes and Mesa samples covers most of the range of the Imperial Valley subsurface sedimentary section.

To provide some indication of the potential for generalization, a third suite of samples of Berea, Navajo (quartzite), and Raven Ridge sandstones was added. These samples, like most from areas other than the Imperial Valley, were taken from quarried blocks. They represent rock types or environments to which a sedimentary basin relationship might be expected to apply.

A final miscellaneous group of rock cores included three pieces of two limestones, a dolomite, a shale, and two pieces of a manufactured porcelain. The purpose for these final samples was to determine whether the predictive equations derived for sands and sandstones could be used for other rock types, or to indicate differences which might exist.

Physical properties which were measured include thermal conductivity (K), bulk density (D_B), porosity (Φ), permeability (k), saturated electrical resistivity (ρ) and electrical conductivity (σ), and compressional velocity (V_P). Shear velocity (V_S) and free fluid index (*FFI*) were also determined for the core samples (see Table 1). The conditions of investigation were 100% saturation with a sodium chloride brine of 160 000 ppm (fluid resistivity $\rho_w = 0.05$ ohm$\cdot$m). Temperatures near room condition (24°C) were used with uniaxial pressures of 200 bar.

DATA ANALYSIS AND RESULTS

In any empirical approach, many models are possible. The meager guidance of previous theory and investigations discussed below does not suggest or justify the use of anything beyond linear models. The statistically modest quantity of data indicates that sophisticated analytical methods would not be worthwhile. For these reasons, a straightforward multiple linear regression was considered adequate. For purposes of comparing different combinations of variables, goodness of fit, correlation coefficients, standardized partial regression coefficients, and F-tests were used (Davis, 1973).

Examination of Table 1 shows that the manufactured porcelain is an unusual material to consider as a rock analogue. For example, this porcelain is characterized by both very high porosity and high velocity; in addition, the measured thermal conductivity is twice that of any other sample. In spite of these apparently conflicting data, reasonable correlations could be obtained for a suite of materials including all core samples plus the porcelain, for example, a predicted thermal conductivity $K_{prd} = f(V_S, FFI)$ with a multiple correlation coefficient of $R = 0.91$. This is a forced fit because for the core samples alone, R drops drastically to 0.52. Since the correlations should not depend significantly on one type of sample, the porcelain was eliminated from further multiple regressions. The discussion of porcelain was presented to demonstrate a typical statistical pitfall.

Multiple regression techniques require uncorrelated variables, thus in the stepwise elimination of nonessential
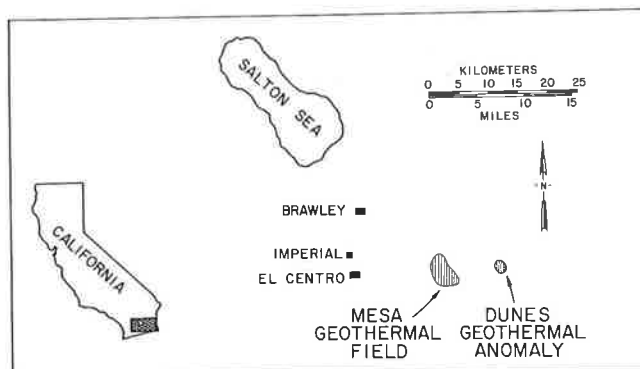
Figure 5. Imperial Valley location map, showing the Mesa geothermal field and the Dunes geothermal anomaly.

Table 1. Data from laboratory measurements of cores.

