Integer Instruction That Works: Best Practices for Instruction of Integers for All Students Including LEP Learners

Math, LEP – Grades 5-8

Frustrated by the fact that your students incorrectly apply the rule that two negatives make a positive to addition problems? Finding it difficult to provide effective mathematics instruction to the growing population of students with limited English proficiency (LEP)? Learn how to apply the best practices and teaching strategies that will allow students to meet required state and district standards as well as develop meaning that will remain after the test is over. Explore how manipulatives, models, graphic organizers, and reading/writing strategies assist in meeting the needs of all students, including LEP learners.

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Connie Colton is a National Board certified teacher teacher. She has spent the past 16 years teaching mathematics for the Omaha Public School District in Omaha, Nebraska. Currently, Connie is the math department chairwoman at McMillan Magnet Center. She serves as a new teacher mentor, a magnet school representative, the math competition coach and an EXCELS School Improvement team leader. Connie provides “Best Practices Workshops” monthly for new mathematics teachers. She was formerly a member of her school district’s Criterion Referenced Test writing committee and currently writes curriculum for the district. Connie has been trained in the Carnegie Algebra Tutorial Program and is Banneker Community of Excellence in Mathematics and Science (CEMS) Certified. She holds a bachelor’s degree in secondary education with a field endorsement in mathematics from the University of Nebraska at Omaha. Connie is currently pursuing a master’s degree in secondary education with an emphasis in mathematics and curriculum development.

Absolute Value and Adding Integers

Notes	Examples
absolute value: a number's distance from zero, is **never** negative absolute value symbol is $\lvert x \rvert$	**examples:** $\lvert -5 \rvert = 5$ $\lvert 5 \rvert = 5$ $\lvert 0 \rvert = 0$
addition **traditional notes read**: when the numbers have "like" signs (both positive or both negative) - add the absolute values and keep the sign when the numbers have "unlike"signs (one positive and one negative) - subtract the absolute values and keep the sign of the number with the larger absolute value **alternate notes**: use **combine** or **cancel** in place of add and subtract since a positive and a negative are opposites, and opposites cancel 1 positive and 1 negative cancel 1 positive and 1 negative = zero for adding: 1) cancel all partners (if any) 2) count what is left	**traditional examples:** -4 + -3 adding absolute values: $\lvert -4 \rvert + \lvert -3 \rvert = 4 + 3 = 7$ then keep the sign, so = -7 -9 + 5 subtract the absolute values: $\lvert -9 \rvert - \lvert 5 \rvert = 9 - 5 = 4$ then keep the sign of the larger absolute value, so = -9 **combine and cancel examples:** -4 + -3 – – – – – – – = -7 (no partners, so **combine** or count all) -9 + 5 – – – – – – – – – + + + + + = -4 (**cancel** 5 partners or zeroes, count what is left) -6 + 6 – – – – – – + + + + + + = 0
Summary: (in your own words) *Include the value on the right is always larger.	

Adding Integers

Definition

absolute value:
a number's distance from zero
is **never** negative
absolute value symbol $|x|$

traditional method for adding:
-if the signs are alike, add the absolute value and keep the sign
-if the signs are different, subtract the absolute values and keep the sign of the number with the larger absolute value

alternate method for adding:
combine and cancel rule
cancel zeroes, count what is left
if no zeroes, **combine** and count all

Characteristics

in life opposites cancel:
-if you earn $1 and spend $1
you have no money
-if you gain a yard and lose a yard you have no gain or loss
-if the temperature rises 1 degree and falls 1 degree
the temperature shows no change

in math opposites also cancel: positive 1 and negative 1 make 0 because they are opposites

Examples

-4 + -3 – – – –
– – – = -7
no partners, so **combine** or count all

-9 + 5 – – – – – – – – –
+ + + + + = -4
cancel 5 partners or zeroes, count what is left

-6 + 6 – – – – – –
+ + + + + + = 0
after cancelling partners, none are left

Non-Examples

you may have heard a rule that 2 negatives make a positive, that is not true when adding integers
ex: $-4 + -3 \neq 7$ (remember it = -7)

you may think that since the operation is add, you should always add the digits, that is not true with integers, sometimes you actually subtract the digits
ex: $-9 + 5 \neq 14$ or -14 (remember it = -4)
ex: $-6 + 6 \neq 12$ or -12 (remember it = 0)

