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MEASURING MATHEMATICAL REASONING: WHOLE NUMBERS AND FRACTIONS

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Texas Process Standards for Mathematics

- (B)…evaluating the **reasonableness** of the solution
- (C)…including **mental math, estimation and number sense** to solve problems
- (D) **communicate** mathematical ideas, **reasoning**, etc.
- (F) analyze mathematical relationships to **connect and communicate mathematical ideas**
- (G) display, **explain, and justify mathematical ideas and arguments using precise mathematical language**

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Purpose

- Identify types of reasoning that are present in low-performing elementary students when they explain their answers to whole number and fraction items on a measure of mathematical reasoning.

- Re-examine and redefine these reasoning types to create clear, concise, and operationally defined categories of mathematical reasoning that can be used in intervention research.

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Participants

Table 1
Student demographic information (N = 105)

Demographic Category	**Frequency**	**Percentage**
Gender		
- Boys	58	55.24%
- Girls	45	42.86%
- Unknown	2	1.90%
Grade		
- 3	16	15.24%
- 4	13	12.38%
- 5	57	54.29%
- 6	19	18.10%
Educational Classification		
- Learning Disability	31	30%
- Dyslexia	6	6%
- OHI	6	6%
- Other Disability	9	2%
- Low Performing	53	56%

Measure

***The Math Reasoning Inventory (MRI;* Burns, 2012)**

- A formative online assessment based on the theoretical ideas related to mathematical proficiency (Kilpatrick, et al., 2001).

- Administered to students in a one-on-one setting and assesses mathematical reasoning through interviews focusing on core numerical reasoning strategies and understanding.

- Each question is answered using mental math, followed by responding to the question "How did you figure that out?" or "How did you decide?"

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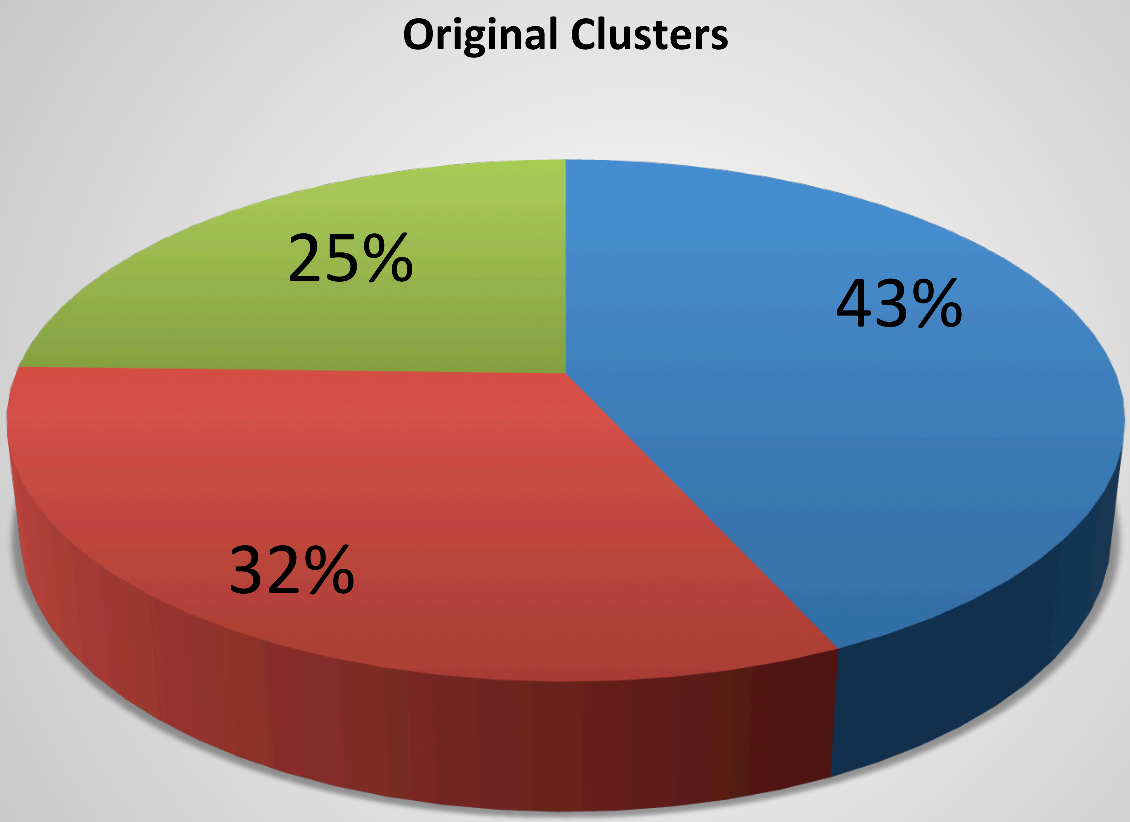
Data Analysis

- Using student responses on 1,928 items, a cluster analysis was performed on the 36 given reasoning strategies from the MRI, and results are reported in terms of three clusters.

Results- Phase 1



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Results- Phase 1

- Three clusters represented 36 categories of answers.

- Data from the first analysis combined with a synthesis of current research on mathematical reasoning (Bergholm, 2012; Bergqvist,2005; Dreyfus & Eisenberg, 1996) allowed for initial operational definitions of types or levels of reasoning to be developed.

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Operational Definitions

Faulty

The student uses reasoning that is incorrect (a logical fallacy), guesses at the solution, or provides incomplete or no reasoning as to how they arrived at the answer.

Example:

"4/10, because 3 is greater than 4, and 4 is greater than 10, so 4/10 is greater than ¾."

Algorithmic

The student applies a set of rules that guarantees a correct solution will be reached, and the remaining reasoning parts are trivial for the reasoner.

Example:

“3/4. I did 10x3 and got 30, and did 4x4 and got 16, and I know that 30 is greater than 16, so 3/4 is greater.”

Conceptual

The student uses reasoning that is founded on the intrinsic mathematical properties of the components of the task with or without describing the procedure.

Example:

“3/4, because I know that 2/4 is the same as 1/2, and 3/4 is greater than 2/4, and 5/10 is the same as 1/2, and I know that 4/10 is less than 1/2, so 3/4 is greater than 4/10.”

Operational Definitions in Action: Student Video Clips

- Faulty Reasoning
- Algorithmic Reasoning
- Conceptual Reasoning
- Conceptual Reasoning

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Results – Phase 2

- Based on these definitions, each item was recoded and assigned to one of three categories (1 = faulty; 2 = algorithmic; 3 = conceptual).

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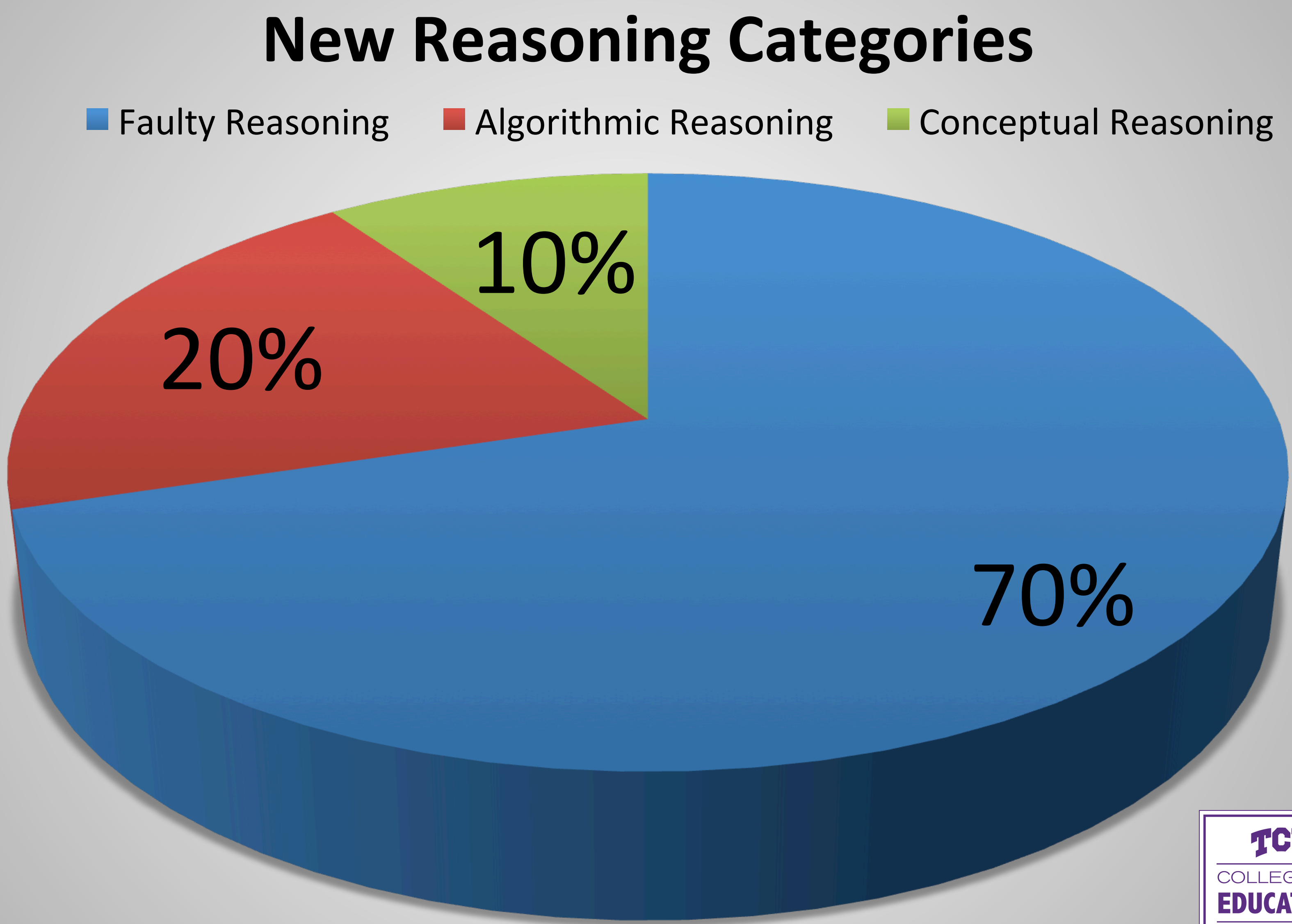
Results- Phase 2



Conclusions & Next Steps

- If we expect teachers to assess and teach mathematical reasoning, we must create a definition that is both theoretically and empirically sound.

- This definition is most useful if it is “tiered” and represents different levels of student reasoning.

- Further exploration of “faulty reasoning” category is needed.
  - Current definition is too broad
  - Difference between IDK and an entrenched misconception

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