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Research in Mathematics Education

MMaRS Assessment Inventory Development

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Technical Report 20-27

MMaRS Assessment Inventory Development

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Fall 2020

Published by
Southern Methodist University
Department of Education Policy & Leadership
Simmons School of Education & Human Development
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Dallas, TX 75275-0114
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This research was supported by the National Science Foundation, Grant #1721100. Opinions expressed herein do not necessarily reflect those of National Science Foundation or individuals within.

Acknowledgements: We would like to thank Moraima Castro-Faix for assistance with the search and compilation of assessments for the inventory.

Abstract

We developed the assessment inventory so that we would have a place to collect and to store items created to measure the early numeracy constructs of spatial reasoning and numerical relational reasoning. These items will help inform the design of assessment items for the MMaRS project by allowing us to identify common item features that are used to assess the constructs. We began by creating a workspace in FileMaker Pro that had data entry fields for each type of information that we needed about the items. Next, we identified a variety of item sources, which included published tests, academic research, and the cognitive interview items used in the larger MMaRS project. Finally, each item from these sources was systematically entered into the database and verified by multiple researchers. At the end of this process, we had 339 spatial reasoning items and 261 numerical relational reasoning items in the inventory.

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MMaRS Assessment Inventory Development

Introduction and Purpose

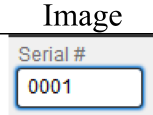
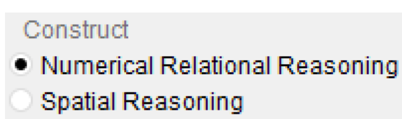
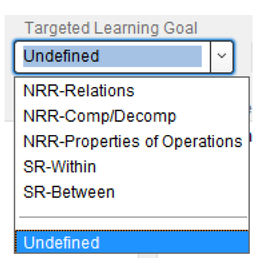
The purpose of this technical report is to describe the creation of an inventory of previously development assessments and test items focused on the constructs of numeric relational reasoning and spatial reasoning. We describe the design of the inventory, how we identified assessments and items to be included in the inventory, the data entry process, and the review and verification process. We also summarize the number of items by construct that exist in the completed inventory.

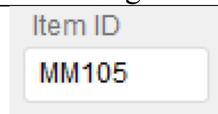
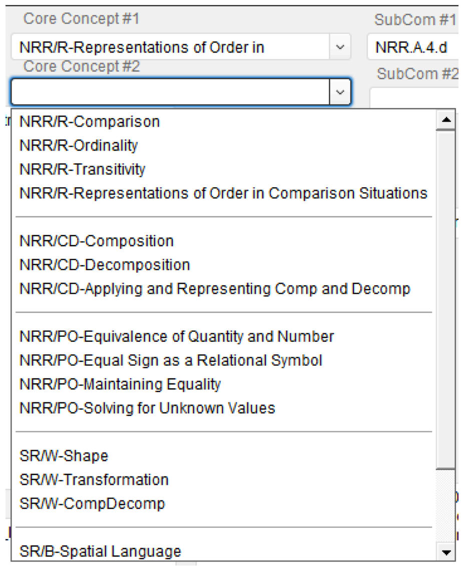
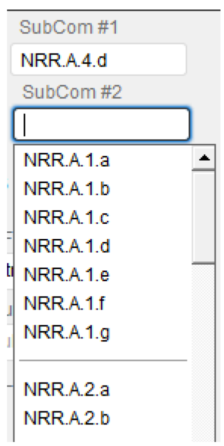
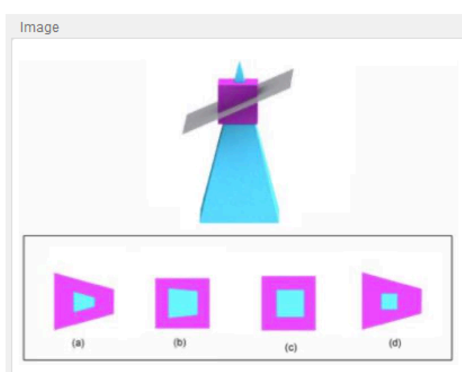
The purpose of the inventory was to design a location where we could collect and store items that our project team as well as other researchers created to measure these early numeracy constructs. These items are intended to inform the design of assessment items for the MMaRS project in that we can examine the items for common components and elements that are used to assess the constructs.