Subtracting Integers

Notes	Examples
<u>subtraction</u> 1) rewrite all subtraction problems as an equivalent addition problem using: COPY, CHANGE, OPPOSITE then follow the combine/cancel steps for adding: 2) cancel all partners (if any) 3) count what is left	**examples:** 9 – 5 9 + -5 + + + + + + + + + – – – – – = 4 (**cancel** 5 partners or zeroes, count what is left) -4 – -3 -4 + 3 – – – – + + + = -1 (**cancel** 3 partners or zeroes, count what is left) -6 – 8 -6 + -8 – – – – – – – – – – – – – – = -14 (no partners, so **combine** or count all) 2 – -5 2 + 5 + + + + + + + = 7 (no partners, so **combine** or count all)
Summary: (in your own words)	

Subtracting Integers

Definition

<u>subtracting traditional method</u>:
rewrite the subtraction problem as an equivalent addition problem, then:
-if the signs are alike, add the absolute value and keep the sign
-if the signs are different, subtract the absolute values and keep the larger sign

<u>subtracting alternate method:</u>
1) rewrite the subtraction problem as an equivalent addition problem using **copy - change - opposite**
2) **cancel** zeroes, count what is left
3) if no zeroes, **combine** or count all

Characteristics

<u>in life subtracting a negative means adding a positive</u>
ex: taking away a debt means making money
<u>in math subtracting a negative also means adding a positive</u>
9 – -5 = 9 + 5
<u>in life subtracting a positive means adding a negative</u>
ex: taking away a balance means losing money
<u>in math subtracting a positive also means adding a negative</u>
5 – 9 = 5 + -9

Examples

9 – 5
9 + -5 + + + + + + + + +
– – – – – = 4
(**cancel** 5 zeroes, count what is left)

-4 – -3
-4 + 3 – – – –
+ + + = -1
(**cancel** 3 zeroes, count what is left)

-6 – 8
-6 + -8 – – – – – –
– – – – – – – – = -14
(no partners, so **combine** or count all)

2 – -5
2 + 5 + +
+ + + + + = 7
(no partners, so **combine** or count all)

Non-Examples

Do not use the rule that 2 negatives make a positive, that is not true when adding OR subtracting integers
ex: $-4 - -3 \neq 1$ (you must first rewrite as -4 + 3, then cancel zeroes and will = -1)
you may think that since the operation is subtract, you should always subtract the digits, that is not true with integers, sometimes you actually add the digits
ex: $-9 - 5 \neq 4$ or -4 (rewrite as -9 + -5, then no partners, so count all and it = -14)
ex: $6 - -6 \neq 0$ (rewrite as 6 + 6, then no partners, so count all and it = 12)

Multiplying and Dividing Integers

Notes	Examples
<u>multiplication and division</u> 1) multiply or divide the digits 2) count the negative signs 3) move to the opposite side of the number line once for each negative sign (since a negative sign means "opposite") Hint: odd # of negative signs/negative answer even # of negative signs/positive answer	**examples:** (1)negative -2 x 3 = -6 (1)negative 2 x -3 = -6 (2)negatives -2 x -3 = 6 (3)negatives -2 x -3 x -4 = -24 (4)negatives -1 x -2 x -3 x -4 = 24 (5) negatives -1 x -2 x -3 x -4 x -2 = -48 (1)negative -6 ÷ 3 = -2 (1)negative 6 ÷ -3 = -2 (2)negatives $\frac{-6}{-3} = 6$

Summary: (in your own words)

Definition

Rules:

positive x or ÷ positive = positive
negative x or ÷ negative = positive
positive x or ÷ negative = negative
negative x or ÷ positive = negative

(+)(+)=(+) (+)/(+)=(+)
(−)(−)=(+) (−)/(−)=(+)
(+)(−)=(−) (+)/(−)=(+)
(−)(+)=(−) (−)/(+)=(−)

conclusion: two negatives make a positive when multiplying or dividing

hint: can always find the product of just two numbers at a time and follow the table

Characteristics

negative means opposite:
apply logic and 3 groups of -5 should be a total of 15 negatives, also 3(-5) is 3(5) = 15 then finding the opposite of 15 once is negative 15, so = -15