Identification	K $\left(\dfrac{\text{mcal}}{\text{cm·sec·°C}}\right)$	D_B $\left(\dfrac{\text{g}}{\text{cm}^3}\right)$	Φ (%)	k (mdarc)	ρ (ohm·m)	$\rightarrow F$ $\left(\dfrac{\rho}{0.05\ \text{ohm·m}}\right)$	σ $\left(\dfrac{\text{mmhos}}{\text{m}}\right)$	V_P $\left(\dfrac{\text{km}}{\text{sec}}\right)$	V_S $\left(\dfrac{\text{km}}{\text{sec}}\right)$	FFI (%)
				Dunes DWR 1 core data from 100 to 300 meters						
UCR-1	10.0	2.47	9.56	0.76	16.7	334	59.9	5.09	3.17	8.33
UCR-2	10.0	2.49	8.62	0.50	23.6	472	42.4	5.07	3.23	6.52
UCR-3	7.60	2.35	12.9	350	3.25	65.0	308	4.63	2.60	11.1
UCR-4	9.99	2.46	10.4	15.0	8.04	161	124	5.03	3.13	8.61
UCR-5	8.45	2.40	13.1	470	3.24	64.8	309	4.91	2.87	10.0
UCR-6	9.76	2.44	11.3	7.8	5.32	106	188	4.86	3.01	8.68
UCR-7	9.00	2.43	12.4	1.3	5.88	118	170	4.33	2.57	9.12
UCR-8	8.01	2.35	18.5	62.0	2.83	56.6	353	4.01	2.24	11.7
UCR-9	8.13	2.36	16.3	14.0	2.77	55.4	361	3.98	2.22	4.11
UCR-10	8.16	2.36	16.5	39.0	2.72	54.4	368	3.88	2.15	11.2
UCR-11	10.1	2.56	3.93	0.01	193	3860	5.18	5.54	3.49	2.91
UCR-13	10.2	2.47	9.66	0.56	15.8	316	63.3	4.94	3.03	7.45
UCR-14	10.0	2.56	4.16	0.08	497	9940	2.01	5.49	3.52	3.53
UCR-15	10.3	2.55	3.14	0.02	445	8900	2.25	5.54	3.56	2.78
UCR-16	10.8	2.54	5.32	0.02	154	3080	6.49	5.34	3.49	4.21
UCR-17	10.0	2.53	5.90	0.02	58.2	1160	17.2	5.34	3.30	4.77
UCR-18	10.4	2.55	4.64	0.01	90.1	1800	11.1	5.38	3.36	3.23
UCR-19	10.0	2.51	7.64	0.16	23.8	476	42.0	5.14	3.17	5.97
UCR-20	10.5	2.53	6.74	0.02	29.7	594	33.7	5.10	3.17	4.60
UCR-21	10.1	2.53	6.44	0.38	31.7	634	31.5	5.35	3.32	5.07
UCR-22	10.6	2.53	6.35	0.08	41.8	836	23.9	5.29	3.31	4.92
				UCR 115 core data, a few meters from Dunes hole						
115-A	9.50	2.50	8.09	0.59	21.2	424	47.2	5.09	3.50	6.53
115-B	10.3	2.53	3.99	0.01	584	11700	1.71	5.25	3.52	3.21
115-C	9.67	2.53	4.13	0.03	402	8040	2.49	5.31	3.73	3.37
115-D	10.1	2.60	3.86	0.02	336	6720	2.98	5.31	3.56	2.87
				Core varieties, mostly quarried from reservoir rocks						
				Sandstones						
BER	10.1	2.37	17.3	79	1.34	26.8	746	3.79	2.05	15.6
MESA-2	7.66	2.51	8.39	0.07	4.41	88.2	227	3.94	2.08	6.5
NAV-1	12.5	2.44	12.1	83	2.55	51.0	392	4.65	2.64	11.0
NAV-2	12.5	2.44	11.9	100	2.55	51.0	392	4.66	2.65	10.8
RAV	9.78	2.44	13.5	42	1.89	37.8	529	4.77	2.22	11.3
				Other						
ALH-2	9.33	2.28	21.0	460	1.19	23.8	840	3.81	2.03	18.9
IND-1	5.75	2.47	13.5	0.55	4.81	96.2	208	4.75	2.42	11.5
IND-2	5.73	2.47	13.7	0.36	3.94	78.8	254	4.72	2.40	12.0
DOL	11.0	2.84	0.10	0.02	1940	38800	0.516	6.98	3.76	0.06
SHA	8.13	2.65	1.20	0.05	54.2	1080	18.4	5.10	2.84	0.52
POR-1	24.6	3.06	28.1	18.0	1.14	22.8	877	6.92	4.11	28.1
POR-2	24.7	3.06	28.2	19.0	1.65	33.0	606	6.87	4.07	28.2