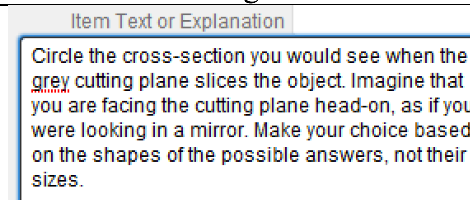
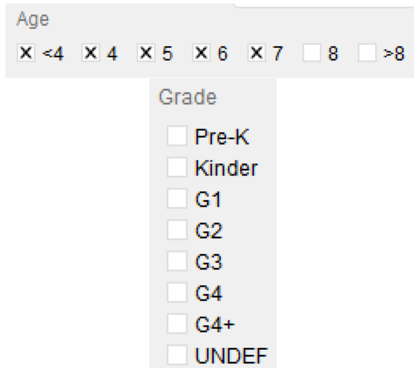
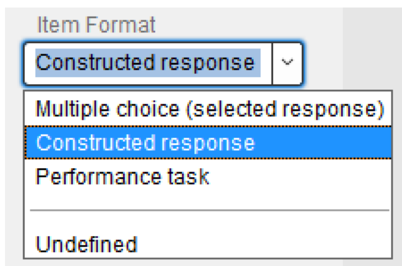
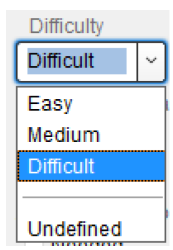
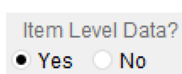
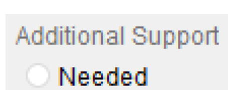
Database Development

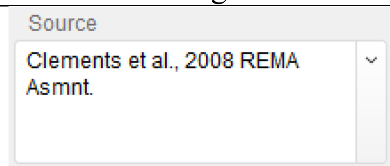
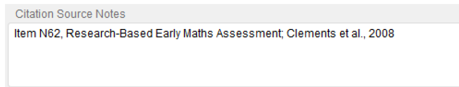
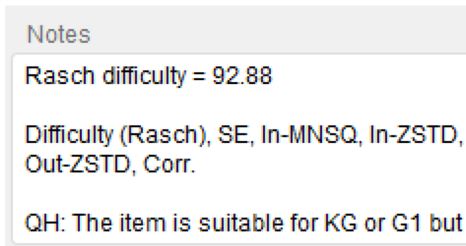
The database was developed using FileMaker Pro, which has flexible layouts and networking capabilities allowing multiple users simultaneously contribute to the database. Specifically, FileMaker Pro allows for graphic images to be easily associated with an item record in addition to text and numbers in each field. Below is a summary of fields in the database. Each record contained empty fields for the pieces of item information described in Table *1*.

Table 1
Database Field Descriptions

Field	Image	Description
Serial number		This is a non-editable field that counts up each time a new record is created.
Construct		Two radio buttons for selecting either Numeric Relational Reasoning (NRR) or Spatial Reasoning (SR)
Targeted Learning Goal		A dropdown menu for noting the five goals; three are for NRR and two are for SR.

Field	Image	Description
Item ID		An identifying number used to track team members entering data.
Core Concept #1 and Core Concept #2		Two fields for a primary and secondary core concept to be recorded. All seventeen core concepts are listed and can be selected using a dropdown menu.
Subcomponent #1 and Subcomponent #2		Two fields for primary and secondary subcomponent. All ninety-five subcomponents are listed and can be selected using a dropdown menu.
Image		This field allows for insertion of an image, a captured screenshot, if applicable to an item.

Field	Image	Description
Item Text or Explanation		This field was for all item prompts, descriptions, or explanations to be entered.
Age and Grade		These two fields captured the age and grade for which the item was designed. In data entry, both or one could be used as some items specified one or both.
Item Format		This field was for capturing how items were presented to students and how they were given the opportunity to provide their answers as either multiple choice (selected response), constructed response or as a performance task.
Difficulty		Where item difficulty was specified from the source, it was selected as either easy, medium, or difficult. The majority of items listed no item level data and were therefore, “Undefined” for this category.
Item Level Data		This field was to indicate if the source of the item listed any item level data that showed the item’s performance statistically based on administration to students.
Additional Support		Internal to the project staff, this field existed to communicate to the lead researcher if questions arose about the content of this item or how to classify within the database.

