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Abstract

The wavelength division multiplexing routing and provisioning problem with uncertain demands and a fixed budget is modeled as a robust optimization problem whose objective is to minimize a regret function. This regret function models the total amount of over or under provisioning in the network resulting from uncertainty in a demand forecast. Uncertain demands are modeled by a set of scenarios with known probabilities, and the regret function is a piece-wise linear convex function. The model is an integer linear program with a large number of continuous variables, but only a pair of integer variables for each link. A realistic test case having 107 links, 67 nodes, and 200 demands was solved in only a few seconds using CPLEX with default settings. In an empirical study, the proposed robust model was superior to a mean-value model, a stochastic programming model, and a worst-case model, using the metrics of minimum regret and minimum unsatisfied demand.

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1 Introduction

In simplest terms, the *wavelength division multiplexing (WDM) routing and provisioning problem* can be stated as follows:

> *Given a network topology and an estimate of the point-to-point demand traffic, determine the routing for each demand and the least-cost WDM equipment configuration required to support the routes.*

This problem can be modeled as an integer linear program (ILP). Modern modeling languages such as AMPL [8, 20], GAMS [7, 23], and OPL [18, 24] can be used to create very detailed design models of a problem instance with only a moderate amount of effort. Modern solvers such as CPLEX [21, 22] can be called by the modeling languages to obtain near optimal solutions to real-world problem instances. The key data that drives these models is the demand forecast. Since the quality of the demand forecasts in this domain is often found to be lacking, network providers and their clients are often concerned that an optimal design based on an erroneous forecast may prove to be a poor investment. If the forecast is too low, the network will not have enough capacity to meet all the demand. Conversely, a network designed for a forecast that predicts more demand than is realized will be over provisioned with expensive, underutilized equipment.

Traditionally, design problems of this type with uncertain demand use a set of forecasts, each with a given probability of occurrence. Such sets generally include optimistic and pessimistic forecasts as well as some intermediate values. It would then be possible to construct a design based on the *mean value* or *worst-case value* for the set of potential traffic demands. However, large error bounds may result from such procedures (see Birge [6]). Other strategies that appear in the literature include *sensitivity analysis* and *stochastic programming.*

While the stochastic programming approach has many disciples (the bibliography [17] lists more than 1000 references), we chose to investigate the *robust optimization methodology* as described in the influential paper of Mulvey, Vanderbei, and Zenios [12]. The idea of robust optimization is to create a design that will be fairly good (i.e., robust) regardless of which demand scenario is realized. The robust methodology uses a regret function to capture this notion of robustness. In this study, the robust model also includes a budget constraint that limits the total cost for WDM equipment that can be deployed to minimize the regret function.

1.1 Survey of Literature

There are several different concepts of *robustness* that appear in the literature (see e.g. Paraskevopoulos, Karakitsos, and Rustem [13], Mulvey, Vanderbei, and Zenios [12], Kouvelis and Yu [10], Ben-Tal and Nemirovski [4, 5], and Averbakh [2].) One of the most popular models is the minmax-regret model for combinatorial optimization problems. A state-of-the-art presentation of this strategy and important applications can be found in [10]. Another popular model is the min-regret model for linear programming models as described by Mulvey et al. [12], and demonstrated in Bai, Carpenter and Mulvey [3]. In our investigation, we adopt the concept and model presented in [3, 12]. Other applications of robust optimization using variations of this strategy may be found in Laguna [11], Soteriou and Chase [15], and Gryseels, Sorbello, and Demeester [9]. A stochastic programming approach for a problem similar to the one we address may be found in Sen, Doverspike and Cosares [14].

1.2 Contributions

A WDM network is composed of fiber, terminal equipment (TE), optical amplifiers (As), and regenerators (Rs). For this investigation, TEs refer to wavelength transponders that perform the optical-electrical-optical conversion. Transponders account for the largest percentage of the node cost for WDM equipment. Other equipment such as cross-connects, couplers, etc. are not modeled and would be added to a final design implemented by a client. If we know the number of transponders required for a given node, then we can easily determine the requirements for the other equipment. In order to simplify the proposed robust model, we only included the transponders. Typically, the extra WDM equipment would increase the total equipment cost by 5 to 10%.

The first contribution is a detailed optimization model that determines the number and location of TEs, As, and Rs required to satisfy a given demand forecast at minimum cost. Using multiple demand scenarios as input, the second contribution is a robust version of this model that minimizes the decision maker's regret under a budget restriction. The final contribution is a demonstration of this methodology on a pair of test problems with realistic characteristics. The largest test problem has 67 nodes, 107 links, 200 demand pairs, and 5 demand scenarios.

2 The Models

In this section, an ILP is presented for the basic provisioning problem. We use an arc-path model that determines the equipment required to route a set of point-to-point demands for a given scenario. In addition, a robust optimization model is presented that determines the equipment needed when all scenarios are considered simultaneously. The robust model uses a convex, piece-wise linear function to model regret and follows the strategy presented in Mulvey et al. [12].