Mesa borehole	Depth (m)	K cell measure	D_B	Φ	"k" est. by SARABAND	ρ deep reading	$\rightarrow F$	σ	V_P	Temp. (°C)	Salin. (ppm)
					Drill cuttings and borehole logged measurements*						
					MESA 5-1 log data (deep), avg of 3 for 10-meter intervals						
5	1680	4.2	2.25	24.0	90	6.4	7.1	160	3.11	150	1400
5	1690	3.6	2.30	21.0	40	7.1	12	140	3.30	150	2300
5	1700	4.2	2.25	19.6	110	4.0	8.9	290	3.16	150	3000
5	1710	3.8	2.23	22.6	60	4.4	6.8	240	2.83	150	2000
5	1730	4.2	2.30	20.3	32	8.0	6.7	170	3.40	150	1000
					MESA 6-1 log data (shallow), avg of 3 for 30-meter intervals						
6	220	3.2	2.05	22.5	800	3.5	15	350	1.85	65	
6	470	4.4	2.10	21.6	80	1.7	17	600	2.27	80	
6	500	3.9	2.17	18.0	100	1.1	25	960	2.15	90	
6	560	3.6	2.15	20.0	70	1.7	20	630	2.28	100	
6	650	3.6	2.20	23.7	100	1.1	14	940	2.17	105	
6	680	3.9	2.17	24.3	200	1.3	14	770	2.32	110	
6	710	4.1	2.20	25.0	200	1.2	12	860	2.29	115	
6	740	3.6	2.20	22.5	100	1.2	15	840	2.26	120	

*D_B obtained from FDC log; Φ from FDC, CNL, and BHC logs; "k" is by SARABAND computer analysis, true k estimate only in a transition zone; ρ and σ from DIL-8 log; F calculated from Φ and ρ using R_w estimate for borehole 5 and the Humble equation for borehole 6; V_P from BHC log, and salinity from SARABAND computer analysis.

physical properties from the regression equations, special attention was given to the cancelling effect of obviously related variables. For example, neither compressional nor shear velocity was highly significant when both were included since each offsets the effect of the other. Each velocity, when taken one at a time, was one of the most significant variables. During the present investigation, electrical resistivity ρ, and $\log \rho$, electrical conductivity σ, formation factor F, $1/F$, and $\log F$ were all examined. The most significant functional forms were commonly ρ, σ and F. Therefore they are used in the present regression analysis. Free fluid index was one of the least significant variables. Final eliminations generally resulted in bulk density or porosity, permeability, or electrical conductivity, in addition to compressional velocity as the meaningful variables.

We obtained predicted thermal conductivity for the most significant variable, $K_{prd} = f(V_P)$, for each of the main groups of samples, and compared linear correlation coefficients, r. For the closely related Dunes core, r was equal to 0.825 ($N = 25$). With the Mesa samples added, r increased to 0.962 ($N = 39$), and the statistical significance improved dramatically. With all sandstones and chips, $r = 0.914$ ($N = 43$) and the significance also dropped, even though only a few samples were added. Finally, consideration of all rock types resulted in a much poorer fit with $r = 0.863$ ($N = 48$), where those types which were not from a similar geological environment fell in an essentially random pattern.

From this brief discussion, it can be seen that the best results were obtained with samples representing a specific geological environment. Three predictive equations for the 39 Imperial Valley samples are:

$$K_{prd} = -1.42 + 2.18 \, V_P, \quad r = 0.962 \tag{6}$$

$$K_{prd} = 2.01 - 0.095 \, \Phi + 1.66 \, V_P, \quad r = 0.966 \tag{7}$$

$$K_{prd} = -0.534 - 0.082 \, \Phi + 0.0019 \, \sigma$$
$$+ 2.11 \, V_P, \quad r = 0.971 \tag{8}$$

where K_{prd} has units of mcal/cm·sec·°C, V_P is in km/sec, Φ is in percent, and σ is in mmho.