Field	Image	Description
Source		This dropdown menu was used during data entry to select the source from which each item was obtained.
Citation Source Notes		For more explicit information about location of the source and/or citation.
Notes		Used for any relevant information about the item that could be useful to researchers at a later date. This could include specifics about the item difficulty, clarity on administration instructions, or notes about item level data.

Identification of Relevant Sources

Researchers identified several potential sources for relevant items to add to the database. Internally, during meetings with team members, and consultants, brainstormed items were captured on sticky notes. Later, these loosely created items were added to the assessment inventory. Additionally, all items from the cognitive interviews were added to the inventory. Items from external sources were also utilized, including the sources listed in the next section. Also, technical advisors to the project were asked to recommend sources of items. Recommendations included researchers' websites, publications about research studies, or personal work.

Sources Used to Develop Assessment Inventory

The items in the assessment inventory were gathered from the following sources.

MMaRS Cognitive Interview Protocol
MMaRS researchers developed and administered assessment items in the form of cognitive interviews. The items each corresponded to one or more subcomponents of the spatial or numerical relational reasoning learning progressions.

International Common Assessment of Numeracy (ICAN)
ICAN is an open-source numeracy assessment tool that is available in 11 languages. The assessment may be administered one-on-one or in a large-scale setting. The items are meant for students grades three and below and cover topics in number knowledge, geometry, measurement, and data display.

Georgia Early Numeracy Project: Diagnostic Assessments
The Georgia Early Numeracy Project has two diagnostic tests: The Global Strategy Stages Assessment (GloSS) & The Individual Knowledge Assessment of Numeracy (IKAN). GloSS is a diagnostic interview meant to be administered individually. the IKAN is also an interview, but it centers around counting and also includes a written assessment. The GloSS is meant to come before IKAN. These assessments are designed to work in tandem to give K-5 teachers insight into students' number knowledge and strategy choices. Students are meant to complete questions until they become too difficult.

Spatial Intelligence & Learning Center (SILC)
The SILC website has a repository of tests on a variety of spatial reasoning topics including mental folding, mental rotation, perspective-taking, and determining the two-dimensional shapes of cross-sections of three-dimensional objects.

Erikson Institute's Early Math Collaborative
This resource describes the "big ideas" involved in number sense, counting, number operations, patterns, spatial relationships, and shape. Examples are given of concepts associated with each big idea.

Young Mathematicians Pattern Block Puzzle Games
These activities encourage children to use the rotation, reflection, and de/composition of two-dimensional shapes to complete puzzles. In addition to simple puzzle figures, there are activities in which children can explore different ways to create the same figure and others that focus on symmetry. These activities can be used at home or in school, including as formative assessments.

Examples from Consultants
These items were items discussed and referenced during meetings with consultants during the project. Consultants would draw on prior experiences and make connections to how an item could represent a particular subcomponent. These items were typically hand drawn with notes of relevance to support future work.

Data Entry

We entered assessment and activity items found in the sources described into the assessment inventory database individually. If there were images associated with an item, we would take a screenshot from the source, reorganize the components as needed (e.g. using Paint or PowerPoint to make one image with the prompt and answer option images), and upload it with the entry for the item. We would also copy and paste the instructions into the data base in order to ensure that the items will be administered in the way the developers intended. We then identified the construct, target learning, goal, core concepts, and subcomponents that were associated with the item. There were occasionally multiple core concepts and/or subcomponents. We also indicated the ages or grades of the students who were intended to take the item, the item format (multiple choice, constructed response, observation, or performance task), and notes about the source. If an answer key was available, the original item number and a URL for the answer key document were also provided. See Figure 1 for an example of what the data entry looked like for a spatial reasoning item.