-2(-4) is 2(4) = 8 then finding the opposite of 8 twice comes back to positive 8, so = 8

-2(-3)(-4) is 2(3)(4) = 24 then finding the opposite of 24 three times comes back to negative 24, so = -24

remember:
odd # negatives=negative answer
even # negatives=positive answer

Multiplying and Dividing Integers

Examples

(1)negative -2 x 3 = -6
(1)negative 2 x -3 = -6
(2)negatives -2 x -3 = 6
(3)negatives -2 x -3 x -4 = -24
(4)negatives -1 x -2 x -3 x -4 = 24
(5) negatives -1 x -2 x -3 x -4 x -2 = -48

(1)negative -6 ÷ 3 = -2
(1)negative 6 ÷ -3 = -2
(2)negatives $\frac{-6}{-3} = 6$

remember:
odd # negatives=negative answer
even # negatives=positive answer

Non-Examples

now is the time to apply the rule that 2 negatives make a positive, not to use the cancel and combine method

ex: in 4 • -3 we do not care that the positive number is larger than the negative number, we also do not cancel 3 negatives and 3 positives, instead we multiply and find the opposite once, =-12

ex: in -5 • -4 the solution is not negative because the numbers are negative, instead we multiply 5 • 4 to make 20, then find the opposite twice, which comes back to positive 20, or use the rule 2 negatives make a positive

Integers and Order of Operations Lesson Plan

Notes to the teacher:
Common errors in integers occur when students try to memorize a set of rules that have no meaning. This lesson attempts to create understanding by emphasizing the following concepts:
1. Negative means opposite
2. Opposites cancel
3. Every subtraction problem has an equivalent addition problem

Common errors in order of operations occur when students interpret PEMDAS (or Please Excuse My Dear Aunt Sally) to mean add before subtract, and multiply before divide. This lesson emphasizes this concept as well as the "invisible" operation multiplication.

Anticipatory Set:
Have students sketch a set of stairs 12 steps high on their paper, labeling their steps 1 - 12. Students begin at ground level for question 1 only, and use their stairs to answer the following:
1. You climb 9 stairs, descend 5 stairs, where are you?
2. You then climb 2 stairs and descend 3 stairs, where are you?
3. Next you climb 8 stairs and descend 4 stairs, where are you?
4. Finally you climb 3 stairs and descend 10 stairs, where are you?
5. Write a mathematical sentence that represents the steps you took.
6. Try to rewrite your mathematical sentence without using subtraction signs.

Objective:
Students will add, subtract, multiply, and divide integers including proper use of order of operations.

Materials:
1. Warm-up
2. Scissors
3. (1) laminated copy each of + and – integer chips per student

*You will need to create a sheet of (8) 1" x 10" disconnected boxes. Within each box, draw lines at every inch to create (10) 1" x 1" squares on each strip. Run a copy of this document before continuing. On 1 copy, draw a + sign in each of the squares, and on the other copy draw a – sign in each of the squares. You may choose to run the positives and negatives on different color paper, but it is not necessary. Laminate enough copies for each student to have 1 page each of

negatives and positives. Students are to cut (2) strips on each sheet into individual chips, and leave the other (6) strips intact to use as tens.

4. Student copy of ***Integers and Order of Operations*** (included), one per student

Procedure:

- Pass out supplies while reviewing the warm-up
- Have students cut around the outside edges of all (8) strips on each page, then cut only (2) strips on each page into the individual squares
- Pass out student copies of *Integers and Order of Operations*
- Have students use the manipulatives to develop the sense of combining like "things" and canceling unlike "things". Avoid using the words add and subtract. Begin with the following examples:
  Earn a dollar; spend a dollar
  Take 1 step forward; 1 step backward
  Gain 5 yards on one play; lose 5 yards on the next play
  Temperature rises 10 degrees; falls 10 degrees
- Use the "earn a dollar, spend a dollar" example to explain that each pair of one positive chip and one negative chip equals a zero, or cancels each other out.
- Have students begin working the problems using the chips to create a visual understanding of when numbers are combined, when they are cancelled, and how to decide whether positives or negatives will be left over after creating partners of each positive and negative.
- The most common error students make with the chips is in creating partners out of 2 negatives and then canceling these 2 negatives even though they do not create a zero.
- Work through all addition problems, insisting students use the chips to develop the visual image needed for understanding.