Some other regression equations had equal or higher correlation coefficients, but were eliminated because they had constants which could conceivably cause unreasonably low or negative predictions; had coefficients an order of magnitude higher or lower than that for which the variable might account, implied that variables were simply canceling each other; or had physically incorrect signs on coefficients. Of the three regression equations above, (6) furnishes an excellent fit for one variable, although the negative constant coefficient is problematic. Equation (7) provides some improvement with no obvious defects. Equation (8) is reasonable and gives a higher multiple regression coefficient than Equations (6) or (7); however, Equation (8) has questionable application in the field because of the dependence of electrical conductivity, σ, on the single salinity saturating fluid used. As a result of these considerations, Equation (7) is deemed the most useful.

An indication of the scatter in the data can be seen in Figure 6, which is a multiple regression plot for Equation (7). A distinct separation exists between core sample data and log data. Although there is no suggestion of forced linearity, it would be desirable to have samples which fell into the intermediate range. The standard deviation for this
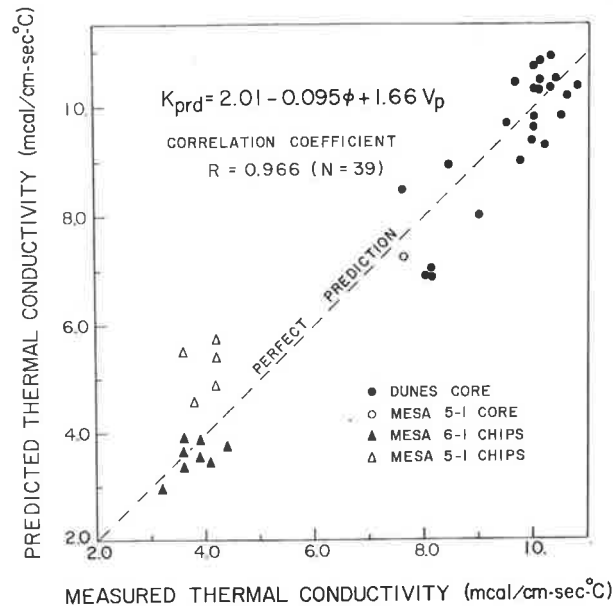

Figure 6. Measured versus predicted thermal conductivity for a multiple linear regression on the Imperial Valley samples, using Equation (7).

regression is ±0.7 mcal/cm·sec·°C, which implies a reliability of approximately 10% for the predicted thermal conductivity.

DISCUSSION

Many empirical investigations have involved thermal conductivity, but only a few have been concerned with the derivation of predictive equations from several physical parameters. Since a number of these empirical studies require specific data or constants which are not usually available, they were disregarded. The following are published relationships which did not require additional laboratory measurements beyond an analysis of geophysical borehole logs.

One of the earliest studies (Thornton, 1919), using data on insulators from ice to wood, resulted in the relation

$$K = V_P^2 D_B^2 \times 10^{-4}, \tag{9}$$

where velocity V_P is in cm/sec, the saturated bulk density D_B in g/cm³, and thermal conductivity K in cal/cm·sec·°C. Dakhnov and Kjakonov (1952) used rock values from the literature to provide

$$K = D_B (3.1/4680) \tag{10}$$

With the same approach, for classes of feldspathic, salt, and other rock types Karl (1965) obtained

$$K = 0.8 \times 10^{-8} \, V_P \tag{11a}$$

$$K = 2.0 \times 10^{-8} \, V_P \tag{11b}$$

and

$$K = 1.3 \times 10^{-8} \, V_P \tag{11c}$$

respectively. Tikhomirov (1968) examined dry and partially saturated individual samples of many rock types, and com-

bined the results into one equation,

$$K = 1.30 \exp (0.58 \, D_D + 0.40 \, S_w) \qquad (12)$$

where S_w is the fractional water saturation and D_D is the bulk density in the dry state. Using core from a wide region of the Siberian lowlands, Moiseyenko and coworkers (1970) derived the relation

$$K = [1.17 + 0.83 \, (3.42 - 0.55 \, \Phi)] \, 10^{-3} \qquad (13)$$

where the term in parentheses is for the dry conductivity, and Φ is the porosity in percent. For a group of rock-forming silicate minerals, Horai (1971) obtained