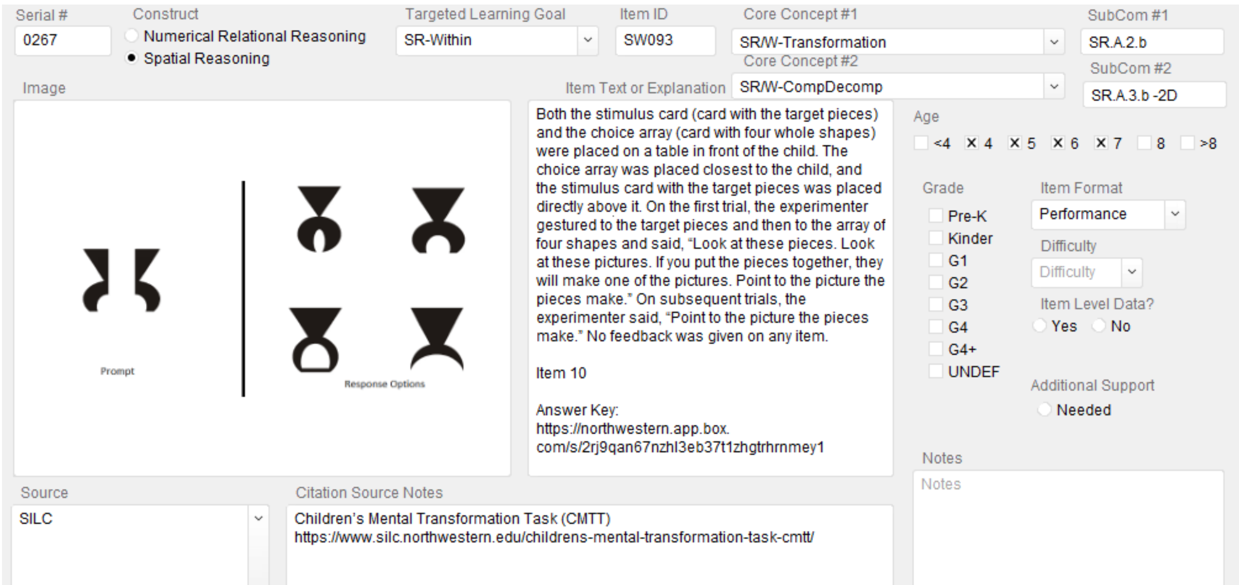


Figure 1. Item Data Example

Data Verification Process

Two graduate students were hired to input items into the database. Each student first entered ten items, then they reviewed each other's work. They reviewed with the lens that if someone unfamiliar with the item were to read the information captured in the database, could they correctly decipher the item's intent and delivery. Also verified were the alignment to MMaRS Targeted Learning Goal, Core Concept(s), and Subcomponent(s). After this initial review, any disagreement or confusion was discussed with the lead researcher. Once the lead researcher and two students were concordant about how these 20 items were captured, they continued with the remaining items. Weekly afterwards, the lead researcher would randomly review 20% of entered items against the original source item. This verification was maintained throughout the building of the item database.

Additionally, due to some confusion about alignment of several items with MMaRS Targeted Learning Goals, Core Concepts, and Subcomponents, the lead researcher and another researcher from the team reviewed items flagged as needing additional support.

After we compiled all items into the inventory, we found that the initial item format definitions needed revision to fit the MMaRS project's needs better. A researcher worked with the PI to curate a priori definitions for three item types: multiple choice/selected response, constructed response, and performance task (See Table 2 for details). A research assistant who works on the project but had not been involved in the inventory's initial development applied the definitions systematically to the 632 items located therein. Through the process, they suggested updates to the definitions and extracted examples and non-examples of items that illustrated each item format.

Table 2
Item Format Definitions and Examples

Item Format	Definition	Example	Non-example
Multiple Choice/ Selected Response	Examinee is presented with a set of 2 or more constrained response options from which to choose; one of which is the correct response. Can be individually administered or as a group. Examples include True/False, multi-select, matching from a specific set of options.	NRR.C.10.b: Without adding 67 + 86, can you tell if the statement below is true or false? 67 + 86 = 68 + 85. How do you know? SR.B.7.d: Perspective taking: The Lego person took a picture; which photo did they take? [given three response options]	NRR.B.6.a: The boy has 12 toys altogether. You can see 2 of his toys, the rest are in the box. How many toys are in the box? SR.A.2.c: Activities with shapes: If I place a mirror next to this shape, what would you see?
Constructed Response	Examinee must produce (construct) their own response to a prompt when no set of options is provided. Offers the test taker an opportunity to explain or document of their answer. Examples include fill in the blank, short answer, essay.	NRR.C.11.e: __+3 = 7 __ + 4 = 5 + 2 6 - __ = 7 - 3 SR.A.1.c-2D: Show the student a shape. What shape is this? How can it be described?	NRR.B.6.d: Provide 15 counters (players). Say: These 15 players have to spread out evenly on the court. How many players should be in each third of the court? SR.A.3.b-2D: choose an animal puzzle and fill in the outline with pattern blocks.
Performance Task	Examinee responds to an assessor’s prompt by generating a response that is physical (i.e., moving manipulatives), oral (verbally answering the assessor), or by gathering evidence (i.e., project, exhibit, portfolio). For our purposes, performance tasks are often individually administered (one-on-one) in early grade assessments	"NRR.B.6.b: There are 12 eggs in (and?) 2 baskets. What are all the ways you can put them in the baskets? SR.B.6.c: Provide a picture of pattern blocks (smaller than actual size). Ask students to create a similar construction with pattern blocks.	NRR.C.11.d: Fill in the blank with the value that makes the number sentence true: 7+3=___+4 SR.B.7.c: Imagine you are standing at the cat and facing the tree. Point to the car. draw another arrow in the circle showing the direction you imagined pointing.

For accuracy of item type capture and reliability between researchers' view of what each item type represented, the researcher conducted a 20% verification of item format definition application. For every 100 items reviewed by the research assistant, the researcher reviewed 20 randomly selected items to ensure item type agreement. For adequate calibration and to prevent coder drift (Marston et al., 1978) in applying the definitions, the pair conducted each verification cycle fully before the research assistant reviewed the next 100 items. When there was disagreement, the team met to review definition interpretations and come to a full agreement. Further, the item development team met after the first three cycles to review the definition types and to ask for clarification, including how relevant response formats would be in this process. The team agreed that item format guided response type and format and less emphasis should be placed on the response process for inventory purposes. The number of item format types by core concept can be seen in Table 3.

Table 3
Item Type Count by Core Concept

Core Concept	Constructed Response	Selected Response	Performance Task	Undefined	Total
NRR.A.01/R-Comparison	3	9	3		15
NRR.A.02/R-Ordinality	15	2	1		18
NRR.A.03/R-Transitivity		5	5		10
NRR.A.04/R-Representations of Order in Comparison Situations	14	3	2		19
NRR.B.05/CD-Composition	38	1	9		48
NRR.B.06/CD-Decomposition	25		9		34
NRR.B.07/CD-Applying and Representing Comp and Decomp	18				18
NRR.C.08/PO-Equivalence of Quantity and Number	5	9			14
NRR.C.09/PO-Equal Sign as a Relational Symbol	4	3			7
NRR.C.10/PO-Maintaining Equality		6	1		7
NRR.C.11/PO-Solving for Unknown Values	14				14
SR.A.01/W-Shape	5	10	10		25
SR.A.02/W-Transformation	4	69	7		80
SR.A.03/W-CompDecomp	4	56	86	1	147
SR.B.05/B-Spatial Language	6	3	3		12
SR.B.06/B-Understanding Models and Maps	1	29	9	2	41
SR.B.07/B-Perspective Taking	14	14	6	1	35
Undefined	53	8	24	3	88
Total	223	227	175	7	632

Summary of Assessment Inventory

We collected and organized items that fit within the constructs of interest, which were spatial reasoning and numerical relational reasoning. Within each construct, there are targeted learning goals, each with their own core concepts. Each of these core concepts is further broken down into sub-components. In the following sections, we present the number of items catalogued in the assessment inventory that are associated with each sub-component. It should be noted that some items were associated with multiple sub-components in a given core concept, so the sum of the counts in each table may not align perfectly with the number of items related to a given core concept. Similarly, some items address multiple core concepts, so the sum of the items associated with each concept does not necessarily equal the total number of items in the inventory.

Spatial Reasoning

The spatial reasoning learning progression has two targeted learning goals: reasoning spatially within objects and reasoning spatially between objects.

Reasoning Spatially Within Objects

The core concepts in the reasoning spatially within objects targeted learning goal have to do with the properties, transformations, and de/composition of shapes. Altogether, there are 25 items associated with the Shape core concept, 95 items for the Transformation core concept, and 185 items representing the Composition and Decomposition core concept. Tables 4-6 provide the number of items in the assessment inventory associated with each of the subcomponents in these core concepts.

Table 4
Item Count for Shape Sub-Components

Sub-Component	Sub-Component Description	Count
SR.A.1.a (2D)	Sort similar *two-dimensional* shapes regardless of size, orientation, or dimensionality.	9
SR.A.1.a (3D)	Sort similar *three-dimensional* shapes regardless of size, orientation, or dimensionality.	3
SR.A.1.b (2D)	Given the name of a *two-dimensional* shape, recognize the shape.	6
SR.A.1.b (3D)	Given the name of a *three-dimensional* shape, recognize the shape.	1
SR.A.1.c (2D)	Name *two-dimensional* shapes.	3
SR.A.1.c (3D)	Name *three-dimensional* shapes.	3
SR.A.1.d (2D)	Classify *two-dimensional* shapes and describe their defining attributes.	7
SR.A.1.d (3D)	Classify *three-dimensional* shapes and describe their defining attributes.	5

Table 5
Item Count for Transformation Sub-Components

Sub-Component	Sub-Component Description	Count
SR.A.2.a	Recognize a two-dimensional figure that has been translated.	8
SR.A.2.b	Recognize a two-dimensional figure that has been rotated.	44
SR.A.2.c	Recognize a two-dimensional figure that has been reflected.	6
SR.A.2.d	Recognize three-dimensional shapes or figures that have been rotated.	7
SR.A.2.e	Recognize the three-dimensional result of folding a two-dimensional figure.	28

Table 6
Item Count for Composition and Decomposition Sub-Components

Sub-Component	Sub-Component Description	Count
SR.A.3.a (2D)	Recognize the result of mentally translating *two-dimensional* figures together.	7
SR.A.3.a (3D)	Recognize the result of mentally translating *three-dimensional* figures together.	1
SR.A.3.b (2D)	Compose a *two-dimensional* composite figure using transformations (i.e., translations, reflections, rotations, and combinations of these).	106
SR.A.3.b (3D)	Compose a *three-dimensional* composite figure using transformations (i.e., translations, reflections, rotations, and combinations of these).	6
SR.A.3.c (2D)	Compose a *two-dimensional* composite figure in more than one way (e.g., a hexagon can be composed of two trapezoids or six triangles).	28
SR.A.3.c (3D)	Compose a *three-dimensional* composite figure in more than one way (e.g., a hexagon can be composed of two trapezoids or six triangles).	1
SR.A.3.d	Find embedded figures within larger figures.	4
SR.A.3.e	Recognize the two-dimensional cross section created by cutting a three-dimensional shape into two parts.	34
SR.A.3.f	Decompose a two-dimensional composite figure in such a way that the parts can be used to create another given figure.	1
SR.A.3.g	Compose a two-dimensional composite figure and iterate it to compose another figure.	1

Reasoning Spatially Between Objects

The core concepts in the reasoning spatially between objects targeted learning goal have to do with the students' abilities to link various spatial representations and to identify and communicate the positions of objects. Altogether, there are 12 items associated with the Spatial Language core concept, 41 items for the Understanding Models and Maps core concept, and 35 items representing the Perspective-Taking core concept. Tables 7-9 provide the number of items in the assessment inventory associated with each of the subcomponents in these core concepts.

Table 7
Item Count for Spatial Language Sub-Component

Sub-Component	Sub-Component Description	Count
SR.B.5.a	Identify an object's spatial position in relation to other objects.	8
SR.B.5.b	Place an object when given positional language.	2
SR.B.5.c	Describe an object's location in relation to other objects using positional language.	2

Table 8
Item Count for Understanding Models and Maps Sub-Components

Sub-Component	Sub-Component Description	Count
SR.B.6.a	Recognize a three-dimensional representation (e.g., model) of a three-dimensional space.	1
SR.B.6.b	Scale distances and figures based on the size of the representation (e.g., place an object on a line based on the relative placement of an object on a smaller line).	2
SR.B.6.c	Recognize a two-dimensional representation (e.g., model or map) of a three-dimensional space.	30
SR.B.6.d	Create a map to represent a three-dimensional space, such as a classroom.	3
SR.B.6.e	Use a map to find locations of objects, including one's own location.	2
SR.B.6.f	Identify the grid reference system coordinates of an object on a grid.	3
SR.B.6.g	Describe and follow routes on maps.	2
SR.B.6.h	Identify the location of an object on a grid when given map coordinates.	1

Table 9
Item Count for Perspective-Taking Sub-Components

Sub-Component	Sub-Component Description	Count
SR.B.7.a	Recognize the view from one's own perspective.	3
SR.B.7.b	Understand that changes in perspective changes the view.	22
SR.B.7.c	Describe relative spatial positions of objects from different perspectives (e.g., "the chair would be closest to me if I stood over there").	4
SR.B.7.d	Recognize views from different perspectives (e.g., identifies what photo could be taken from a specific viewpoint of a concrete or pictorial representation of a three-dimensional space or object).	7
SR.B.7.e	Construct a three-dimensional object or space given at least two images of top, front, or side views.	3

Numerical Relational Reasoning

The numerical relational reasoning learning progression has three targeted learning goals: relations, composition and decomposition, and properties of operations.

Relations

The core concepts in the relations targeted learning goal have to do with the relative sizes of numbers and how the relationships between those magnitudes is expressed. Altogether, there are 16 items associated with the Comparison core concept, 18 items for the Ordinality core concept, 10 items representing the Transitivity core concept, and 20 items connected to the Representation of Order in Comparison core concept. Tables 10-13 provide the number of items in the assessment inventory associated with each of the subcomponents in these core concepts.

Table 10
Item Count for Comparison Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.A.1.a	Compare two quantities to find which is more/less using matching and counting strategies.	8
NRR.A.1.b	Compare two unspecified weights using balances to find which weighs more/less.	1
NRR.A.1.c	Compare two quantities to find which is more/less using mental images.	2
NRR.A.1.d	Compare two numbers using mental number lines to determine which is more/less.	1
NRR.A.1.e	Compare two numbers using written number lines to determine which is more/less.	1
NRR.A.1.f	Compare two numbers using open number lines to determine which is more/less.	1
NRR.A.1.g	Compare two numbers using symbols: >, <.	3

Table 11
Item Count for Ordinality Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.A.2.a	Without counting, use tools to find a unit more/less than a given number.	4
NRR.A.2.b	Without calculating, mentally find a unit more/less than a given number.	15

Table 12
Item Count for Transitivity Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.A.3.a	Compare two unspecified lengths (a) and (b) to a given reference length (c) to determine which is longer/shorter (a) or (b).	2
NRR.A.3.b	Order unspecified quantities in a word problem.	2
NRR.A.3.c	Order three unspecified weights using balances.	1
NRR.A.3.d	Order three numbers using number relationships with tools.	3
NRR.A.3.e	Order three numbers using number relationships without tools (i.e., mental strategies).	1
NRR.A.3.f	Order three numbers in a word problem.	1

Table 13
Item Count for Representation of Order in Comparison Situations Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.A.4.a	Find how much more/less between two quantities using matching and counting strategies.	5
NRR.A.4.b	Find how much more/less between two quantities using tools.	1
NRR.A.4.c	Find how much more/less between two numbers in a word problem using tools.	1
NRR.A.4.d	Find how much more/less between two numbers in a word problem.	9
NRR.A.4.e	Compare two numbers to find which is [closest to/furthest from] a benchmark.	2

Composition and Decomposition

The core concepts in the composition and decomposition target learning goal have to do with breaking down and combining numbers. Altogether, there are 48 items associated with the Composition core concept, 36 items for the Decomposition core concept, and 18 items connected to the Applying and Representing Composition and Decomposition core concept. Tables 14-16 provide the number of items in the assessment inventory associated with each of the subcomponents in these core concepts.

Table 14
Item Count for Composition Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.B.5.a	Compose a number with single objects.	8
NRR.B.5.b	Compose a number with two parts.	23
NRR.B.5.c	Compose a number with three or more parts.	16
NRR.B.5.d	Compose a number with two or more parts using different number combinations.	2
NRR.B.5.e	Compose a number with two or more parts using concepts of place value.	1

Table 15
Item Count for Decomposition Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.B.6.a	Decompose a number into two parts.	6
NRR.B.6.b	Decompose a number into two parts using equipartitioning.	9
NRR.B.6.c	Decompose a number into three or more parts.	3
NRR.B.6.e	Decompose a number up to 25 into three or more parts using equipartitioning.	17
NRR.B.6.d	Decompose a number into two or more parts using different number combinations.	4
NRR.B.6.f	Decompose a number with two or more parts using concepts of place value.	5

Table 16
Item Count for Applying and Representing Composition and Decomposition Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.B.7.a	Given one part of a number, identify the missing part.	13
NRR.B.7.b	Given a unit, identify the missing part.	3
NRR.B.7.c	Given one part of a number, identify two or more missing parts.	1
NRR.B.7.d	Given one part of a number, identify two or more missing parts using different number combinations	1
NRR.B.7.e	Write an expression to represent the decomposition of a number.	1

Properties of Operations

The core concepts in the properties of operations target learning goal involve understandings of equality. Altogether, there are 14 items associated with the Equivalence of Quantity and Number core concept, 7 items for the Equal Sign as a Relational Symbol core concept, 8 items connected to the Maintaining Equality core concept, and 14 items representing the Solving for Unknown Values core concept. Tables 17-20 below provide the number of items in the assessment inventory associated with each of the subcomponents in these core concepts.

Table 17
Item Count for Equivalence of Quantity and Number Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.C.8.a	Given equivalent sets of quantities, recognize that the quantity of each set remains the same regardless of size, color, or arrangement. (conservation of number)	2
NRR.C.8.b	Given a quantity broken into two parts, recognize that order does not change the quantity. (commutative property)	3
NRR.C.8.c	Given a quantity, recognize that the quantity remains the same after joining/removing a part then removing/joining the same part. (undoing or additive inverse)	4

Sub-Component	Sub-Component Description	Count
NRR.C.8.d	Given two associated parts and another part, recognize that the quantity of the three parts remains the same if the parts are reassociated. (associative property)	2
NRR.C.8.e	Given a quantity, recognize an equivalent expression that demonstrates one or more property of operations.	1
NRR.C.8.f	Recognize two equivalent expressions that demonstrate one or more property of operations.	1
NRR.C.8.g	Recognize two equivalent expressions that demonstrate decomposition and at least one property of operations.	1

Table 18
Item Count for Equal Sign as a Relational Symbol Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.C.9.a	Recognize the equality between two quantities using a balance.	1
NRR.C.9.b	Write a true equation using an equal sign to represent the relationship between given quantities on a balance or in a pictorial representation.	2
NRR.C.9.c	Recognize true and not true equations with different equation structures: operations on the left side (a + b = c); no operations (a = a); operations on the right side (c = a + b).	1
NRR.C.9.d	Recognize true and not true equations with different equation structures: operations on the both sides (a + b = c + d); multiple instances of a number.	4

Table 19
Item Count for Maintaining Equality Sub-Components

Sub-Component	Sub-Component Description	Count
NRR.C.10.a	Given a contextual situation with known quantities, use one or more properties of operations to recognize when equality is maintained.	1
NRR.C.10.b	Given a contextual situation with unknown quantities, use one or more properties of operations to recognize when equality is maintained.	3
NRR.C.10.c	Given a contextual situation with known quantities that models one or more properties of operations, write a true equation to represent the situation.	1
NRR.C.10.d	Recognize true and not true equations with known numbers using one or more properties of operations.	3

Table 20
Item Count for Solving for Unknown Values Sub-Component

Sub-Component	Sub-Component Description	Count
NRR.C.11.a	Solve for an unknown value in a true equation using a relational definition of equal sign.	3
NRR.C.11.b	Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value using concrete objects.	2
NRR.C.11.c	Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value in a true equation.	1
NRR.C.11.d	Apply one or two properties of operations or property of equality to solve for an unknown value in a true equation.	5
NRR.C.11.e	Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value using concrete objects.	3
NRR.C.11.f	Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation.	1
NRR.C.11.g	Apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation.	1

Conclusions

We developed the assessment inventory so that we would have a place to collect and to store items created to measure the early numeracy constructs of spatial reasoning and numerical relational reasoning. These items will help inform the design of assessment items for the MMaRS project by allowing us to identify common item features that are used to assess the constructs.

We began by creating a workspace in FileMaker Pro that had data entry fields for each type of information that we needed about the items. Next, we identified a variety of item sources, which included published tests, academic research, and the cognitive interview items used in the larger MMaRS project. Finally, each item from these sources was systematically entered into the database and verified by multiple researchers. At the end of this process, we had 339 spatial reasoning items and 261 numerical relational reasoning items in the inventory.

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