Example:

1. -8 + 10 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -8 + 10 =____

Students should circle "different" and "cancel" in the 1st row, and "more" and "positives" in the 2nd row. Because students identified there were more positives, this should lead to understanding why the answer is positive later when they work without the chips.

- Before beginning the subtraction portion of the lesson, demonstrate equivalent subtraction and addition problems that students will understand such as:
  $9 - 6 = 9 + -6$ $8 - 3 = 8 + -3$ $7 - 2 = 7 + -2$ $10 - 4 = 10 + -4$

(Since most students do not *believe* that addition and subtraction problems can be equivalent, this will be the most difficult part of building conceptual understanding of subtracting integers. Many students will simply be able to correctly use the "copy, change, opposite" steps to arrive at the correct answer, but will still not truly understand the concept.)

- Have students work the subtraction problems by first rewriting the subtraction problem as an addition problem, and then using the chips to find the solution. DO NOT have students set out the model for the subtraction problem, and add zeros as the textbooks traditionally do. This only confuses students!

Example:

13. -8 – 10 Copy/change/opposite makes the equivalent addition problem _____
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -8 – 10 =____

Students should answer -8 + -10 in the 1st row, circle "alike" and "combine" in the 2nd row, circle "all" and "negatives" in the 3rd row, and solve the problems with their chips.

- For the multiplication and division section of the lesson, very few alternative teaching methods are available. Students either remember this correctly or incorrectly at testing time.
- A few additional suggestions are as follows:

a negative x a positive is like flipping a quarter to tails (or negative)
a negative x a negative is like flipping a quarter twice, returning to heads (or positive)
a negative x a negative x a negative is like flipping a quarter 3 times, landing on tails (or negative)

a negative x a positive is finding the opposite once
(the opposite of 5 is -5)
a negative x a negative is finding the opposite twice
(the opposite 5 is -5, and the opposite of -5 is 5)
a negative x a negative x a negative is finding the opposite 3 times
(the opposite 5 is -5,the opposite of -5 is 5, and the opposite of 5 is -5)

- For the order of operations section of the lesson, the emphasis in the selected problems is on the most common mistakes students make. These mistakes include always adding before subtracting, always multiplying before dividing,

multiplying the base and exponent, and adding or subtracting before multiplying in problems such as $9 - 4(6)$.

- Allow students to perform only 1 operation in each step, and then rewrite the remaining problem in its entirety before performing the next operation.
- Having students underline or circle the step they are performing will help prevent students from performing multiple operations in 1 step.
- Collect the manipulatives before students begin the mixed review portion of the lesson.
- Demonstrate how students can use + and – signs in place of the chips to calculate answers.
  Example: -8 + 6 would look like + + + + + + + + showing that 2 positives
  – – – – – – should be left over
- Correct the mixed review for immediate feedback
- The mixed review may then be used the next day as a lesson opener to check for retention.

Summary:

The mixed review actually serves as a lesson summary, but you may choose to have students share anything that the day's lesson clarified for them. Verbalizing procedures helps students cement concepts into long-term memory more than just applying the procedures can.

Class_____ Name_____
Period_____ Date_____

Integers and Order of Operations

Adding Integers Directions:

1. Count the appropriate chips to match the given problem.
2. Circle the correct choice in each of the underlined sections.
3. Solve the problem.

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1. -8 + 10 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -8 + 10 =____

2. 6 + -13 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. 6 + -13 = ____

3. -5 + -12 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -5 + -12 =____

4. -9 + 16 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -9 + 16 = ____

5. -28 + 15 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -28 + 15 =____

6. 16 + -33 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. 16 + -33 = ___

7. -25 + -32 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -25 + -32 =___

8. -19 + 16 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. -19 + 16 = ___

9. 22 + -17 The numbers are alike/different so I should combine/cancel.
   I have more/all negatives or more/all positives. 22 + -17 = ___

10. -18 + -19 The numbers are alike/different so I should combine/cancel.
    I have more/all negatives or more/all positives. -18 + -19 =___

11. -21 + 27 The numbers are alike/different so I should combine/cancel.
    I have more/all negatives or more/all positives. -21 + 27 = ___

12. -35 + 13 The numbers are alike/different so I should combine/cancel.
    I have more/all negatives or more/all positives. -35 + 13 = ___

Integers and Order of Operations (cont)

Subtracting Integers Directions:

1. Rewrite the subtraction problem as an equivalent addition problem using copy/change/opposite.
2. Count the appropriate chips to match the given problem.
3. Circle the correct choice in each of the underlined sections.
4. Solve the problem.