$$V_P = 6.07 + 0.15 \, K \qquad (14)$$

and

$$V_S = 3.37 + 0.08 \, K \qquad (15)$$

where the thermal conductivities are in mcal/cm·sec·°C, and the compressional and shear velocities in km/sec. If these equations are solved for K in the velocity range of normal rocks, however, they return meaningless negative thermal conductivities. In an experiment with unconsolidated sands, Somerton, Keese, and Chu (1974) found that

$$K' = 0.735 - 0.0130 \, \Phi + 0.363 \, K'_S \, \sqrt{S_w} \qquad (16)$$

where the prime will imply a result in Btu/ft·hr·°F, and K'_S is the thermal conductivity of the solid or component grains. Of most direct interest for the present study is the work by Anand, Somerton, and Gomaa (1973) which yields for dry sandstones

$$K'_D = 0.340 \, D_D - 0.032 \, \Phi + 0.53 \, k^{0.10}$$
$$+ \, 0.013 \, F - 0.031 \qquad (17)$$

and for saturated samples,

$$K' = K'_D \left[1.0 + 0.30 \left(\frac{K'_L}{K'_G} - 1.0 \right)^{0.33} \right.$$
$$\left. + \, 4.57 \left(\frac{\Phi}{100 - \Phi} \frac{K'_L}{K'_D} \right)^{0.48m} \left(\frac{D_B}{D_D} \right)^{-4.3} \right] \qquad (18)$$

where the permeability, k, is in millidarcies; K, Φ, and D are the thermal conductivity, porosity, and density, respectively, with subscripts D, L, and G, for dry rock, saturating liquid, and gas (air), respectively; m, an empirical parameter, is the cementation factor of Archie's formula

$$F = A/\Phi^m \qquad (19)$$

with A another empirical parameter.

Most of the relationships presented above are deficient since they are not based on sets of variables measured for the same samples; instead values from the literature which are related by rock type were considered. The investigators have noted this problem and recommend that multiple measurements on the same samples be a future goal. Investigators who have made multiple measurements often note that velocity should have been a useful parameter, but was not available. When correlations are based on values for dry rock, the reductions for saturated samples tend to be involved and are not always effective for prediction. Results using different equations usually do not agree. Finally, there are not enough studies with their initial data published to determine limitations, areas of overlap, or reliability of extension to other samples.

A comparison of the results of the empirical equations described above with our results is listed in Table 2. None of the equations are completely satisfactory although it must be expected that Equation (7), which is partially based on these samples, will give the best fit. Thornton (1919) used many materials besides rock to obtain Equation (9); and as noted in Table 2, values range widely. Equation (10) yields exceedingly low values, probably because Dakhnov and Kjakonov (1952) used bulk densities of nonporous rocks. Of these published relationships, reasonable agreement of trend is provided by Equation (11c) from Karl (1965), although it is consistently low. Equation (12) derived by Tikhomirov (1968) from many consolidated rock types does not appear to allow for unconsolidated material. Equation (13) of Moiseyenko et al. (1970) from core samples smooths out to very low values. Equation (16) of Somerton, Keese, and Chu (1974) with the assumed solid conductivities used, does not differentiate the unconsolidated materials, even though the equation was derived for these. Finally, Equation (18) by Anand, Somerton, and Gomaa (1973) tends to give very high values especially when applied to the nonreservoir type samples.

None of these relations furnishes completely satisfactory predictions over the range of interest, and none is expected

Table 2.   Comparison of prediction equations.

Sample Identification	Equation numbers for empirical relations								Measurement‡
	9	10	11c	12*	13	16*	18*† F_{mea}/F_{est}	7	
	Thermal conductivity (mcal/cm·sec·°C)								
UCR-1§ (Dunes)	16	1.6	6.6	7.7	3.6	9.2	42/15	9.6	10
115-A§ (Dunes)	16	1.7	6.6	7.9	3.6	9.3	52/18	9.7	9.5
BER§ (Berea)	8.1	1.6	4.9	6.6	3.2	8.8	11/11	6.7	10
5-1680‖ (Mesa)	4.9	1.5	4.0	6.2	2.9	7.0	7.8/8.7	4.9	4.2
6-220‖ (Mesa)	1.4	1.4	2.4	5.6	3.0	7.1	9.1/9.1	2.9	3.2

*Requires assumed-dry density $D_D = D_B - 0.01 \, \Phi$, saturation $S_w = 1.0$, and/or solid conductivity $K_S = 4.5$ Btu/ft·hr·°F if sample is predominately quartz and 3.5 if significant clay in sample. Values based on discussion of Somerton, Keese, and Chu, (1974). †Requires an assumption for gas conductivity (air) $K_G = 0.055$, liquid conductivity (sea water) $K_L = 1.4$, and $m = 1.73$; values taken respectively from Ingersoll, Zobel, and Ingersoll (1954), Ratcliffe (1960), and Timur, Hempkins, and Worthington (1972). Since the measured values of formation factor F_{mea} for the thermally altered samples are quite high, a comparison is also made using F_{est} values estimated from the relation $F = 1.13 \, V^{-1.73}$, $V = 0.01 \, \Phi$ of Timur, Hempkins, and Worthington (1972). ‡Thermal conductivity value obtained from measurement of the sample in a divided-bar apparatus. §Core samples. ‖Unconsolidated material.

to give satisfactory results in the Imperial Valley geological environment. Therefore, we return to Equation (7), derived herein.

APPLICATION

Our ultimate goal is to determine thermal conductivity from common borehole logging parameters. Empirical relations are of little value unless they can predict reasonable thermal conductivities. We have made an initial attempt to predict thermal conductivities for the 300- to 700-m interval of a borehole from the Mesa geothermal field. Casing at 310 m and a convective thermal regime below 670 m determine the limits of useful investigation. Equation (7) has been applied.

The logs from which hand-digitized versions of Figure 7 were made, and a computer evaluation generated from them, provided the basic data for the predicted thermal conductivities. Measured values were taken from drill chips of the unconsolidated sediments, using a cell apparatus in the divided-bar (Sass, Lachenbruch, and Munroe, 1971). Problems exist in the accuracy of this method, for example, the effect of drilling mud, the uncertainty in depth, and the incompleteness of sampling. An ideal relationship with excellent measurements throughout could not be expected to provide exact agreement. As a minimum, however, both predicted and measured values should reflect similar changes with depth, with reasonable explanations for the differences.

From Figure 8, showing the deviations in repeatability for cell measurements (the line arbitrarily passes through the first measurement) with the differences of sampling
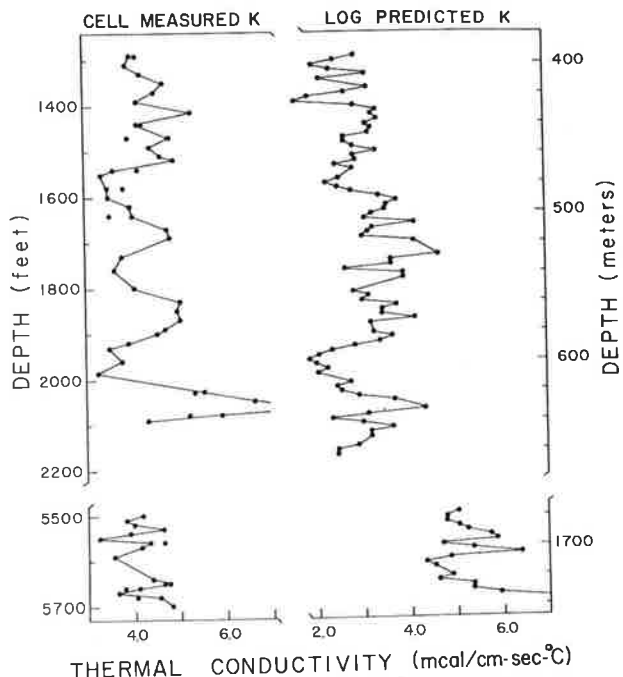

Figure 8.   Thermal conductivities for the Mesa 5-1 geothermal well. Two cell data points at the same depth indicate repeated measurements. Log predictions are from Equation (7).
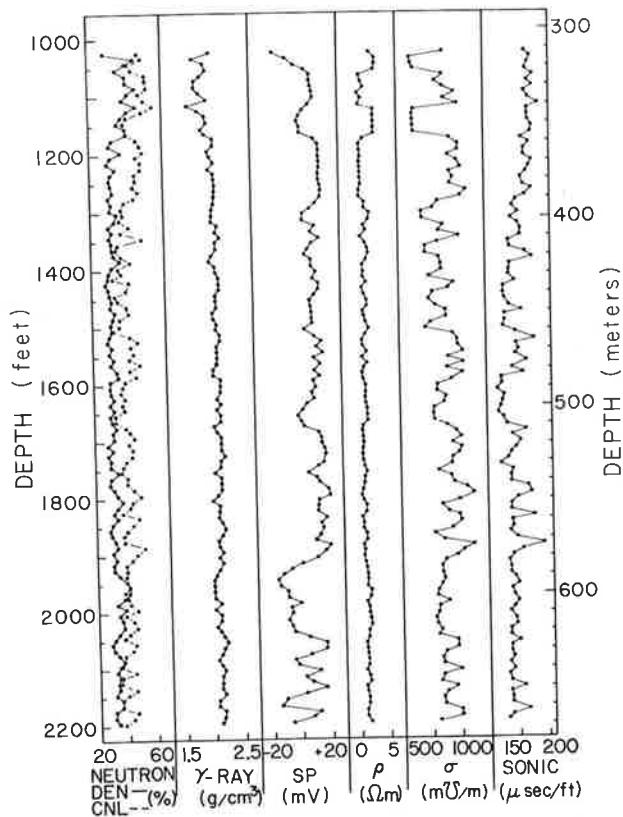
depth, it can be seen that agreement between measured and predicted trends with depth is quite good. However, there is a distinct difference in the mean value using the two methods, from a mean of near 4.5 compared to about 3 mcal/cm·sec·°C. Field washing of the samples, which is certain to have depleted the clay content, probably caused the shift in the mean value of the two methods. Since computer evaluation of the geophysical logs indicates a relatively high content of clay (about 30%) in this shallow interval, higher measured values are expected.

For the deep zone, the shift is reversed from a mean near 4 to about 5 mcal/cm·sec·°C. This is partly a cored section from which samples were taken for the prediction analysis and the values measured on the core samples were about 7 mcal/cm·sec·°C. Deviations in this zone appear to be caused by a relative loss of sand from the drill chips reaching the surface. The core includes sections of clay-cemented sand; computer evaluation of the geophysical logs indicates less clay than in the shallower section, yet the grab samples consist of a considerable amount of shale fragments. There are also shifts with depth of about 10 m in this deep zone, but the gross patterns appear to follow closely. These shifts are almost certainly a result of poor control of sampling and the sampled depth as depth increased.

CONCLUSIONS

A divided-bar design has been developed which is rapid and easy to use. Under test, the design appears to meet or exceed the limits of most divided-bar apparatuses described in the literature. It has been used to estimate the pressure effects on thermal conductivity for three common reference standards and to investigate the effects of sample thickness. The pressure dependence of thermal conductivity for fused silica appears reliably established. Further work

Figure 7.   Digitized geophysical logs for the Mesa 5-1 geothermal well. Data points obtained from original logs by averaging over 3-m intervals.

on the two phenomena, pressure effects and sample thickness, are expected to clarify many subtle problems associated with the divided-bar technique.

A study of the possibility of using borehole logging parameters to predict thermal conductivity has been completed. There is every indication that useful empirical relationships can be obtained. Application of a predictive equation to a geological sequence similar to the one from which it was derived may be reliable. Although a relationship might remain useful in comparable environments, limitations must be determined. There seems little hope of more general predictive relationships being successful; however, typical geological settings can probably be characterized.

Experimental data and an empirical equation for the Imperial Valley of southern California have indicated satisfactory prediction of thermal conductivity in an initial application. In fact, we conclude that the indirect method of prediction may be more accurate than direct cell measurements on drill chips. This may well be true for most unconsolidated sedimentary environments.

ACKNOWLEDGMENTS

REFERENCES CITED

Anand, J., Somerton, W. H., and Gomaa, E., 1973, Predicting thermal conductivities of formations from other known properties: Soc. Petroleum Engineers Jour., v. 13, p. 267-273.

Bird, D. K., 1975, Geology and geochemistry of the Dunes hydrothermal system, Imperial Valley of California [M.S. thesis]: University of California, Riverside, 123 p.

Birch, F., 1950, Flow of heat in the Front Range, Colorado: Geol. Soc. America Bull., v. 61, p. 567-630.

Dakhnov, V. N., and Kjakonov, D. J., 1952, Thermal investigation of fissures (Russian text): Moscow; cited in Karl, 1965, Sec. 2.1.3.

Davis, J. C., 1973, Statistics and data analysis in geology: New York, Wiley and Sons, 550 p.

Debye, P., 1914, Equation of state and quantum hypothesis, with an appendix on thermal conductivity (German text), in Von M. Planck et al., editors, Lectures on the kinetic theory of matter and electricity: Leipzig, Teubner, p. 19-60.

Goss, R. D., 1974, Empirical relationships between thermal conductivity and other physical parameters in rocks [Ph.D. thesis]: University of California, Riverside, 216 p.

Horai, K., 1971, Thermal conductivity of rock-forming minerals: Jour. Geophys., v. 76, p. 1278-1308.

Horai, K., and Simmons, G., 1969, Thermal conductivity of rock-forming minerals: Earth and Planetary Sci. Letters, v. 6, p. 359-368.

Huntington Alloys, 1970, Inconel alloy 625 data catalog: Huntington, W. Va., International Nickel Company, 19 p.

Ingersoll, L. R., Zobel, O. J., and Ingersoll, A. C., 1954, Heat conduction with engineering, geological and other applications: Madison, Wisconsin, University of Wisconsin Press, 325 p.

Karl, R., 1965, Physical rock parameters, wave velocity, thermal conductivity, (German text): Freiberger Forschungshefte, Series C, v. 197, p. 7-76.

Kittel, C., 1971, Introduction to solid state physics: New York, Wiley and Sons, 4th ed., 766 p.

Moiseyenko, V. I., Doroginitskaya, L. M., Leont'ev, Ye. I., and Sokolova, L. S., 1970, Dependence of the heat conductivity of the clastic rocks of the West Siberian Lowlands on other physical parameters (Russian text), in Methods of determining the thermal properties of rocks: Akademy Nauk SSR Sibirsk, Otedeleniye Geologiya i Geofizika, no. 2, p. 106-110.

Ratcliffe, E. H., 1959, Thermal conductivities of fused and crystalline quartz: British Jour. Appl. Physics, v. 10, p. 22-25.

——, 1960, The thermal conductivities of ocean sediments: Jour. Geophys. Research, v. 65, p. 1535-1541.

Sass, J. H., Lachenbruch, A. H., and Munroe, R. J., 1971, Thermal conductivity of rocks from measurements on fragments and its application to heat-flow determinations: Jour. Geophys. Research, v. 76, p. 3391-3401.

Somerton, W. H., Keese, J. A., and Chu, S. L., 1974, Thermal behavior of unconsolidated oil sands: Soc. Petroleum Engineers Jour., v. 14, p. 513-521.

Thornton, W. M., 1919, The thermal conductivity of solid insulators: cited in Karl, 1965, Sec. 2.2.

Tikhomirov, V. M., 1968, The thermal conductivity of rocks and its relationship to density, moisture content, and temperature (Russian text): Neftianoe Khoziaistvo, v. 46, p. 36-40.

Timur, A., Hempkins, W. B., and Worthington, A. E., 1972, Porosity and pressure dependence of formation resistivity factor for sandstones: Canadian Well Logging Society Trans., Fourth Formation Evaluation Symposium, Sec. D.