2.1 Sets

The network topology is represented as a graph $G = [N, E]$, where N denotes the set of nodes and $E \subseteq N \times N$ denotes the set of links. For each $n \in N$, A_n denotes the set of links adjacent to node n. The origin/destination node pairs $o, d \in N$ corresponding to the point-to-point demands are given by $D \subseteq N \times N$. For each $(o, d) \in D$, J_{od} denotes the set of possible paths from o to d that can be used to route this demand. For each $n \in N$ ($e \in E$), P_n (L_e) denotes the set of paths containing node n (link e). The set of scenarios for a problem having $\bar{s}$ scenarios is denoted $S = \{1, \ldots, \bar{s}\}$.

2.2 Constants

For this model we assume that a maximum of 192 DS3s can be carried on each wavelength (λ) and that a fiber has 80 channels (2 λ's/channel). We assume that when signal regeneration is required regenerators are installed on each bi-directional channel. When required by optical reach limitations, optical amplifiers can boost the signal of an entire fiber. The constants used in our models along with the specific values used for the first of our two test problems, DA, may be found in Table 1. The equipment costs used in the study approximate current market prices, but do not represent specific prices offered by any particular vendor.

2.3 Decision Variables

The various types of decision variables used in the models are defined in Table 2. By requiring four of these variables to assume integer values, the number of optical amplifiers and number of regenerators will assume integer values. For our test cases, the number of TEs are always large and we simply round them up to the nearest integer.

2.4 The Basic Routing and Provisioning Model

For each scenario s, there is a basic provisioning model whose objective is to minimize the *total cost for provisioning* the network. The network has TE equipment located at each node and optical amplifiers and regenerators associated with the links as need. The objective is as follows:

$$\text{minimize} \quad \sum_{n \in N} C^{TE} \ell_n^s + \sum_{e \in E} (C^R r_e^s + C^A a_e^s) \tag{1}$$

There are six sets of constraints that define this model. The first set of constraints ensure *demand satisfaction* and are given as follows:

$$\sum_{p \in J_{od}} x_p^s = R_{od}^s, \qquad \forall (o,d) \in D \tag{2}$$

The second set of constraints *convert path capacity to link capacity* and are defined by

$$\sum_{p \in L_e} x_p^s = z_e^s, \qquad \forall e \in E \tag{3}$$

The third set of constraints *convert link capacity to TEs* and are as follows:

$$z_e^s \leq M^{TE} t_e^s, \qquad \forall e \in E \tag{4}$$

The fourth set of constraints *accumulate TEs on links to give the number of TEs at each node.* These are simply accounting constraints and a can be substituted out of the model. They are

$$\sum_{e \in A_n} t_e^s = \ell_n^s, \qquad \forall n \in N \tag{5}$$

The fifth set of constraints *convert link capacity into fibers and channels* and are simply

$$z_e^s \le M^A f_e^s, \qquad \forall e \in E \tag{6}$$

$$z_e^s \le M^R c_e^s, \qquad \forall e \in E \tag{7}$$

The sixth type of constraints *convert fiber and channels into amplifiers and regenerators.* They are defined as follows:

$$G_e^A f_e^s = a_e^s, \qquad \forall e \in E \tag{8}$$

$$G_e^R c_e^s = r_e^s, \qquad \forall e \in E \tag{9}$$

The basic routing and provisioning model for scenario s is the ILP defined by (1)-(9). The only change for different scenarios is the right-hand-side for the demand-satisfaction constraints (2).

2.5 The Robust Model

We use the general modeling framework as described by Mulvey et al. [12] to construct our robust model. The key to the robust model is the construction of a regret function to capture the trade-off between too little network capacity and too much excess capacity. For the WDM routing and provisioning problem, the client experiences regret when either the network can not meet a substantial part of the demand or when the network has been over provisioned and most of the network only uses a small amount of its available capacity. Using z_{ods}^+ (z_{ods}^-) to represent under (over) provisioning, the demand constraints can be modeled as follows:

$$\sum_{p \in J_{od}} x_p^s = R_{od}^s - z_{ods}^+ + z_{ods}^-, \qquad \forall (o,d) \in D, \forall s \in S$$

Using P_s to denote the probability of scenario s, Mulvey et al. [12] recommend using some type of quadratic function of the form:

$$\sum_{s \in S} P_s \sum_{(o,d) \in D} [(z^+_{ods})^2 + (z^-_{ods})^2].$$

For our problem, under provisioning is viewed as resulting in more regret than over provisioning and we use a non-symmetrical, piece-wise linear regret function with four pieces each for under and over provisioning.

Let $R_{\max}$ denote the largest demand value, then $0 \leq z^+_{ods} \leq R_{\max}$ and $0 \leq z^-_{ods} \leq R_{\max}$ for all o, d and s. Let c^u_k (c^o_k) denote the linear cost for under (over) provisioning for linear piece k of the regret function where we assume that $c^u_{k+1} > c^u_k$ ($c^o_{k+1} > c^o_k$) for $k = 1, 2, 3$. Under this assumption the regret function is convex. Then,

$$z^+_{ods} = \sum_{k=1,\ldots,4} \bar{z}^+_{odsk},$$
$$z^-_{ods} = \sum_{k=1,\ldots,4} \bar{z}^-_{odsk}, \text{ and}$$

the regret function is

$$\sum_{(o,d) \in D} \{\sum_{s \in S} P_s [\sum_{k=1,\ldots,4} (c^u_k \bar{z}^+_{odsk} + c^o_k \bar{z}^-_{odsk})]\}.$$

For this problem, clients are also concerned about a budget restriction. Our robust model is to minimize the regret subject to a budget restriction and structural constraints defined for the basic model. A mathematical description of the *robust design model* for the WDM routing and provisioning problem is as follows:

$$\text{minimize} \sum_{(o,d) \in D} \{\sum_{s \in S} P_s [\sum_{k=1,\ldots,4} (c^u_k \bar{z}^+_{odsk} + c^o_k \bar{z}^-_{odsk})]\} \tag{10}$$

subject to

(accumulation of regret function pieces)

$$z^+_{ods} = \sum_{k=1,\ldots,4} \bar{z}^+_{odsk}, \qquad \forall (o,d) \in D, \forall s \in S \tag{11}$$

$$z^-_{ods} = \sum_{k=1,\ldots,4} \bar{z}^-_{odsk}, \qquad \forall (o,d) \in D, \forall s \in S \tag{12}$$

(budget constraint)

$$\bar{E} = \sum_{n \in N} C^{TE} \ell_n + \sum_{e \in E} (C^R r_e + C^A a_e) \leq \text{Budget} \tag{13}$$

(demand constraints)

$$\sum_{p \in J_{od}} x_p^s = R_{od}^s - z_{ods}^+ + z_{ods}^-, \qquad \forall (o,d) \in D, \forall s \in S \tag{14}$$

(conversion of path flows to link flows)

$$\sum_{p \in L_e} x_p = z_e, \qquad \forall e \in E \tag{15}$$

(conversion of DS3s on links to TEs)

$$z_e \leq M^{TE} t_e, \qquad \forall e \in E \tag{16}$$

(conversion of TEs at nodes)

$$\sum_{e \in A_n} t_e = \ell_n, \qquad \forall n \in N \tag{17}$$

(conversion of DS3s on links to fibers and channels)

$$z_e \leq M^A f_e, \qquad \forall e \in E \tag{18}$$

$$z_e \leq M^R c_e, \qquad \forall e \in E \tag{19}$$

(conversion of fibers and channels to amplifiers and regenerators)

$$G_e^A f_e = a_e, \qquad \forall e \in E \tag{20}$$

$$G_e^R c_e = r_e, \qquad \forall e \in E \tag{21}$$

(bounds on individual pieces)

$$0 \leq \bar{z}_{odsk}^+ \leq R_{\max}/4, \qquad \forall (o,d) \in D, \forall s \in S, k = 1, \ldots, 4 \tag{22}$$

$$0 \leq \bar{z}_{odsk}^- \leq R_{\max}/4, \qquad \forall (o,d) \in D, \forall s \in S, k = 1, \ldots, 4 \tag{23}$$

(bounds on fibers)

$$f_e \leq c_e, \qquad \forall e \in E \tag{24}$$

All other variables have non-negativity restrictions and f_e and c_e are restricted to be integer.

3 Alternative Models

The robust optimization model is only one of several models that can be used to help design a network when the demand forecast is uncertain. Other possibilities include the *mean-value* *model*, the *stochastic programming model*, and the *worst-case model*. When applied to the WDM routing and provisioning problem, these models differ only in the objective function to be optimized and possibly the formulation of the demand constraints.

Let the mean of the demand scenarios be given by

$$\bar{R}_{od} = \sum_{s \in S} P_s R^s_{od}, \qquad \forall (o,d) \in D.$$

The *mean-value model* will determine the least-cost design that satisfies the mean of the demand. Mathematically, this model is

$$\begin{aligned}
&\text{minimize } \bar{E} \\
&\text{subject to} \\
&\sum_{p \in J_{od}} x_p = \bar{R}_{od}, \qquad \forall (o,d) \in D \\
&\text{and } (13), (15) - (24)
\end{aligned}$$

The *stochastic programming model* for this problem uses a penalty, say d, to represent the cost of infeasibility (see [19]). Mathematically this model is closely related to the robust optimization model and may be stated as follows:

$$\begin{aligned}
&\text{minimize } \bar{E} + \sum_{s \in S} P_s [\sum_{(o,d) \in D} d(z^+_{ods} + z^-_{ods})] \\
&\text{subject to} \\
&\text{and } (13) - (24)
\end{aligned}$$

Note that an identical penalty is used for both over and under provisioning. The *worst-case model* selects a design that minimizes the largest possible value of a combination of equipment and infeasibility cost. Mathematically, the model is

$$\text{minimize } \{\max_{s \in S}[\bar{E} + \sum_{(o,d) \in D} d(z^+_{ods} + z^-_{ods})]\}$$

subject to

and (13) – (24)

Note that the probabilities for the various scenarios do not appear in this model.

4 Demonstration of the Robust Design Methodology

To illustrate the practical application of the robust design methodology for the WDM assignment and provisioning problem, two test problems have been solved using the various models presented in this manuscript. All models have been implemented using the AMPL modeling language [8, 20] with a direct link to the solver in CPLEX [21, 22] 6.6.0. All test runs were made on a Compaq AlphaServer DS20E with dual EV 6.7 (21264A) 667 MHz processors and 4096 MB of RAM. AMPL data files for the DA and KL test problems are available on line at `http://www.engr.smu.edu/~olinick/papers/`. The network topology for the DA problem is illustrated in Figure 1 and its characteristics are given in Tables 1 and 3. The model includes four candidate paths for each of the 200 demand pairs. These paths are the four shortest loopless paths and the demand values were randomly generated from a uniform distribution with the ranges specified in Table 4. The non-symmetrical regret function, which is based on an approximation for $(z^+_{ods})^2 + (z^-_{ods})^2/4$, is illustrated in Figure 2.

The runs for the individual scenarios are summarized in Table 5, where the numbers under the column titles TEs, Rs, and As denote the total number of TEs, regenerators, and optical amplifiers required for the optimal network design for that scenario. Total equipment cost varied from \$1.8 B to \$5.6 B. We used an optimality gap of 5% for CPLEX and each of the five runs required less than a second of CPU time.

Both AMPL and CPLEX use preprocessors that attempt to reduce the size of the problem instance prior to application of the integer optimizer. The resulting problem sizes after preprocessing for the various models are reported in Table 6. The individual scenario models have the smallest row count while the robust model has the largest. The number of integer variables corresponding to the number of fibers on each link and the number of channels on each link are approximately the same for all models (107)(2) = 214. The column increases over the individual scenarios are primarily additional continuous variables. The robust model uses the four-piece approximation for each z^{+}_{ods} and each z^{-}_{ods} resulting in (200)(5)(2) = 2000 additional rows and (200)(5)(2)(4) = 8000 additional columns prior to preprocessing.

Using the equipment cost from Table 5, three budgets were selected, the largest value ($5.630 B) corresponds to a generous budget, the smallest value ($1.848 B) corresponds to a very tight budget, and $3.787 B corresponds to a mid-range budget. Results from runs using these three budgets with the various models may be found in Table 7. Two metrics were used to help compare the various models, the total unrouted demand summed over all scenarios and the regret as illustrated in Figure 2. The regret as reported in the column entitled Scaled Regret has been scaled for each budget so that the best value for regret is 1.0. For the generous budget, the regret for the mean-value model is 40% higher than for the robust model and approximately 10% more demand was unrouted. For the mid-range budget, the mean-value model was much closer to the robust model, but still inferior with respect to both metrics. For the tight budget, the mean-value model failed, but the stochastic programming model does fairly well with respect to both metrics. The worst-case model designs were poor using these metrics for all three budgets. All runs used a 5% optimality gap that permits early termination of the CPLEX optimizer. For these 12 runs, no model required more than 7 seconds of CPU time to obtain a solution guaranteed to be within 5% of optimality. Also, the runs for the mid-range budget used the optimal solution for the generous budget as an initial starting solution. Likewise, the runs for the tight budget began with the solutions for the mid-range budget. In other runs, the robust model for budget number two required over 1200 seconds using a 1% gap.

After determining the robust design for each of the three budgets, we then attempted to route the demand for each of the five scenarios individually. Unrouted demand represented by the variables z^{+}_{ods} is converted into the equivalent amount of equipment for which we have under provisioned. Likewise, excess capacity represented by the variables z^{-}_{ods} is converted into the equivalent equipment for which we have over provisioned. Summing over all nodes (links) gives an under and over provisioning of TEs (Rs and As). These sums are reported in Table 8. The tighter the budget, the more under provisioning that occurs. Also note that under and over provisioning occurs simultaneously due to the placement of the equipment in the network.

The network topology and data characteristics for the problem named KL may be found in Figure 3 and Table 3, respectively. Solutions for the individual scenarios are shown in Table 9 and the model sizes are given in Table 6. The runs with the four models may be found in Table 10. Note that the robust model for the second budget required 200 seconds to obtain a solution guaranteed to be within the 5% optimality gap. Since we are solving ILP's with a branch-and-bound technique, it is always possible that changing a single number in a problem instance (such as the budget) can result in a dramatic increase in the solution time. Unlike the robust models proposed by Mulvey et al. [12] that involve only continuous variables, our models are mixed integer linear programs. Hence, a simple regret model such as our four-piece linear function is needed to ensure that the problem instances are tractable.

The under and over provisioning for the robust solution for problem KL is given in Table 11. The only unexpected number is the under provisioning for TEs with the mid-range budget. This number is smaller than the under provisioning for TEs for the generous budget. Ordinarily as the budget is tightened, we would expect the under provisioning for all equipment types to increase.

5 Summary and Conclusions

This manuscript presents a robust optimization model for the WDM routing and provisioning problem. Under a budget restriction, a robust design gives the number and placement of TEs, regenerators, and optical amplifiers that minimize a regret function. Regret is modeled using a convex, piece-wise linear function. Four variables (corresponding to the 4 linear pieces that make up the regret function) are used to represent under provisioning and 4 variables are used to represent over provisioning for every (o, d)-s combination. Our robust optimization model has $2|E|$ variables (corresponding to the number of fibers and channels on a link) that must be assigned integer values. This ensures a design that can be constructed using a discrete number of regenerators and optical amplifiers and distinguishes our robust optimization model from the linear programming formulation of robust optimization presented by Mulvey et al. [12]. The number of TEs are represented by continuous variables that are rounded to the nearest integer. Since we anticipate large values for these, rounding results in only a slight increase in total regret.

Alternative models for this problem include the mean-value model, the stochastic programming model, and the worst-case model. A major weakness of the mean-value model is that it does not guarantee a feasible solution for all problem instances. This occurred for both test cases under the tight budget restriction. For any given budget, the stochastic programming model, the worst-case model, and the robust optimization model always produce a design. Both the stochastic programming model and worst-case model require a unit cost for under and over provisioning that is not required for the robust optimization model. In addition, the worst-case model ignores the scenario probabilities which means that a very unlikely scenario could be the one that drives the solution for this model. For our test cases, regret was always highest for the worst-case model. Even though this investigation makes a compelling case for using the robust optimization model for this problem, the strategy presents challenges for both the analyst and the client. A regret function that accurately reflects the client's position regarding under and over provisioning must be defined. Instances of integer linear programs may require an excessive amount of computational time. As we observed with the KL problem, changing the budget resulted in a 200-fold increase in computational time. Using an ILP model can result in anomalies as illustrated in Table 11 with under provisioning of TEs for the mid-range budget. However, none of the aforementioned issues should prohibit use of the robust optimization model.

Network restorability using a dedicated protection scheme (e.g. 1+1 protection) can be accommodated with the present model by replacing shortest paths with shortest cycles. The issue of a shared protection scheme has not been addressed in this investigation and issues concerning an optical node bypass (glassthrough) are subjects of a future investigation.

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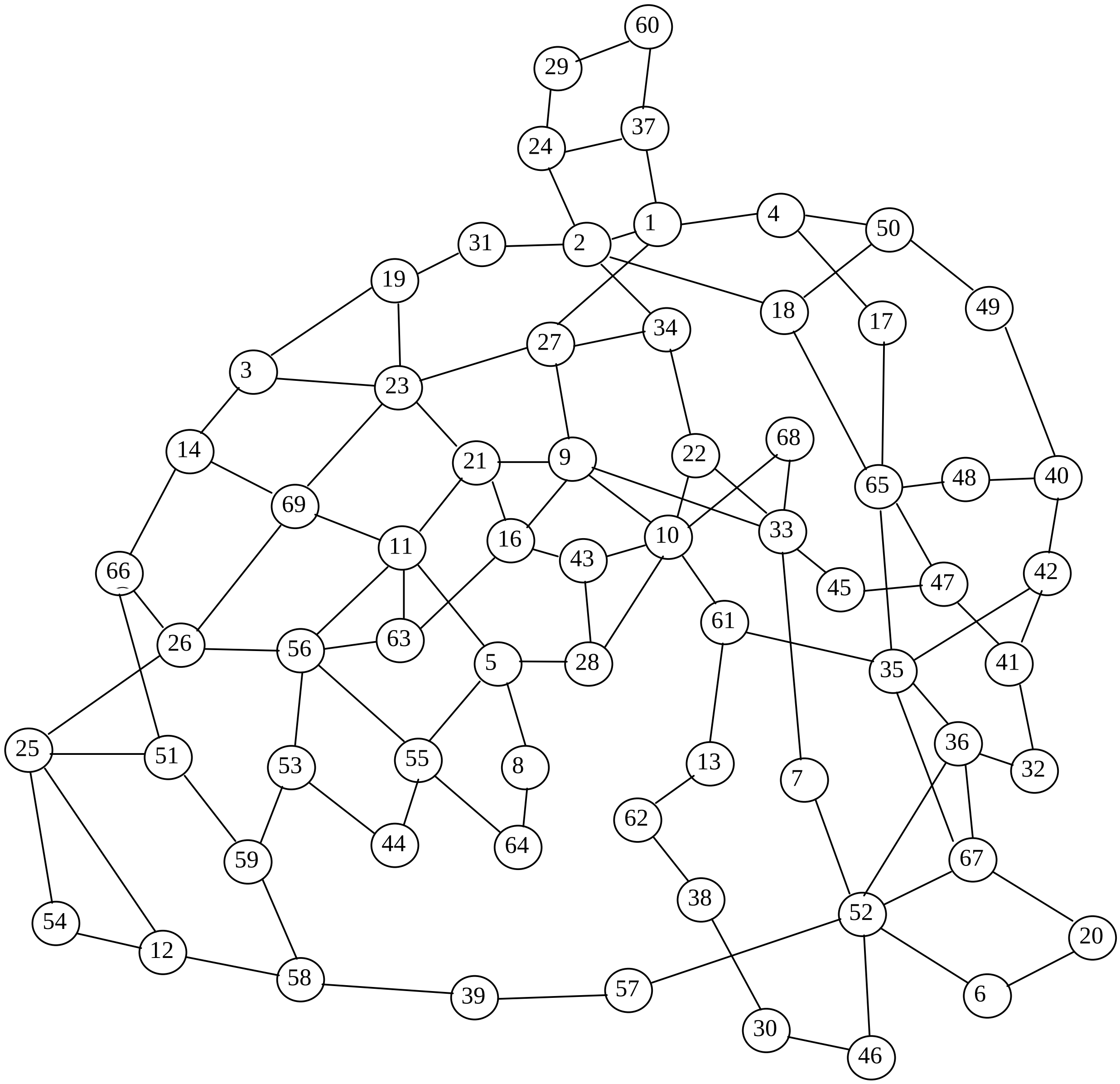

Figure 1: Network for DA Problem

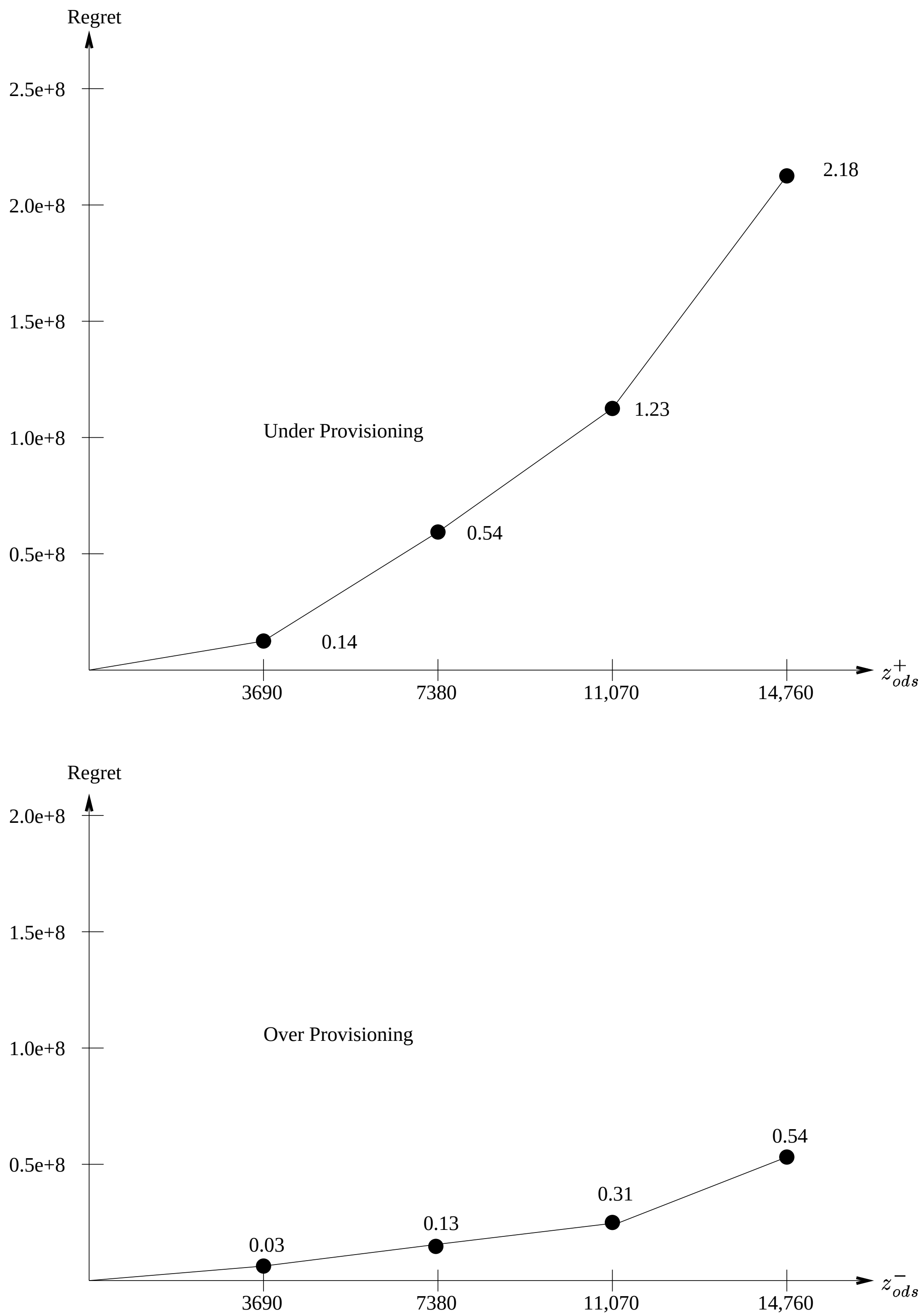

Figure 2: Regret Function Problem DA

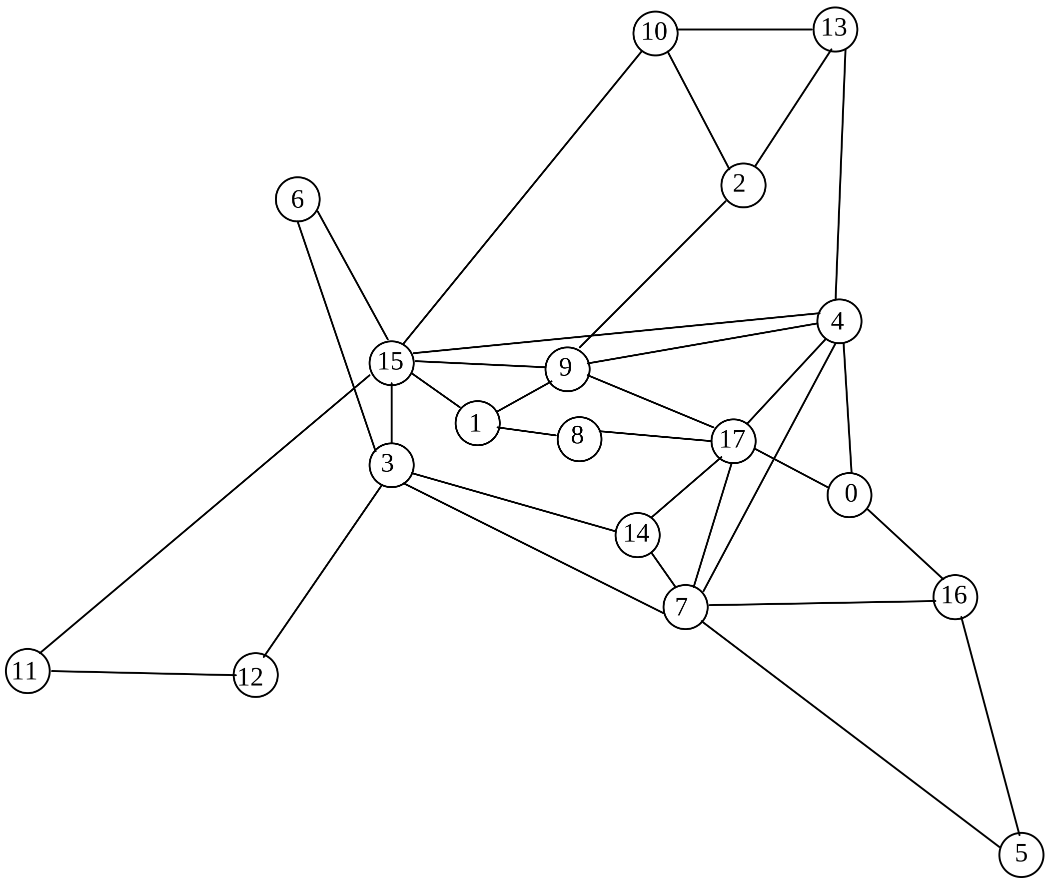

Figure 3: Network for KL Problem

Table 1: Description of Constants in Problem DA

Constant	Value or Range	Description
R^s_{od}	300-1500	traffic demand for pair (o, d) in scenario s in units of DS3s
M^{TE}	192	number of DS3s that each TE can accommodate
M^R	192	number of DS3s that each regen can accommodate
M^A	15,360	number of DS3s that each optical amplifier can accommodate
C^{TE}	\$50,000	unit cost for an TE
C^R	\$80,000	unit cost for a regen
C^A	\$500,000	unit cost for an optical amplifier
F_e	24	max number of fibers on link e
R	80km	max distance that a signal can traverse without amplification, also called the optical reach
Q	5	max number of amplified spans above which signal regeneration is required
B_e	2km-1106km	the length of link e
G^A_e	0-11	number of amplifier sites on link e
G^R_e	0-2	number of regenerator sites on link e

Table 2: Decision Variables

Variables		Variable Type	Description
Scenario s Model	Robust Model		
x_p^s	x_p	continuous	number of DS3s assigned to path p
ℓ_n^s	ℓ_n	continuous	number of TEs assigned to node n
t_e^s	t_e	continuous	number of TEs assigned to link e
a_e^s	a_e	continuous	number of optical amplifiers assigned to link e
r_e^s	r_e	continuous	number of regens assigned to link e
f_e^s	f_e	integer	number of fibers assigned to link e
c_e^s	c_e	integer	number of channels assigned to link e
z_e^s	z_e	continuous	number of DS3s assigned to link e
	z_{ods}^+	continuous	positive infeasibility for demand (o, d) and scenario s
	z_{ods}^-	continuous	negative infeasibility for demand (o, d) and scenario s
	z_{odsk}^+	continuous	k^{th} linear piece for z_{ods}^+
	z_{odsk}^-	continuous	k^{th} linear piece for z_{ods}^-

Table 3: Characteristics of Test Problems

Name	DA	KL
Source	[1]	[16]
Total Nodes	67	18
Total Links	107	35
Total Demand Pairs	200	100
Number of Paths/Demand	4	4
Total Demand Scenarios	5	5

Table 4: Demand Range in DS3s for Problem DA

Scenario	Probability	Demand Range	Average Demand
1	0.15	800-9600	5200
2	0.20	2400-10800	6600
3	0.30	4000-12000	8000
4	0.20	4400-16800	10,600
5	0.15	4800-21600	12,200

Table 5: Solution for Individual Scenarios for Problem DA

Scenario	Prob.	TEs	Rs	As	CPU Seconds	Equipment Cost
1	0.15	24,996	3962	563	0.5	$1.848 B
2	0.20	39,456	6502	864	0.5	$2.925 B
3	0.30	51,882	8074	1101	0.5	$3.791 B
4	0.20	65,086	10,122	1355	0.6	$4.742 B
5	0.15	76,848	12,447	1584	0.5	$5.630 B
Expected Value	—	51,749	8,208	1096	—	$3.792 B

Table 6: Model Sizes after Preprocessing for Test Problems

Problem	DA			KL		
Model	Rows	Columns	Nonzeros	Rows	Columns	Nonzeros
Individual Scenarios	353	953	6122	270	570	3566
Mean Value	512	1112	7062	365	665	3855
Stochastic Programming	1326	3126	12,290	769	1669	641
Worst Case	1331	3127	14,300	774	1670	7425
Robust Optimization	1510	9309	18,764	911	4810	9816

Table 7: Comparisons for Various Methods for Problem DA Using an Optimality Gap of 5%

Budget	Method	TEs	Rs	As	Equip. Cost	CPU Seconds	Unrouted Demand	Scaled Regret
$5.630 B	Mean Value	51,800	8117	1081	$3.780 B	0.7	15.5%	1.40
	Stoch. Prog.	44,373	7446	918	$3.273 B	1.8	20.4%	1.82
	Worst Case	39,098	5495	757	$2.773 B	4.6	27.2%	3.75
	Robust Opt.	63,122	10,813	1425	$4.734 B	2.7	5.2%	1.00
$3.787 B	Mean Value	51,800	8117	1081	$3.780 B	0.2	15.5%	1.11
	Stoch. Prog.	44,373	7446	918	$3.273 B	0.6	20.4%	1.44
	Worst Case	39,098	5495	757	$2.773 B	2.1	27.2%	2.95
	Robust Opt.	52,159	8108	1061	$3.787 B	4.5	12.6%	1.00
$1.848 B	Mean Value	—	—	—	No Feasible Solution	0.3	100%	—
	Stoch. Prog.	25,583	3696	515	$1.832 B	3.9	42.3%	1.15
	Worst Case	27,180	2960	505	$1.848 B	6.6	42.3%	1.51
	Robust Opt.	25,856	3575	539	$1.848 B	5.6	43.3%	1.00

Table 8: Under and Over Provisioning for Problem DA

Budget	Provisioning	Scenario					Totals
		1	2	3	4	5	
$5,630 B	under TE	11	381	2908	7838	16,536	27,674
	under R	0	0	91	504	2076	2671
	under A	1	4	49	126	254	434
	over TE	38,137	24,048	14,149	5875	2811	85,020
	over R	6851	4311	2830	1195	442	15,629
	over A	863	565	373	196	95	2092
$3,787 B	under TE	253	566	3969	13,832	25,322	43,942
	under R	119	101	724	2080	4419	7443
	under A	12	17	124	307	535	995
	over TE	27,482	13,334	4312	971	699	46,798
	over R	4205	1647	698	6	20	6576
	over A	513	217	87	16	15	848
$1,848 B	under TE	2731	14,180	26,145	39,157	50,922	133,135
	under R	720	3082	4552	6555	8880	23,789
	under A	82	341	572	822	1051	2868
	over TE	3663	653	191	0	3	4510
	over R	352	147	45	0	0	517
	over A	52	10	4	0	0	66

Table 9: Solution for Individual Scenarios for Problem KL

Scenario	Prob.	TEs	Rs	As	CPU Seconds	Equipment Cost
1	0.15	12,767	7275	638	0.3	$1.539 B
2	0.20	17,493	11,691	958	0.3	$2.288 B
3	0.30	24,020	15,783	1178	0.3	$3.052 B
4	0.20	29,295	19,196	1455	0.2	$3.727 B
5	0.15	35,732	23,606	1760	0.3	$4.554 B
Expected Value	—	23,837	15,545	1196	—	$3.033 B

Table 10: Comparisons for Various Methods for Problem KL Using an Optimality Gap of 5%

Budget	Method	TEs	Rs	As	Equip. Cost	CPU Seconds	Unrouted Demand	Scaled Regret
$4.554 B	Mean Value	25,124	15,350	1221	$3.094 B	1.1	15.4%	1.41
	Stoch. Prog.	20,264	14,168	996	$2.644 B	0.6	20.8%	1.94
	Worst Case	17,977	11,812	872	$2.279 B	1.1	27.8%	4.05
	Robust Opt.	27,520	21,348	1614	$3.890 B	1.0	5.6%	1.00
$3.032 B	Mean Value	23,978	15,382	1198	$3.028 B	0.5	15.4%	1.11
	Stoch. Prog.	20,264	14,168	996	$2.644 B	0.2	20.8%	1.52
	Worst Case	17,977	11,812	872	$2.279 B	0.4	27.9%	3.20
	Robust Opt.	23,967	15,548	1181	$3.032 B	200.0	13.1%	1.00
$1.539 B	Mean Value	—	—	—	No Feasible Solution	0.1	100%	—
	Stoch. Prog.	12,154	7456	666	$1.537 B	2.7	42.7%	1.19
	Worst Case	12,782	7222	645	$1.539 B	1.9	44.4%	1.71
	Robust Opt.	13,562	7172	575	$1.539 B	5.6	43.3%	1.00

Table 11: Under and Over Provisioning for Problem KL

Budget	Provisioning	Scenario					Totals
		1	2	3	4	5	
$4.554 B	under TE	965	1484	3993	7311	12,263	26,016
	under R	294	722	1793	3500	6676	12,985
	under A	22	48	112	259	442	883
	over TE	15,718	11,511	7494	5536	4060	44,319
	over R	14,367	10,379	7358	5652	4418	42,174
	over A	998	704	548	418	296	2964
$3.032 B	under TE	102	295	2302	6207	12,602	21,508
	under R	52	263	1932	3963	8592	14,802
	under A	14	34	139	296	615	1098
	over TE	11,151	6619	2099	729	696	21,294
	over R	8369	4164	1741	359	578	15,211
	over A	565	265	150	30	44	1054
$1.539 B	under TE	1372	5473	10,518	15,733	22,161	55,257
	under R	918	5272	8641	12,024	16,434	43,289
	under A	120	440	603	880	1185	3228
	over TE	2166	1542	60	0	0	3768
	over R	815	753	30	0	0	1598
	over A	57	57	0	0	0	114