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13. -8 – 10 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -8 – 10 =____

14. 6 – -13 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. 6 – -13 = ____

15. -5 – -12 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -5 – -12 =____

16. -9 – 16 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -9 – 16 = ____

17. 15 – 28 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. 15 – 28 =____

18. 16 – -33 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. 16 – -33 = ___

19. -25 – -32 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -25 – -32 =___

20. -19 – 16 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -19 – 16 = ___

21. -27 – -27 _____ Copy/change/opposite makes the equivalent addition problem
The numbers are alike/different so I should combine/cancel.
I have more/all negatives or more/all positives. -27 – -27 = ___

Integers and Order of Operations (cont)

Multiplying and Dividing Integers Directions:

1. Multiply or divide the numbers ignoring the positive/negative signs.
2. Since each negative sign means the opposite, decide how many times you must find the opposite to decide if the final answer is positive or negative.
3. Solve the problem.

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22. $-4 \bullet 10 =$ _____
23. $-60 \div 12 =$ _____
24. $13 \bullet -3 =$ _____
25. $-36 \div -6 =$ _____
26. $-12 \bullet -12 =$ _____
27. $64 \div -16 =$ _____
28. $-15 \bullet 5 =$ _____
29. $-99 \div -11 =$ _____
30. $-2 \bullet -3 \bullet -3 =$ _____
31. $-42 \div -14 =$ _____
32. $-6 \bullet 3 \bullet -4 =$ _____
33. $84 \div -21 =$ _____

Order of Operations Directions:

1. Underline the first step according to the order of operations.
   (Please Excuse My Dear Aunt Sally or PEMDAS)
   - Parentheses
   - Exponents
   - Multiply or Divide *left to right*
   - Add or Subtract *left to right*
2. Find the answer to the underlined portion only.
3. Rewrite the remaining problem.
4. Continue to underline, solve, and rewrite until the problem is solved.

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34. $14 - 3 + 7$
    $=$ _____
    $=$ _____
35. $16 + 7 - 4$
    $=$ _____
    $=$ _____
36. $19 - 5 + 6$
    $=$ _____
    $=$ _____
37. $6 \div 3 \bullet 2$
    $=$ _____
    $=$ _____
38. $18 \div 9(2)$
    $=$ _____
    $=$ _____
39. $10 \bullet 15 \div 3$
    $=$ _____
    $=$ _____
40. $6 + 4 \div 2$
    $=$ _____
    $=$ _____
41. $45 - 9(5)$
    $=$ _____
    $=$ _____
42. $12 \bullet 3 + 18$
    $=$ _____
    $=$ _____

Integers and Order of Operations (cont)

43. $18 - 2(9 - 4)$

= _____

= _____

= _____

44. $24 - 2^3 \div 4$

= _____

= _____

= _____

45. $25 - 3^2 + 4$

= _____

= _____

= _____

46. $17 + 3(8 - 5)$

= _____

= _____

= _____

47. $5 \bullet 6 - 3 \bullet 8$

= _____

= _____

= _____

48. $28 + 4^2 \div 2$

= _____

= _____

= _____

Mixed Review

1. $-1 + -3$ = _____
2. $-3 - 8$ = _____
3. $-4 \bullet -3$ = _____
4. $10 - 17$ = _____
5. $-20 \div -5$ = _____
6. $-5 + 12$ = _____
7. $-16 - -8$ = _____
8. $2 - -19$ = _____
9. $-3 \bullet 12$ = _____
10. $-72 \div -9$ = _____
11. $13 + -7$ = _____
12. $-4 \bullet -5 \bullet -6$ = _____
13. $3^2 - 5 + 10$ = _____
14. $3 + 2(7 - 5)$ = _____
15. $15 - 3(10 - 2^3)$ = _____

Combination Notes

Notes	Graphic Representations

Summary: