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Numeric Relational Reasoning (NRR) Cognitive Interviews: Properties Qualitative Analyses

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Numeric Relational Reasoning (NRR) Cognitive Interviews: Properties Qualitative Analyses

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Abstract

The purpose of the current report is to describe the qualitative analyses of the properties of operations targeted learning goal of the Numeric Relational Reasoning (NRR) learning progression part of the larger Measuring Early Mathematics Reasoning and Skills (MMaRS) project. In this report, we describe the data procedures, analyses, and results. The results from these analyses will help inform updates to the NRR learning progression.

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Numeric Relational Reasoning (NRR) Cognitive Interviews: Properties Qualitative Analyses

Introduction

The purpose of this report is to describe the qualitative analyses conducted of the Numeric Relational Reasoning (NRR) Cognitive Interviews (CIs) of the Properties of Operations Targeted Learning Goal for the Measuring Early Mathematical and Reasoning Skills (MMaRS) project. Based on the hypothesized NRR learning progression (LP) for the Properties of Operations Targeted Learning Goal, we developed CI protocols and implemented those to inform the LP's conceptualization and empirical recovery. See the Numeric Relational Reasoning Interview Protocol Development (Tech. Rep. No. 20-04) and the Numeric Relational Reasoning Cognitive Interview Administration (Tech. Rep. No. 20-05) technical reports for development and administration details. Details on the qualitative analyses for the Relations and Composition/Decomposition Targeted Learning Goal can be found in the Numeric Relational Reasoning Qualitative Analyses Technical Report (Tech. Rep. No. 20-29). This report mirrors the Spatial Reasoning Qualitative Data Analysis Technical Report (Tech. Rep. No. 20-21). Therefore, some figures have been omitted and can be found Spatial Reasoning technical report.

Research Questions

We designed the cognitive interviews to address four research questions related to empirically recovering the NRR learning progressions. We included sub-questions within each overarching research question. Question 3 and 4 required information from the cognitive interview video and transcription data with gestures included. This report details the methods and results of analyses to address Questions 3 and 4.1 for the Properties of Operations Targeted Learning Goal.

RQ 1: Developmental Appropriateness

1.1 Do the entry and exit KSAs align with teachers' expectations of pre-requisite and target skills?
1.2 Does teachers' frequency of teaching KSA align with progression?
1.3 Does student performance and engagement indicate floor or ceiling effects that align with entry and exit KSAs?

RQ 2: Ordering

2.1 Are teachers' perceptions of the appropriateness aligned with the hypothesized order?
2.2 Do students demonstrate increasingly sophisticated reasoning aligned with the hypothesized ordering?
2.3 Do students appear comfortable with tasks and task elements?

RQ 3: Conceptions

3.1 Do students demonstrate reasoning that is consistent with the hypothesized conceptions?
3.2 What misconceptions and/or errors do students make? Is there a pattern leading to greater competence?

RQ 4: Interconnectedness

4.1 In what ways are students' KSAs interconnected?
4.2 In what ways does prior impact students' responding?

Table 1 describes the data used by research question.

Table 1

Data used by research question

Research Question		Data Use
1		
	1.1	Teacher Survey Data
	1.2	Teacher Survey Data
	1.3	Quantitative Data; Fidelity Data
2		
	2.1	Teacher Survey Data
	2.2	Quantitative Data (c-prop, p-values)
	2.3	Fidelity Data
3		
	3.1	Quantitative and Qualitative Data
	3.2	Classification of Incorrect CI Responses and Qualitative Data
4		
	4.1	Qualitative Data
	4.2	--

Methods and Processing

Data Processing

The primary data source for qualitative analyses were student audio and video recording, workbooks, and observer protocols. For data handing in the field and upon secure delivery to offices at the university, see the Numeric Relational Reasoning Cognitive Interview Methods and Quantitative Data Analysis Technical Report (Tech. Rep. No. 20-05).

Once data was in the Research in Mathematics Education (RME) office space, a team member sorted the interview materials, including the student assent, student workbook, observer copy of the interview protocol, and fidelity observation form, before filing and locking all documents in a secure space. Video and audio files were securely uploaded to BOX, the university Institutional

Review Board (IRB) approved, secure, cloud-based file storage. Audio files were transcribed through Rev.com, an approved, outside transcription service, and uploaded to the student-level folders in Box.

Following the audio file transcription, a group of internal team members watched the videos and inserted student actions and gestures, and any pertinent interviewer actions, using a process we called non-verbal transcription. They simultaneously removed non-mathematical conversation and added subcomponent names to divide the transcription into sections of text for further analysis.

After non-verbal transcription was completed, we used the new transcript files containing non-verbals for qualitative coding and analyses. These files were uploaded to NVivo and used as the sole data source for coding, except when non-verbals were not specific enough to determine student thinking and strategies. Those exceptions were detailed in the codebook audit trails and included viewing pictures and student work or reviewing videos when deemed necessary by the coding team. Specific methods and procedures, including training, non-verbal transcribing, deductive a priori coding, and open coding, are detailed below.

Non-Verbal Transcriptions

The purpose of non-verbal transcription was to allow coders to better understand the numeric relational reasoning cognitive interviews without consulting multiple data sources. We created transcripts of the interviews that included relevant non-verbal actions and gestures by the student and interviewer. By including these in the written transcript, we sought to lower the coders' cognitive load while searching for themes.

Three team members were onboarded to facilitate the insertion of non-verbal gestures and actions into the audio-only transcripts. They received a project overview and online training. They were connected with a researcher working on the project as an internal point of contact with whom to complete transcript insertions. The MMaRS researcher oversaw the flow of training, verified the accuracy of non-verbal coding on the sample video, and released the data to transcribers after training. Each coder was assigned one core concept (i.e., relations, composition/decomposition, properties of operations) and completed a sample video of the assigned protocol from our earlier try-out interviews. The researcher verified the entire try-out transcript and provided feedback to align coding practices with anticipated results.

Coders were given core concepts and subcomponents specific coding gesture insertion instructions to write within the transcript. Cross core concept protocol instructions were as follows:

- Insert, in bold font, the skill code name exactly as it appears on the protocol (e.g., if a = a, insert a = a).
- Gestures on the "student" line: If a student makes a gesture during their talk turn in the transcript, you do not have to refer to the student. For example, if the student points to a number and the student is talking, you make insert [points to number].

- Gesture on the "interviewer" line: If a student makes a gesture during the interviewer talk turn, indicate in the non-verbal insertion that the student is the one gesturing. For example, if the interviewer is talking and the student points to a number, you may insert [student point to number].

- Gestures by the interviewer: We are not generally interested in what the interviewer gestures, but if you believe something is integral to the reader's understanding, please note [interviewer points to…] regardless of talk turn at which the gesture and insertion occur.

The researcher provided individual feedback specific to the assigned protocol, as the targeted learning goals and core concept varied, and the information needed was not consistent across interview types. As part of the analysis plan, the researcher verified 20% of 20% of the videos' nonverbal transcriptions, or three to four subcomponents in each of the videos. This verification process provided continuous feedback loops so that the researcher could refine the process throughout the coding process. Coders benefited from becoming proficient quickly and reduced their time spent per video while inserting higher-quality non-verbals. After non-verbal transcription was complete, coders pre-coded the same interview groups in which they had inserted non-verbals.

During the non-verbal transcription process, coders identified and marked each subcomponent in the transcripts using naming conventions, which delineated the targeted learning goal, core concept, subcomponent, and in some cases, the microconceptualization that the task assessed; task names additionally specified what property (e.g., commutative property) or equation (Figure 1).

Figure 1

Subcomponent items with multiple equations structures

Time: SID #: 10. Maintaining Equality

Skill	Actions	Questions/Student Responses
NRR.C.9.c. [equation structures] NRR.C.9.d. [equation structures]	• Give child one card at a time. • Check equation structures that child classifies correctly. • Ask follow-up questions starting with the cards placed in the yellow pile. • Work space:	*I'm going to give you a number sentence [equation]. If you think the number sentence is true, then place the card in this true area. If you think this is not true, then place the card in the not true area.* [Motion to those areas on the workmat.]. *If you are not sure about a card, then put it in the center [unsure] area.*

True	?	Not True

0-5 ▲	a + b = c	a = a	c = a + b	a + b = c + d	a + a + … + a = b + b + … + b
True	-	4 = 4	-	2 + 2 = 3 + 1	-
Not True	3 + 1 = 3	-	3 = 1 + 4	-	1 + 1 = 2 + 2

0-10 ♦	a + b = c	a = a	c = a + b	a + b = c + d	a + a + … + a = b + b + … + b
True	-	8 = 8		1 + 5 = 4 + 2	-
Not True	6 + 1 = 6	-	6 = 2 + 8	-	2 + 2 = 4 + 4

0-20 ♥	a + b = c	a = a	c = a + b	a + b = c + d	a + a + … + a = b + b + … + b
True	-	16 = 16	-	11 + 5 = 14 + 2	-
Not True	12 + 1 = 12	-	12 = 4 + 16	-	4 + 4 = 8 + 8

0-50 ♠	a + b = c	a = a	c = a + b	a + b = c + d	a + a + … + a = b + b + … + b
True	-	25 = 25	-	21 + 5 = 24 + 2	-
Not True	24 + 2 = 25	-	24 = 8 + 32	-	10 + 10 = 20 + 20

0-99 ♣	a + b = c	a = a	c = a + b	a + b = c + d	a + a + … + a = b + b + … + b
True	-	50 = 50	-	51 + 5 = 50 + 6	-
Not True	49 + 2 = 50	-	48 = 12 + 60	-	20 + 20 = 40 + 40

0-199 ★	a + b = c	a = a	c = a + b	a + b = c + d	a + a + … + a = b + b + … + b
True	-	150 = 150	-	151 + 5 = 150 + 6	-
Not True	149 + 2 = 50	-	160 = 6 + 166	-	40 + 40 = 80 + 80

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Coders also used a strikethrough to remove irrelevant talk turns (e.g., student talking about Halloween) as the first step in data trimming. However, they did not remove any text at this stage as researchers wanted to preserve the full transcript if needed later.

While completing non-verbal transcriptions, security protocols were implemented for IRB and data security purposes. Coders used a tracking spreadsheet to communicate completion, track time spent on tasks, and communicate about verification and assignments. Videos and transcripts were accessed through BOX. All videos were viewed in preview mode, and non-verbal transcription was completed in the Word online version.

Pre-Coding

To prepare the transcripts with non-verbals (henceforth named combined transcripts) in NVivo for qualitative coding, the researcher who oversaw the non-verbal transcription process combined all individual student transcripts by protocol into single files for the labeling, trimming, and import into NVivo. During the process, additional confirmation was completed to remove extraneous talk turns that were irrelevant to the process (e.g., talking with the child about family, dogs, cats, etc.). Some irrelevant talk turns that occurred within the discussion were left in the transcript for coders to reference context. The lead coder/project researcher uploaded the combined transcripts into NVivo for pre-coding.

The non-verbal coders engaged in a two-part training with a MMaRS researcher to learn pre-coding procedures. The researcher led coders in reviewing the protocols before introducing them to and training them on procedures for NVivo pre-coding. They trained on a sample transcript and completed their first coding assignment on-site with the researcher's support as the lead coder. Once fully trained, coders were assigned a set number of subcomponents, after which the researcher verified 20% of 20% of text coded before they proceeded to the following group of subcomponents.

To complete pre-coding, the coder first attached all text associated with the subcomponent to the subcomponent node. While codes are defined as "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study" (Miles & Huberman, 1994, p. 56), noes are the structures used within NVivo to organize codes. For each subcomponent, coders attached specific text to the following nodes: (a) content question & response, and (b) reasoning questions and response.

After coders completed attaching all associated text to the subcomponent and further coding the associated text to the relevant nodes, a researcher divided the entire protocol into various sections (based on the 20% rule) and identified various verification points at the end of each section. Coders were required to stop after completing all skill codes within each section and wait for the researcher's permission to start coding the next section. The researcher verified that all available transcript text was attached to the appropriate subcomponent and that within subcomponents, all text as coded to the relevant nodes. After verification, the researcher informs coders to make adjustments (if needed) or move to the next coding section. Once pre-coding was complete for all interview protocols across NRR LPs, the NVivo projects were ready for qualitative coding and analyses.

Coding

The hypothesized learning progression and its bands of developmental appropriateness aligned with stepping stones of knowledge and understanding that occur over time, liked to the grounded theory approach of qualitative inquiry (Creswell & Poth, 2018). We developed a systematic procedure using the constant comparison between our data and codes as they were developed (Corbin & Strauss, 2015) to developed preliminary codes that led to axial codes and final themes (Saldaña, 2016).

To move through open coding, data trimming was needed for focused analysis of mathematical reasoning in student responses to ensure that findings were grounded in meaningful student thoughts, words, and actions. This necessitated a two-cycle coding process of first a deductive, a priori schema, followed by open-coding to search for emergent themes.

A Priori Coding

We used the project's research goals to develop a priori structural codes (DeCuir-Gunby et al., 2011). The later open-coding process involved "breaking data apart and delineating concepts to stand for blocks of raw data" (Corbin & Strauss, 2008, p.195). Open-coding was an iterative process in which coders created codes and then used axial coding to analyze them. The holistic process led to the development of data-driven codes and involved five steps to inductively create codes for a codebook (DeCuir-Gunby et al., 2011). We:

1. Reduced the raw information in pre-coding and a priori coding (as described below)
2. Identified subsample codes
3. Compared codes across subsamples
4. Created codes
5. Determined the reliability of codes through debrief and reconciliation meetings

A coding team, consisting of one lead coder, one secondary coder, and a facilitator, engaged in this initial part of the coding process. Both coders had in-depth content and age-range or developmental appropriateness expertise. The facilitator provided training and NVivo software support during the a priori coding. The facilitator gradually released responsibility for compiling individual files to obtain reliability and maintaining master files post reconciliation to the lead coder; the lead coder's file was already serving as the master, but verification and maintenance were additional steps. They applied a priori codes to reasoning responses in which students provided no explanation a tangential explanation, or a mathematical explanation, regardless of correctness (See Table 2). The coding structure was applied to all three NRR targeted learning goals in NVivo.

Coders maintained an a priori codebook for each targeted learning goal, relations, composition/decomposition, and properties of operations. Within the codebook were spreadsheets that coders completed while coding: (a) a timeline to complete the work, (b) status tracking of each task including the time to code, and (c) an audit trail to note questions or inconsistencies found in the data (Brinkmann & Kvale, 2015). The audit trail also provided a space in which the team developed coding rules. These including coding to the deepest level of student thought within a single or multiple reasoning question and response, chunking each unique reasoning question separately, and alerting the team if there is text that does not align with the protocol item. These rules then applied to each targeted learning goal.

There was additional tab for coding agreement that the coding facilitator and the lead coder used to track and record coding agreement between coders (See Appendix A for detailed agreement information). Twenty percent of subcomponents were double coded to verify coder agreement with a target of at least 80% coding agreement among coders. The facilitator led a coding agreement check process using NVivo coding comparison queries and reported agreement and kappa coefficients as evidence of agreement (Saldaña, 2015). After each subcomponent marked for common coding was complete, the team debriefed to resolve any disagreements and discourage coder drift (Marston et al., 1978) from the deductive scheme. The lead coder recorded all disagreements and outcome of the debrief in the agreement tab as needed.

Using explicit code definitions as guides, coders moved through a process of elimination to arrive at the most accurate code for each child's reasoning response. If the child did not provide any reasoning about an item, that response was coded as "no reasoning". If the child reasoned using some type of logic that could be extracted as mathematical or connected to real life, the response was coded as "mathematical reasoning". If the child told a story or talked about an idea that was not connected to math, the response was coded as "tangential reasoning" (See Table 2 for examples). After coding all subcomponents using a priori schemes, the reconciled file was distributed to the coder with a new codebook and folder structure in the secure BOX drive for open coding.

Open Coding

Next, a coding team engaged in an open coding process looking of emergent themes using only the data reduced to "mathematical reasoning" from a priori coding. While themes are generally reporting patterns found in a data set, they also capture some information related to the research question. Given the fluid nature of interviews, an iterative process was developed to move through identifying themes, yet there is no specific rule on the number of evidential instances required to develop a theme (Braun & Clarke, 2006). The goal of open coding was to identify common themes across student reasoning working on the same subcomponent task. By identifying themes across subcomponents and defining them through codes, the team sough to support the conceptualization of ordering, find interconnections, and identify student misconceptions and errors.

The lead and secondary coder engaged in an iterative process of open coding. They independently coded in NVivo to each develop pre-codes, or first pass themes, based on independent review. Coders next compared their preliminary codes in debrief meetings and reconciled NVivo project files while determining final code names and definitions. The coding facilitator guided the development of the process and codebook procedures.

The open coding process focused on identifying patterns of student strategies used in response to the reasoning question for specific subcomponents. Coders crafted a detailed timeline to code between one and three subcomponents, debrief, and finalize the codebook for each core concept before moving to the next. The team moved sequentially through the NRR Properties targeted learning goal. The timeline for coding completion and synthesis writing was maintained in the the codebook, with a status spreadsheet that coders updated throughout the process. For each core concept, there was a new set of tabs in the codebook that corresponded with matching folders in BOX. This structure was to retain all NVivo files and summary statements in a single location with the codebook for quick retrieval and reference.

Independent Preliminary-Coding. Using the codebook and NVivo file simultaneously, coders independently created preliminary codes within the Mathematical Reasoning node in NVivo for each core concept. Preliminary codes were based on strategies or patterns of response that emerged broadly across student responses for the subcomponent protocol item. To enable later conversations, each coder developed a description based on preliminary jottings (Saldaña, 2016) of the code and identified at least one example from student transcripts. The code name, description, and sample transcript text were recorded in the codebook on coders' individual spreadsheets with the number of students by grade level who used that strategy.

Temporary Combined Codebook. After each coder independently created their preliminary codes, the lead coder retrieved those pre-code names, definitions, and student count and compiled the information to facilitate reconciliation in the debrief meeting. Before the debrief meeting, the lead coder aligned similar pre-codes in the temporary combined codebook tab within the codebook workbook (see Tech. Rep. No. 20-21 for example). The alignment served to guide the conversation on similarities or differences between early code iterations. When on coder distinguished fine-grain codes from the data the other used coarser codes, the latter's big idea codes could open encompass multiple, specific pre-codes.

Coder Debrief and Codebook Comparison. The team relied on "dialogical intersubjectivity", a process in which the two coders hosted an intensive discussion to achieve group consensus on codes (Brinkmann & Kvale, 2015; Harry et al., 2005; Sandelowski & Barroso, 2007). Coders held these debrief meetings to ensure the quality of coding and from agreement with common codes for a protocol item. Coders compared NVivo files, codes, and definitions, developing code names and substantive definitions before concluding each meeting.

Through the meeting, code names were revised and NVivo files were updated with new code names and corresponding text to capture the reconciliation. When revisiting codes, definitions were jointly determined and examples were located in transcript data to illustrate the heart of the overall themes that codes were to represent. Further, the discussion of interconnection between subcomponents emerged, which led to conversations about which skills might be interrelated and require additional analysis concerning the given subcomponent.

Finalization of Codebook. At the end of each meeting, the lead coder recorded the final code names, definitions, examples from the data, and any exclusion criteria in the final codebook. Student counts by code were revised, and the examples of student talk and gesture were included. The lead coder's NVivo file was also updated with the new codes and uploaded to BOX as a reconciled file.

Results of the Qualitative Analyses

In this section, we describe the results by research question, including associated sub-questions. The results of this qualitative analysis address children's conceptions of the content, including misconceptions and errors, and interconnections of knowledge, skills, and abilities with and across the cognitive interview protocol items.

RQ 3: Conceptions

RQ 3.1

We examined the reasoning that students used through qualitative analyses as one source of evidence to inform the conceptualization of the hypothesized learning progressions. We describe reasoning patterns as a standalone source of data, not yet informed by or reconciled with correctness data. These patterns and other results are synthesized in the Bulleted Summaries found in Appendix B.

Subcomponent Synthesized Description. Using a two-step process, the lead coder created synthesized descriptions of student actions that aligned with increasingly complex or mature ways of thinking about the given construct. The first step was an independent draft of axial codes based on codebook descriptions. The second step involved a team review, or member check for interpretive convergence (Saldaña, 2016), to ensure accurate capture of the analysis outcome. The coders read each subcomponent, the elements that varied, the codebook codes, synthesized descriptions, and inferred student misconceptions and errors. Together, they created final statement that included the elements that varied when possible to streamline later iterations of learning progressions. See the bulleted statements in Appendix B for example of each component.

Some skills evolved more linearly in anticipated patterns of developmental progression, while others seemingly developed simultaneously. Misconceptions and errors also emerged from these data and were delineated and aligned with codes from the codebook of response patterns. Also, each synthesized statement in the progression of correct thinking with final codes from the codebook, informed the statement, and supported the ordering or concurrent skill development.

Microprogressions of skills. We observed smaller progressions of skills within subcomponents that we call microprogressions. These small progressions are the steps reasoning strategies student engaged with while solving problems related to properties of operations. Table 2 demonstrate this microprogression for NRR.A.1.a. We observed a hierarchical nature to how students reasoned with properties of operations. For example, students had to recognize that the value stayed the same. However, reasoning could consist of either matching to compare or reasoning through conservation.

Table 2

Performance Level Descriptions in the Conservation of Number Task

Performance Level	Descriptor
i. Conservation Recognizer	Student recognizes that the two quantities are the same does not reason about how they are the same
iia. Match Comparer	Student reasons conservation of number by matching corresponding objects
iib. Conservation Reasoner	Student reasons that changes in location does not yield different numbers without counting the objects.

Reasoning with individual properties. The properties of operations targeted learning goal was originally designed to start with core concepts that were more concrete (e.g., tangible quantities) to more abstract (e.g., symbolic representations). Within this progression, the multiple properties of operations (i.e., commutative, additive inverse, associative) were nested. Results from open coding revealed that students engaged with these properties of operations similarly across the different representations. For example, students would often reason similarly if they were given quantities to represent the commutative property or an equation that represents the commutative property. The findings suggest that the development from concrete representation to more abstract representations can occur within each properties due to similar reasoning strategies occurring amongst the different representations of the same property.

RQ 3.2

When analyzing patterns of each subcomponents' linear development, or the microprogression, we analyzed misconceptions and/or errors made by students. Errors in thinking that represented less sophistication in student reasoning evidenced patterns upon which educators could develop scaffolding for learning. In contrast, misconceptions provided a direct path to intervention to correct those conceptions. Misconceptions can be found in Appendix B.

Student Misconceptions or Errors. For those codes that did not represent developmental steps in a microprogression for a given subcomponent, coders analyzed student transcripts to find misconceptions or errors in thinking. These were characterized by individual or common examples from transcripts and were further detailed in each code's descriptors. Errors and misconceptions aligned with the levels of thinking in the synthesized descriptions, which could be later used by practitioners to facilitate academic feedback or scaffolded practice. By identifying the level of sophistication at which a misconception was associated, one could map to where academic feedback was needed for children to correctly conceptualize the skill.

Student conceptualization of the equal sign. When we presented students equations, we observed patterns in their behavior consistent of the work conducted by Stephens et al. (2013). We posit that students conceptualize equations in three different ways. First, the operational view where the equal sign is seen as to do something. We observed multiple instances in our data where student would consider the equal sign as symbol for performing some operations. The next view is the relational-computations view. In this view students are able to conceptualize the equal sign as a balancing symbol but believe that computation is the only way to ensure equality. For example, 3 + 2 = 4 + 1 because the sums are equal. Lastly, the relational-structural view considers the relation among the numbers on both sides of the equal sign and how the equal sign operates as a balance between the numbers. For example, 3 + 2 = 4 + 1 because 4 is one more than 3 and 1 is 1 less than 2.

RQ 4: Interconnectedness

RQ 4.1

The ways in which students' KSAs are interconnected was evidenced through their words, actions, and gestures related to reasoning responses during cognitive interviews. Each progressively sophisticated task in the subcomponents within each core concept relied upon anticipated microprogressions of interrelated KSAs. For example, within the Equivalence of Quantity and Number core concept, we observed students using total number equivalence across four unique subcomponents (see Figure 2). This provides evidence that students can reason similarly across multiple skills.

Figure 2

Use of Total Number Equivalence Across the Equivalence of Quantity and Number Core Concept

Strategy Code	**Strategy Description**	**Example (only unanticipated strategies will have an example)**	NRR.C.8.a	NRR.C.8.b	NRR.C.8.c	NRR.C.8.d	NRR.C.8.e. Multiple Properties	NRR.C.8.e. Multiple Properties	NRR.C.8.e. Multiple Properties	NRR.C.8.e. Multiple Properties	NRR.C.8.f.	NRR.C.8.g
							Assessed as AP	Assessed as CP	Asssessed as Literal Translation	Assessed as No Property		
Total Number Equivalence	Student reasons equality by recognizing that the total did not change or location did not change		2	2		2	2					

Next Steps

In conjunction with the results from the Numeric Relational Reasoning Cognitive Interview Quantitative Analyses and the Numeric Relational Reasoning Teacher Survey results, the Numeric Relational Reasoning learning progression can be reconciled to reflect the evidence gathered. After reconciliation, items will be developed and tested before large scale implementation.

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Appendix A – Reliability Data for a priori coding

Protocol Item	Code (Reasoning)	Kappa	NVivo Agreement	M & H Agreement Needed (0=no; 1 = yes)	Number of Agreements (A)	Number of Disagreements (dA)	% Agreement =A/(A+dA)	Debrief Required (0=no; 1 = yes)
NRR.C.8.a.	Mathematical Reasoning	0.87	99.63	0				
	No Reasoning	0.87	99.9	0				
	Tangential Reasoning	0.81	99.96	0				
NRR.C.8.f.	Mathematical Reasoning	0.94	99.67	0				
	No Reasoning	0.51	99.97	1	0	1	0.0%	1
	Tangential Reasoning	1	100	0				
NRR.C.9.a.	Mathematical Reasoning	0.85	99.39	0				
	No Reasoning	0	99.93	1	0	2	0.0%	1
	Tangential Reasoning	0.13	99.75	1	1	3	25.0%	1
NRR.C.10.c.	Mathematical Reasoning	0.92	99.81					
	No Reasoning	0.68	99.88	1	1	2	33%	1
	Tangential Reasoning	0	99.83	1	1	2	33%	1
NRR.C.11.c.	Mathematical Reasoning	0.94	99.92	0				

	No Reasoning	0.82	99.98	0				
	Tangential Reasoning	1	1	0				

Appendix B – Bulleted Summaries

A bulleted style summary was created with axial codes of student reasoning and synthesized descriptions as related to the skill. This summary also included student misconceptions and errors that were discovered through the cognitive interview qualitative analysis to be considered for learning progression reconciliation and later assessment item writing. See Figure X for visual example, with details explained below for each field.

Naming Conventions. All naming conventions for the core concept name and number were included for continuity across documents, with an added field for a short description of the subcomponent.

Original subcomponent Statement. The subcomponent statement of student actions was included from the detailed learning progression descriptions, including the target grade band and developmental level.

Open Coding Themes. Synthesized themes captured in open coding were detailed with refined names. Some direct student examples were included if necessary to illustrate what students had done as reasoning. These codes served as the axial codes that aligned with all other steps in the bullet summaries.

Subcomponent Synthesized Description. In two steps, the lead coder first created synthesized descriptions of student actions that aligned with increasingly complex or mature ways of thinking about the given construct. Some skills evolved more linearly in progressions while others may develop simultaneously, while still others were not developmental in nature and were errors in student thinking or misconceptions that required academic feedback to facilitate student growth. For each synthesized statement, final codes from the codebook were aligned and included to inform the statement and support the ordering or concurrent skill development.

Student Misconceptions or Errors. For those codes that did not represent developmental steps in a microprogression of the subcomponent, coders analyzed student transcripts to find misconceptions or errors in thinking. These were characterized by individual or common examples from transcripts and were further detailed in the descriptors of each. They often aligned with the levels of thinking that were listed in the synthesized descriptions, which could be later used by practitioners to facilitate academic feedback or scaffolded practice.

Equivalence of Quantity and Number	Properties of Operations: Equivalence of Quantity and Number					
	ORIGINAL SUBCOMPONENT STATEMENT & ELEMENTS THAT VARY	**AXIAL CODES**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION INITIAL**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION RECOMMENDATION (Cass/Robyn) *summarize/synthesize G (axial coding)***	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
	Subcomponent: Given equivalent sets of quantities, recognize that the quantity of each set remains the same reguardless of size, color, or arrangement (conservation of number) Elements that varied: number ranges	(a) Total Number Equivalence: student reasons that combining the sets of objects in different ways does not change the total amount in the contextual situation. (b) Different Location Inequality: student reasons that the amount of animals is more, less, or the same by comparing the amount of objects in the set to each other with or without referring to the number of objects. (c) Matching: student reasons equality by matching different objects that correspond	I. Given sets of equivalent quantities, recognize that each set remains the same regardless of color or arrangement (conservation of number)	i. Given a contextual situation involving sets of equivalent quantities, student recognizes that each set remains the same regardless of color or arrangement (conservation of number) ii.a. Given a contextual situation involving sets of equivalent quantities, student reasons equality by matching objects of different type, color, and arrangement by matching objects that correspond (c) ii.b. Given a contextual situation involving sets of equivalent quantities, student reasons that	Based on the corresponding codes from the data set, the item accurately accessed students' knowledge of the conservation of number. The contextual situation statement was added to differentiate this subcomponents that did not have a contextual situation to assess the subcomponent. Students were able to match objects to reason equality and then progress to recognition without direct	(a) Student does not recognize that different type, and arrangement of objects does not change the amount of object in the set [code b]

				arranging objects in different ways does not changes the total amount in a contextual situation (a)	matching to determine that arrangement does not change the total amount.	

	Subcomponent: Given a quantity broken into two parts, recognize that order does not change the quantity (commutative Property) Elements that varied: number ranges	(a) I-R-Perceptual Subitizing - Recognizing that length or density impacts the number of objects (b) Commutative Property - Given a contextual situation, students are able to tell that the two quantities are the same because it does not matter what order you introduce them in (c) Number Comparison - Student reasons by comparing to two values without using the operations (d) Total Number Equivalence - Student reasons equality by recognizing that the total did not change or location did not change (e) Add all - Student reasons by adding all the quantities	i. Given a contextual situation with quantities of objects broken into parts, recognize that order does not change the quantity	i. Given a contextual situation with quantities of objects broken into parts, student recognizes that order does not change the quantity (commutative property) ii.a Given a contextual situation with quantities of objects broken into parts, student reasons by comparing to two values without using operations (c) ii.b Given a contextual situation with quantities of objects broken into parts, student reasons equality by recognizing the total amounts did	Contextual situation was added to distinguish this subcomponent from other subcomponents that did not assess with a story and based on developmental needs of students to understand these properties within context. Commutative Property was added back into the statement to add specificity.	(a) Student does not recognize that length or density does not impact the number of objects [code a] (e) Student reasons about the equality by adding all the quantities presented [code e]

				not change or location did not change (d) iii. Given a contextual situation with quantities of objects broken into parts, student reasons that the two quantities are the same because the order of presentation does not matter (b)		

	Subcomponent: Given a quantity, recognize that the quantity remains the same after joining/removing a part then removing/joining the same part. (undoing or additive inverse) Elements that varied: Quantities/number ranges	One Tree Codes ~ these are used for bulleted summaries/misconceptions (a) One: Two Operations: Student can reason equality of quantity when a subset of the quantity is subtracted and then added back in the one tree setting using both operations. Students who use this reasoning have an idea of the additive inverse but do not specifically reference zero. (b) One: Single Operation: Student can reason equality of quantity when a subset of the quantity is subtracted and then added back in the one tree setting using only one operation. (c) One: Separation of sets: Students reason by comparing two subsets of values within the quantity when given only one tree to reason with. Comparing the two subsets instead of reasoning about the	i. Given a quantity in two parts, reasons that the quantity remains the same after joining/removing a part then removing/joining the same part. (additive inverse; a+b-b). (a)	i. Given a quantity in two parts and a contextual situation, student reasons that the quantity remains the same after removing a part, then rejoining the same part (additive inverse; a-b+b). (a)	Contextual situation was added to distinguish this subcomponent from other subcomponents that assessed the additive inverse without a story. The undoing was removed in favor of more appropriate mathematical language and the expression corresponding to the contextual situation was added. Evidence of students recognizing this without reasoning did not exist, so we removed the idea of recognition being a part of the SSD.	(a) student is unable to reason using both operations simultaneously when given quantities [code: b] (b) student compare the two quantities, without reasoning about the operations of the situation [code c] (c) student combines the quantities provided, without reasoning about the operations of the situation [code d]

		whole situation is a misconception. (d) One: adding all: student focuses on adding all the numbers when given one tree. Student doesn't reason about the whole situation. This is a misconception. Two Tree Codes are in the codebook and not pulled over into this document, it was an issue with the protocol item and should not be used in the future.				

	Subcomponent: Given two associated parts and another part, recognize that the quantity of the three parts remains the same if the parts are reassociated. Elements that varied: number ranges **protocol was associative**	(a) Total Number Equivalence: student reasons that combining the sets of objects in different ways does not change the total amount in the contextual situation. (b) Different Location Inequality: student reasons that the amount given is more, less, or the same by comparing the amount of objects in the set to each other with or without referring to the number of objects.	i. Given two associated parts and another part, reasons that the quantity of the three parts remains the same if the parts are reassociated. [(a+b)+c = a+(b+c)]	i. Given two associated parts and another part in a contextual situation, student reasons that the quantity of the three parts remains the same if the parts are reassociated. [(a+b)+c = a+(b+c)] (a)	Contextual situation was added to distinguish this subcomponent from other subcomponents that assessed the associative property without a story. The corresponding equation to the contextual situation was also added to provide a guide for the assessment of this subcomponent. Evidence of students recognizing this without reasoning did not exist, so we removed the idea of recognition being a part of the SSD.	(a) student compares the quantities presented instead of reasoning about the total remaining the same [code b]

	Subcomponent: Given a quantity, recognize an equivalent expresson that demonstrates one or more properties of operations. Elements that varied: number ranges		I. Given a contextual situation, recognize that an expression represents the situation using properties of operations.		Contextual situation was added to distinguish this subcomponent from other subcomponents that assessed the various property without a story. Students were asked to select a card with a number sentence that related to the contextual situation provided. Each card represented a different property, no property, or a literal translation. Therefore statements were separated according to the card the student chose and the reasoning that followed from the selection of that card. One or more properties was separated into multiple	

					subcomponents because student reasoning was different depending on the property represented in the item.	
	Literal Translation	(a) I-R-Comp-Decomp - Student reasons by composing two numbers to make a quantity in the expression (b) Number-object Correspondence - Student reasons equality of expression by making a one-to-one correspondence between quantities and numbers in the expression	i. Student reasons equality by making a one-to-one correspondence between the quantities presented and the numbers in the expression (b) ii. Student reasons by composing and/or decomposing	i. Given a contextual situation and expression, student reasons about the equality by making a one-to-one correspondence between the quantities presented and the numbers in the expression (b) ii. Given a contextual situation and expression, student reasons by composing and/or decomposing two		

			two number to make an equivalent expression (a)	number to make an equivalent expression (a)		
	Commutative Property	(a) Symbols - Student reasons about the equation by focusing on the operations and numbers as symbols (b) Number Comparison - Student reasons by comparing to two values without using the operations	i. Student reasons by comparing the two values in the expression without using the operation (b)	i. Given a contextual situation and expression representing the commutative property, student reasons by comparing the two values in the expression without using the operation (b)	Note that this item students were given 3 quantities, (e.g., 2+2+1) and the commutative property express kids were given was also associative (e.g., 1+4); so this is a recommendation to include a literal translation + commutative (e.g., 1+2+2) although we don't have evidence of this.	(a) Student does not understand how symbols and numbers work together to form an equation (e.g. student does not know what the equal sign represents and uses it as an operation. [code a]

	Associative Property	(a) Regrouping - Student reasons associative property by regrouping the quanities (b) Total Number Equivalence - Student reasons equality by recognizing that the total did not change	i. Student reasons equality by recognizing the total did not change (b) ii. Student reasons equality by recognizing the quantities can be regrouped (a)	i. Given a contextual situation and expression representing the associative property, student reasons equality by recognizing the total did not change (b) ii. Given a contextual situation and expression representing the associative property, student reasons equality by recognizing the quantities can be regrouped (a)	For this item, students were associating like items together, ducks in field and ducks in pond. Should this be a part of this subcomponent? (combining like objects vs in the next item ~ no property was combining objects in the same location that were not the same object (rabbits/ducks). POTENTIALLY WE SHOULD GO BACK AND RE-REVIEW the student data from the AP and NP items to see if this idea of combining like objects vs combining objects in the same location was prevalent in the student reasoning.	

	No Property	(a) Commutative Property - Students are able to tell that the two number sentences are the same because it does not matter what order you add them in (b) Number Comparison - Student reasons by comparing to two values without using the operations (c) Total Number Equivalence - Student reasons that equations represent the same contextual situation because they equal the same value (d) Place Value - Student reasons equality by adding place value to estimate	i. Student reasons by comparing the two values in the expression without using the operation (b) ii. Student reasons equality by recognizing the total did not change (c) iii. Student reasons equality by estimating using place value (d) iv. Student reasons that two expressions are equivalent because it does not matter what order you add the numbers in (a)		Although the item was labeled as no property, it was the associative property and students were connecting non like items together, ducks and rabbits but the location remained the same. We don't think place value should be a part of this one subcomponent, but it was a code, it's dependent on number range. Since this was labeled no property but it was a combination of properties, we are recommending not making this a subcomponent.	

	Subcomponent: Recognize two equivalent expressions that demonstrates one or more property of operations. Elements that varied: number ranges	(a) Commutative Property: Students are able to tell that the two number sentences are the same because it does not matter what order you add them in (b) I-CD-decomp: Student decomposes the amount of objects in a group to find the total amount and make a comparison when given equations representing the associative. (c) Operation: Student can only reasons with one operation at a time (d) Counting: Student can reason equality by counting the quantities on both sides (with or without the use of a manipulative) (e)Associative Property: Student can reason equality by recognizing that the sum of two numbers is equal to the composed number (f) Same Number Equivalence: Student reasons that the two	i. Recognize two equivalent expressions that demonstrates one or more property of operations.	i.a Recognize two equivalent expressions that demonstrates the commutative property. i.b. When given two equivalent expressions representing the commutative property, student reasons that order does not matter in maintaining equality with or without manipulatives. (a, d) ii.a Recognize two equivalent expressions that demonstrates the associative property. (e) ii.b. When given two equivalent expressions representing the associative property, student	Note that the skill code was to recognize equivalent expressions but the content questions asked if they represent the same amounts or different amounts. Based on the protocol the only difference between 8f and 10d is that 10d has an equals sign and two expressions on a single card, but 8f had expressions on separate cards without an equal sign. Not all students were given the cards that represented the same properties, so codes relate	(a) student uses computation to determine equivalency rather than reasoning about quantities [code: c] (b) student incorrectly computes when adding to reason about equivalency [code: c] © Student reasons that the numbers in two expressions are equal but disregards the operation [code f]

		number sentences are equal because both sentences contain the same numbers early stage of commutative, just not able to say that order doesn't matter.		reasons that numbers can be reassociated while maintaining equality with or without manipulatives. (b, d, e)	more specifically to the cards (properties) were given. Most frequently, student that were given the commutative property cards were able to use it. How were the cards given determined (some associative, some commutative)? Coding was impacted by the cards given. Is this related to property of operations being a "green word" We don't have evidence of the additive inverse for this to be able to expand on this.	

					If we separate out the properties, would recommend splitting into two subcomponents.	

Subcomponent: Recognize two equivalent expressions that demonstrate decomposition and at least one property of operations. Elements that varied: number ranges	(a) Commutative Property: Students are able to tell that the two number sentences are the same because it does not matter what order you add them in (b) I-CD-decomp: Student decomposes the amount of objects in a group to find the total amount and make a comparison when given equations representing the associative. (c) Operation: Student can only reason with one operation at a time (d) Counting: Student can reason equality by counting the quantities on both sides (with or without the use of a manipulative) (e)Associative Property: Student can reason equality by recognizing	i. Recognize two equivalent expressions using decomposition and at least one property of operations	i. Given a contextual situation and an expression decomposing the numbers in the situation, student uses one operation at a time to determine if two expressions are equivalent. Students may or may not use decomposition of numbers (b) (c) (d) ii. Given a contextual situation and an expression decomposing the numbers in the situation, student identifies corresponding quantities in two expression to determine if they are equivalent (f) iii. Given a contextual situation and an expression	Students were not given a symbol of equality but had to determine equality of two expressions Students were told the answer and thus most of the strategies use computation to find the total of the two given expressions. Students use decomposition, not the expressions doing the decomposition as stated in the skill statement. We are not confident that this data fully reflects the initial subcomponent.	(a) student uses computation to determine equivalency rather than reasoning about quantities [code: c] (b) student incorrectly computes when adding to reason about equivalency [code: c] (c) Student reasons that the numbers in two expressions are equal but disregards the operation [code f]

	that the sum of two numbers is equal to the composed number (f) Same Number Equivalence: Student reasons that the two number sentences are equal because both sentences contain the same numbers early stage of commutative, just not able to say that order doesn't matter.	decomposing the numbers in the situation, student identifies two equivalent expressions using at least one property of operations (a) (e)	

	Properties of Operations: Equal Sign as a Relational Symbol					
	ORIGINAL SUBCOMPONENT STATEMENT & ELEMENTS THAT VARY	**Axial CODES**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION INITIAL**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION RECOMMENDATION (Sparks/Audrey/Cass) *summarize/synthesize G (axial coding) ***	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
Equal Sign as a Relational Symbol	Subcomponent: Recognize the equality between two quantities using a balance. Elements that varied:	(a) I-R-Motion Size Comparison - Student reasons equality based on the motion of the balance (b) I-R Number Comparison - Student reasons equality by comparing the number of objects on both sides of the balance (c) I-R Position Size Comparison	I. Student recognizes the equality between two quantities using a balance.	i. Given a picture or concrete representation; student recognizes the equality between two quantities using a balance. ii.a Given a picture or concrete representation; student reasons equality based on the motion of the balance (a) ii.b. Given a picture or concrete representation, student reasons equality based on the position of the balance (e.g. student reasons equality by the balance being level) (c)	Students can reason about the equality using any methods listed within the description. When asked further questions about how they knew the quantities were equal students would refer to the motion of the balance (moving down or up), position of the balance (lower or higher), or comparing the numbers on both sides of the balance. The picture or concrete representations had objects of similar size but different color and this was not determined to be a factor in reasoning. The physical balance was a scaffold, we wonder if all students should be provided the physical balance in one stage and a picture of a balance in	

		- Student reasons equality based on the position of the balance		ii.c. Given a picture or concrete representation, student reasons equality by comparing the number of objects on both sides of the balance (b)	another stage ~ consultants could support with this.	

	Subcomponent: Write a true equation using an equal sign to represent the relationship between given quantities on a balance or in a pictorial representation. Elements that varied:	(a) Add all - Student reasons equality by adding all objects on both sides of the balance (b) Equal sign as balance - Student reasons that the equal sign is a symbols for equal quantities (c) Operation - Student reasons about equality by using one operation at a time.	I. Student recognizes using an equal sign to represent the relationship between given quantities on a balance in a pictorial representation	i. Given a picture or concrete representation of a balance; student identifies an equation that matches the relationship between given quantities. ii. Given a picture or concrete representation of a balance, student reasons that the matching equation represents the equality of the quantities. [code b] iii. Given a picture or concrete representation on a balance, student writes an equation represent the relationship between given quantities.	Assessment of this subcomponent utilized the same picture representation as that subcomponent above (9a). This subcomponent required the creation of an equation but students were not able to do this, so the interviewer scaffolded by providing the equation (3=3). Many students did not recognize the use of or symbolic representation of the equal sign as a symbol of equality. The physical balance was a scaffold, we wonder if all students should be provided the physical balance in one stage and a picture of a balance in another stage ~ consultants could support with this. Since the students were not able to write an equation and the	(a) Student does not recognize that the equal sign represents equality in an equation representing equal quantities and instead adds all quantities represented [code a] (b)Student does not recognize that the equal sign represented equality in an equation representing equal quantities and instead reasons by using one operation (such as addition) at a time [code c]

					scaffold was to provide an equation. We recommend breaking this down so that 9a is concrete, then representational (they identify a matching equation), and then abstract (they write an equation). However, we don't have evidence that most students can write it. Would need to compare to correct/aligned data.	

	Subcomponent: Recognize true and not true equations with different equation structures: operations on the left side (a+b=c); no operations (a=a); operations on the right side (c=a+b) Elements that varied:	(a+b=c) (a) operation: Student reasons equality by one operation at a time (b) I - R - number line: Student reasons by using positions of numbers on a number line (c) add all: Student does not recognize the different operations and adds all of the quantities	i. Recognize true and not true equations where the operations are the left side (a+b=c)	i. Given an equation in the form a+b=c, recognize whether it is true or not true. ii. Given an equation in the form a+b=c, student reasons about the quantities that need to be balanced on both sides (e)	This subcomponent is assessing whether or not students can make a determination about the truth of an equation by reasoning whether both sides of the equation are equal. Early conceptualizations used the descriptions equality of both sides but this was changed to true or not true equations to correspond with how students would be assessed. Code B was a strategy students used when the equation was not true, but we didn't include in the progression because it wasn't consistent ~ (Given an equation in the form a+b=c, student reasons whether the equation is true or not true by using a number line. (b))	(a) Student does not reason about the equality of both sides of the equation and uses operating to prove if the equation is true or not true. [code a] (b) Student does not recognize the different operations and adds all of the quantities [code c] (c) Student does not understand how symbols and numbers work together to form an equation (e.g., student may not know the number symbols (6 is a 9) or know what the

		(d) symbols: Student reasons about the equation by focusing on the operations and numbers as symbols (e) balancing equation: Student reasons inequality by balancing the equation				= sign represents.) [code d]

		(c=a+b) (a) reasons by order: Student reasons about the equation by referring to the order of orientation (e.g. the equal sign being on the left instead of the right) (b) Student does not recognize the different operations and adds all of the quantities. (c) Equal sign as operation:	i. Recognize true and not true equations where the operations are on the right side (c=a+b)	i. Given an equation in the form c=a+b, recognize whether it is true or not true. ii. Given an equation in the form c=a+b, student reasons about the quantities that need to be balanced on both sides. (d)	This subcomponent is assessing whether or not students can make a determination about the truth of an equation by reasoning whether both sides of the equation are equal. Early conceptualizations used the descriptions equality of both sides but this was changed to true or not true equations to correspond with how students would be assessed. Students had a hard time reasoning about the equation when the operation was on the right side. Students would reason about the equation being not true because of the unfamiliar orientation of having the operation on the right. Other students would either add all quantities or use the equal sign as an operation and not as a symbolic relationship	(a) Student does not recognize whether an equation is true or not true because it is in a different orientation (e.g. equal sign on the left of the operation) [code a] (b) Student does not recognize the different operations and adds all of the quantities [code b] (c) Student does not understand how symbols and numbers work together to form an equation (e..g, student does know what the = sign represents and uses it as an operation.) [code c]

		Student reasons with the equals sign as an operation, not as a symbol of equality. In some instances the operation right before the equals sign (d) Balanced equations: student reasons about the value of quantities as a values to balance an equation			between the two sides of the equation.	

		(a=a) (a) Add all: Student does not recognize the different operations and adds all of the quantities (b) equal sign as operation: Student reasons with the equals sign as an operation, not as a symbol of equality. In some instances the operation right before the equals sign (c)	i. Recognize true and not true equations where there are no operations (a=a)	i. Given an equation in the form a=a, recognize whether it is true or not true. ii. Given an equation in the form a=a, student reasons about the quantities that need to be balanced on both sides (c)	This subcomponent is assessing whether or not students can make a determination about the truth of an equation by reasoning whether both sides of the equation are equal. Early conceptualizations used the descriptions equality of both sides but this was changed to true or not true equations to correspond with how students would be assessed. Based on a prior subcomponent about the use of and symbolic representation of the equal sign, this subcomponent was moved down in the progression.	(a) Student does not recognize the different operations and adds all of the quantities [code a] (b) Student does not understand how symbols and numbers work together to form an equation (e.g., student does know what the = sign represents and uses it as an operation.) [code b]

		balanced equation: student reasons about the value of quantities as a values to balance an equation				
	Subcomponent: Recognize true and not true equations with different equation structures: operations on both sides (a+b=c+d); multiple instances of a number. Elements that varied:	(a+a=b+b) (a)Symbols - Student reasons about the equation by focusing on the operations and numbers used as symbols (b)Equal sign as operation - Student reasons			Since this problem was a not true equation related to the subcomponent, we kept this merged and added the format of the equation in parenthesis	

		with equal sign as operation, not as a symbol of equality (c) Balanced equations - Student reasons by trying to move numbers from one side of the equal sign to the other				

		(a+b=c+d) (a) Add all - Student does not recognize the different operations and add all of the quantities (b) Balanced equations - Student reasons by trying to move numbers from one side of the equal sign to the other (c) Equal sign as operation - Student reasons with equal sign as operation, not as a symbol of equality	i. Recognize true and not true equations where there are operations on both sides of the equal sign	i. Given an equation with addition on both sides of the equation (a+a=b+b or a+b=c+d), recognize whether it is true or not true ii.a Given an equation with addition on both sides of the equation (a+a=b+b or a+b=c+d), student reasons by trying to move numbers from one side of the equal sign to the other (b) ii.(b) Given an equation with addition on both sides of the equation (a+a=b+b or a+b=c+d), student reasons by comparing numbers on both sides of the equal sign. (e)	This subcomponent is assessing whether or not students can make a determination about the truth of an equation by reasoning whether both sides of the equation are equal. Early conceptualizations used the descriptions equality of both sides but this was changed to true or not true equations to correspond with how students would be assessed. Based on the progression of codes and student reasoning, this subcomponent was moved down in the progression. Students also had a difficult time reasoning with this equation due to the multiple operations on both sides. This gave rise to the misconception of not recognizing how operation symbols, the equal sign, and numbers	(a) Student does not recognize the different operations and adds all of the quantities [code a] (b) Student does not understand how symbols and numbers work together to form an equation (e.g., student does know what the = sign represents and uses it as an operation. [code c] (c)Student is confused by the amount of symbols used to represent operations the different operations [code d]

		(d) Symbols - Student reasons about the equation by focusing on the operations and numbers used as symbol (e) Number comparison - Student reasons equality by comparing numbers on both sides of the equal sign			work in an equation. Students were able to reason equality by comparing numbers on both sides of the equal sign or trying to move numbers from one side of the equation to the other.	

Maintaining Equality	Properties of Operations: Maintaining Equality					
	ORIGINAL SUBCOMPONENT STATEMENT & ELEMENTS THAT VARY	**AXIAL CODES**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION INITIAL**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION RECOMMENDATION (Cass/Robyn) *summarize/synthesize G (axial coding) ***	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
	Subcomponent: Given a contextual situation with known quantities, use one or more properties of operations to recognize when equality is maintained. Elements that varied: 0-5, 0-10, 0-20, 0-50, 0-99, 0-199 **protocol item was commutative**	(a) One-sided equality - moves the sets of objects that are similar in number and kind to be grouped together rather than separate, do not recognize the two sides as being separate contexts in the situation/story. (b) Two-sided equality - recognize that sets of objects have the same amount of objects despite order of presentation but do not refer to order not	i. determines that the total number of objects on one side of a two-sided, situation/story by recognizing that the total can consist of two separate and unrelated subsets of objects. (a) (d) (e) ii. understand that both sides of a two-sided situation/story are equal (b) (d) (e)	i. Given a contextual situation representing the commutative property, recognize that order of presentation does not change the quantity. (d) (e) ii.a. Given a contextual situation representing the commutative property, recognize that both situations are equal using counting (one by one or in groups) (d) (e) ii.b. Given a contextual situation and quantities that represent the communitive property, recognize that both situations are equal by matching sets	The conceptualization is based on the codes being related to the commutative property due to the item design. However the original subcomponent had "one or more properties Commutative property was added to be consistent with skill code NRR.C.8.b These two items were similar in representation. The progression of reasoning began with recognition that the quantity	(a) student reasons equality but considers order important in maintaining equality (c) (b) when quantities are different objects, student groups the objects together by type disregarding the context (a)

		mattering in determining equality (c) Commutative property - determine the equality of sets of objects using two-sided equality reasoning and refer to order not mattering in determining equality (d) Counting by one-to-one correspondence - to determine the amount of objects in the set, student counts individual objects one-by-one (e) Counting by groups - to determine the amount of objects in the set, student counts by groups rather	iii. understand that both sides of a two-sided situation/story are equal regardless of the order the sets of objects were presented. (c) (d) (e)	together (b) iii. Given a contextual situation representing the commutative property, reason that both situations are equal regardless of the order the sets were presented. (c)	did not change and then moved to understanding that both sides were equal. Reasoning in both contexts was the same however students progressed by not only recognizing the total amount was the same but also that the two situational sides were equal. The final progression of thinking again used the similar reasoning but students were able to state that the order of representation of the objects did not matter. The misconception occurred when students could reason that the two amounts of the two-sided representation were equal but stated that order of representation did matter.	

		than each individual object				

	Subcomponent: Given a contextual situation with unknown quantities, use one or more properties of operations to recognize when equality is maintained. Elements that varied: **Protocol item was additive inverse**	Reasoning Scheme: (r.a) Reasons with fixed known quantity: student reasons with known quantities but disregards unknown quantities (r.b) Reasons with fixed unknown quantity: Student uses operations to reason equality but uses a fixed unknown value as a reference (r.c) Reasons with unknown quantity: Student uses operations to reason equality while maintaining the unknown	i.a. Given a contextual situation representing the additive inverse property in the form of a+b-b, where a is given and b is unknown, student reasons about equality using a fixed unknown value. i.b Given a contextual situation representing the additive inverse property in the form of a-b+b, where b is known and a is unknown, student reasons about equality using a fixed unknown value. ii.a. Given a contextual situation representing the additive inverse property in the	i.a. Given a contextual situation representing the additive inverse property in the form of a+b-b, where a is given and b is unknown, student reasons about equality using an anchor quantity that was not provided. (r.b) i.b Given a contextual situation representing the additive inverse property in the form of a-b+b, where b is given and a is unknown, student reasons about equality using an anchor quantity that was not provided. (r.b) ii.a. Given a contextual situation representing the additive inverse property in the form of a+b-b, where a is given and b is unknown, student reasons about equality using a unknown value as a reference. (r.c) ii.b Given a contextual	since this protocol item is related to additive inverse; the codes are specific to the property. We aren't certain that if students were given another property the codes that support the conceptualization would remain the same. current progression focuses on the reasoning, however, students used various tools to support their reasoning. Should the progression be based on the tool support?	(a) student reasons with known quantities but disregards unknown quantities (r.a)

		Tools Scheme: (t.a) Drawings/tally marks: was able to communicate the addition and/or subtraction of a subset of objects from the total set using other means (e.g. drawings or tally marks) (t.b) manipulatives: was able to communicate the addition and/or subtraction of a subset of objects from the total set using manipulatives; student was always prompted to use manipulatives.	form of a+b-b, where a is given and b is unknown, student reasons about equality using a unknown value as a reference. (a+b-b, where a is known and b is unknown) ii.b Given a contextual situation representing the additive inverse property in the form of a-b+b, where b is known and a is unknown, student reasons about equality using a unknown value as a reference.	situation representing the additive inverse property in the form of a-b+b, where b is given and a is unknown, student reasons about equality using a unknown value as a reference. (r.c)		

		(t.c) symbolic: was able to communicate the addition and/or subtraction of a subset of objects from the total set using symbols (e.g. 6-2=4) (t.d) verbal/words: was able to communicate the addition and/or subtraction of a subset of objects from the total set using verbal descriptions or written words				

	Subcomponent: Given a contectual situation with known quantities that models one or more properties of operations, write a true equation to represent the situation. Elements that varied: 0-5, 0-10, 0-20, 0-50, 0-99, 0-199 ••Protocol item was commutative**	(a) Same-number equivalence - understand that the amounts on both sides of the two-sided situation/story were equal because they contained the same numbers, either in written form or within the number of objects in the sets. (b) Same-sum equivalence - understand the quantities on two-sides of the situation/story are equal because both written number sentence sums were equal (c) Order - understand that the two written number sentences were equal while also recognizing the difference in	i. understands that both sides of the two-sided situation/story are equal because they have the same amount or numbers (a) ii. understands that quantities on both sides of a two-sided situation/story represented as a number sentence are equal because their sums are equal (b) iii. understand that quantities on both sides of a two-sided situation/story represented as a number sentence are equal in sum and this equality does not depend on the order in the number sentence (c)	i. Given a contextual situation with known quantities, representing the commutative property, write an equation to match the situation.	This skill code was also assessing the commutative property with a two-sided situation/story context. The representation was presented using sets of objects broken into parts. The difference between this skill code and NRR.C.8.b or NRR.C.10.a was that students were also provided with the symbolic number representing the quantity of each set of object. Students were also asked to represent each side as a number sentence before determining the equality relationship between the two sides. The progression for this skill code was similar in that students could recognize the total was the same, then determine equality, and finally arrive at the conclusion that different order representation did not matter. The protocol asked students to write a number sentence, identify if the boxes had the same/more/less, and then reason about the number sentences. We don't have the correct/aligned data to know if what they wrote	(a) student does not demonstrate an understanding of equality by adding all the numbers together in two number sentences (d)

		order of the numbers in the two number sentences (d) Adding all - focuses on adding all the numbers together in the number sentence			is correct. It isn't clear from the student workbook that the equations were written by the students vs written by the interviewer. We wrote this to combine the original statement and student reasoning, but we are not sure if this is appropriate because of the lack of clarify on if the kids actually wrote the equations. The reasoning question does not relate well to what they were asked to do. Thus, we just revised the original statement because reasoning about the structure of the equation is in 8b We only have evidence on commutative property. Would recommend writing a statement for each property.	
	Subcomponent: Recognize true and not true equations with known numbers using one or more properties of operations. Elements that varied: 0-5, 0-10, 0-20, 0-50, 0-99, 0-199			We did not have any evidence of the associative property being assessed for this skill code, so we didn't write a statement for it.	Students were provided cards with an equation and then had to determine whether the equation was true or not true. Then students provided their reasoning on that determination. The skill code was divided into the different properties	

					being represented on the cards provide to students one at a time.	
	a+b-b+a commutative	(a) Same-number equivalence - understand that the amounts on both sides of a given equation are equal because they contain the same numbers. (b) Equal sign as an operation - student reasons that the equal sign is an operator and operation should occur. (c) Horizontal Place Value Addition: student incorrectly added digits within numbers, without reference to place value, to	i). understand that both sides of an equation are equal if they contain the same numbers regardless of the order written on either side is different.	i. Student recognizes whether an equation is true or not true that represents the commutative property (e.g., a+b=b+a). ii. When given a true equation that represents the commutative property, student reasons that both sides of an equation are equal if they contain the same numbers regardless of the order written on either side is different (a)	Skill code was changed to reflect that students had to first recognize whether the equation was true or not true and then reason why.	(a) does not reason equality with the equal sign and considers the equal sign as an operator [code b] b) student reasons equality by incorrectly adding horizontally by place value [code c]

	make a determination about equality				
a+b-b=a+0 additive inverse	(a) Unbalanced numbers: student reasons by having the same number of numbers in the equation or believes the number on both sides must be the same. (b) Operations: Student can reason with one operation but cannot extend the reasoning to multiple operations in one number sentence (c) Zero Concept: Student	i. Student recognizes true and not true equations in the form of a+b-b = a+0. ii. When given an equation in the form a+b-b=a+0, student reasons that the additive inverse is the same as zero	i. Student recognizes whether an equation is true or not true that represent the additive inverse when a zero anchor is included (e.g., a+b-b=a+0). ii. When given a true or not true equation that represents the additive inverse with a zero anchor (e.g.,a+b-b=a+0), student reasons that the additive inverse is the same as zero. (d)	The difference between ii and iii is the zero anchor; we kept them separate but are wondering how critical it is to separate the skills or call them out separately Is this associative with additive inverse?	(a) reasons that the amount of numbers must be the same on both sides of the equation [code a] (b) reasons that the numbers on each side of the equation must be the same for it to be equal (e.g., does not recognize that an operation will make both sides equal) [code a] (c) reads equation from right to left [no code, just found in the transcripts. G3_850]

	recognizes that zero is the same as no quantity but does not recognize equality of the whole equation (d) Additive Inverse: Student can reason that the additive inverse is the same as zero (e) Equal sign as an operation: student reasons that the equal sign is an operator and operation should occur.				(d) reasons with one operation but cannot extend the reasoning to multiple operations in one equation [code b] (e) recognizes that zero is the same as no quantity but does not recognize equality of the whole equation [code c] (f) student reasons equality by incorrectly adding horizontally by place value [code e]

a-b+b=a additive inverse	(a) Unbalanced numbers: student reasons by having the same number of numbers in the equation or believes the number on both sides must be the same. (b) Operations: Student can reason with one operation but cannot extend the reasoning to multiple operations in one number sentence (c) Equal sign as operation: Student reasons with the equals sign as an operation, not as a symbol of equality. In some instances the operation right before the	no initial draft, completed together	i. Student recognizes whether an equation is true or not true that represent the additive inverse (e.g., a-b+b=a). ii. When given a true or not true equation that represents the additive inverse (e.g., a-b+b=a), student reasons that the additive inverse is the same as zero (d)	a-b+b=a somehow became a-b+b=a-0 on the 0-99 card. The protocol and the cards did not match; We need to understand what we need to do to best represent the skill we are trying to assess. This only impacted one student. We didn't have evidence of students reasoning about not true equations for this item.	"(a) reasons that the amount of numbers must be the same on both sides of the equation [code a] (b) reasons that the numbers on each side of the equation must be the same for it to be equal (e.g., does not recognize that an operation will make both sides equal) [code a] (d) reasons with one operation but cannot extend the reasoning to multiple operations in one equation [code b] (e) reasons with the equals sign as

	equals sign (d) Additive Inverse: Student can reason that the additive inverse is the same as zero				an operation, not as a symbol of equality. [code c] (f) reasons with the equals sign as an operation only using the operation closest to the equal sign. [code c]
a+b+c=a+d	(a) Two-sided number equality: student determines if the equation is true by referring to the equal sign or the equality of the value on both sides of the equation (b) Number Composer: student determines if the equation is true by referring to the numbers on	i. Student recognizes true or not true equations that represent more than one property of operations. ii. When given a true equation that represents more than one property of operation, student reasons that the total on both sides of the	i. Student recognizes whether an equation is true or not true that represents more than one property of operations. ii. When given a true or not true equation that represents more than one property of operation, student reasons about the equality of the numbers or composition of the numbers. (b) iii. When given a true or not true equation that represents more	Problem students were given had multiple properties of operations, was false, not necessarily friendly numbers, and required combining of numbers. a+b+c=d+2b; We didn't write this into the SSD because we think it was too much going on. So we just went expanding the original SSD to include more than one property of operation, since i, ii, and iii all had one	(a) student refers to the operations or symbols that represent the operations in the equation and does not reason about equality. (c)

		one or both sides of the equation or the amount of numbers in the equation (c) Operations: student determines if the equation is true by referring to the operations or symbols that represent the operations in the equation	equation is equal. (a)	than one property of operation, student reasons that the total on both sides of the equation is equal. (a)	property of operation. we didn't have evidence of students reasoning about not true equations for this item.	

Solving for Unknown Values	Properties of Operations: Solving for Unknown Values					
	ORIGINAL SUBCOMPONENT STATEMENT & ELEMENTS THAT VARY	**AXIAL CODES**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION INITIAL**	**SUBCOMPONENT SYNTHESIZED DESCRIPTION RECOMMENDATION (Sparks/Audrey/Cass) *summarize/synthesize G (axial coding) ***	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
	Subcomponent: Solve for an unknown value in a true equation using a relational definition of equal sign. Elements that varied:	(a) Add all - student focuses on adding all the numbers in the number sentence (b) Equal sign as operation - Student reasons with the equals sign as an operation, not as a symbol of equality (c) Balanced equation - Student reasons equality by recognizing the same quantity on both sides of the equal	i. Given an equation, determine the value of the unknown quantity using the relational definition of the equal sign	i. Given an equation in the form a+b=_+d, determine the value of an unknown quantity ii. Given an equation in the form a+b=_+d, reason about the value of an unknown quantity using the relational definition of the equal sign (c)	Student misconceptions corresponded to students being unaware of the relational definition of the equal sign. Student reasoning corresponded to students using the equal sign as an operation or adding all the quantities rather than reasoning that the unknown was a quantity to balance the two sides of the equation. Equation representation was added to distinguish this skill code from other skill codes that represented other properties	(a) student reasons about the unknown quantity by adding all the numbers in an equation [code a] (b)Student reasons with the equals sign as an operation, not as a symbol of equality [code b]

					(commutative, NRR.C.11.d)	
	Subcomponent: Given a contextual situation modeling a true equation, apply one or more properties of operations or property of equality to solve for an unknown value using concrete objects. Elements that varied: **protocol was	(a) Additive Inverse: students can reason that the additive inverse is the same as zero. (b) Multiple Operations: student can reason with multiple operations but cannot recognize that operations with the additive inverse are zero (c) Single Operation:	i. Given a contextual situation modeling a true equation, apply one or more properties of operations or property of equality to solve for an unknown value using concrete objects. (a) (b)	i. Given a contextual situation and quantities modeling a true equation of the additive inverse, solve for an unknown value by operating. (b) ii. Given a contextual situation and quantities modeling a true equation of the additive inverse, solve for the unknown value using the understanding of zero (e.g., a+b-b = a because adding be	The SSD we wrote is related to additive inverse, because that's what the protocol used. We recommend busting this out into multiple subcomponents, by properties. Beehive problem did not have cards with bees. We still kept the concrete objects from the skill statement. We also recommend that this skill code should be broken	(a) student believes a horizontal equation needs to be rewritten vertically (b) student reasons about the unknown quantity by adding all the numbers in an equation [code d] (c) student reasons using one operation but cannot extend to multiple operations in one equation [code c]

	additive Inverse**	Student can reason with one operation but cannot extend the reasoning to multiple operations (d) Add all: Student does not recognize the different operations and adds all of the quantities		and removing b is zero). (a)	down by property. Essentially 11B was tested as 11C since the concrete objects were not provided and an equation was given	

	Subcomponent: Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value in a true equation. Elements that varied: **protocol was associative**	(a) Adding all: student focuses on adding all the numbers in the number sentence (b) Balanced equation: student reasons about the unknown quantity as a value to balance an equation (c) Equal sign as operation: Student reasons with the equals sign as an operation, not as a symbol of equality (d) Reasons by situation: student reasons about the unknown quantity within and specific to the context of the situation	i. Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value in a true equation.	i. Given a contextual situation modeling a true equation of the associative property, solve for an unknown value reasoning about the reassociation of the numbers. (e)	The SSD we wrote is related to associative, because that's what the protocol used. We recommend busting this out into multiple subcomponents, by properties.	(a) student focuses on adding all the numbers [code a] (b) Student reasons about the unknown quantity as a value to balance an equation but is incorrect [code b] (c) Student reasons about the unknown quantity as a value to balance an equation using estimation [code b] (d) Student reasons with the equals sign as an operation, not as a symbol of equality [code c] (e) student reasons about the unknown quantity within and specific to the context of the situation without

		(e) Associative Property: Student reasons by the associative property to combine two numbers on one side of the equal sign that are equal to one number on the other				reasoning about equality [code d]

	Subcomponent: Apply one or two properties of operations or a property of equality to solve for an unknown value in a true equation. Elements that varied:	(a) Commutative Property - Student can reason equality by recognizing that the order of the numbers does not matter (b) Consecutive Numbers - Student reasons about an unknown quantity using consecutive numbers (c) Equal sign as operation - Student reasons with the equals sign as an operation, not as a symbol of equality (d) Add all - Student adds all the numbers in the number sentence	i. Solve for an unknown value using a property of operations or a property of equality.	i. Given an equation of the form a+b=_+a, solve for an unknown quantity using the commutative property (a)	Contextual situation was removed as students were only presented with an equation with an unknown value. The equation form was also added to distinguish this skill code from other skill codes that provided an equation to assess other skills (equality as relational symbol, NRR.C.11.a)	(a) Student reasons about the unknown quantity by considering the number in the equation as consecutive numbers [code b] (b) Student reasons about the unknown quantity by adding all the numbers in an equation [code d] (c)Student reasons with the equals sign as an operation, not as a symbol of equality [code c] (d) Student reasons about the unknown quantity by incorrectly adding place values or incorrect use of manipulatives representing

		(and student could have miscounted) (e) Horizontal Place Value - Student incorrectly adds values either through place value or manipulatives that represent place value				place value [code e]
	Subcomponent: Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value using concrete objects.	(a) Add all - student reasons by adding all the numbers (b) Equal sign as operation - Student reasons with the equals sign as an operation, not as a symbol of equality (c) Balanced	i. Given a contextual situation modeling a true equation, solve for an unknown quantity using decomposition and properties of equality in a true equation(c) (e)	i. Given a contextual situation modeling a true equation where operations are on both sides, solve for an unknown value using decomposition and properties of operations or equality (c) (e)	The equation (not shown to students) was in the form a+b=d+_. We are wondering whether or not this should be a part of the i statements so it is clear what the skill code is assessing. Students were not given a set of object but only the number representing the quantities on both sides of the two-	(a) Student reasons about the unknown quantity by considering the number in the equation as consecutive numbers [code d] (b) Student reasons about the unknown quantity by adding all the numbers in an

	Elements that varied:	equation - Student reasons equality by recognizing the same quantity on both sides of the equal (d) Consecutive Numbers - Student reasons about an unknown quantity using consecutive numbers (e) Two-sided number comparison - Student reason equality by comparing the corresponding numbers on either sides of the equal			sided situation/story. Skill code states concrete objects but it is suggested that this should be removed as students were only given the number and not the sets of objects. This skill code was also labeled as the associative property but was modified to be more general with properties of equality.	equation [code a] (c)Student reasons with the equals sign as an operation, not as a symbol of equality [code b]

	Subcomponent: Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. Elements that varied:	(a) Add all - student does not recognize the different operations and adds all of the quantities (b) Equal sign as operation - Student reasons with the equals sign as an operation, not as a symbol of equality (c) Balanced equation - Student reasons equality by recognizing the same quantity on both sides of the equal (d) Consecutive Numbers - Student reasons about an unknown quantity using consecutive	i. Given a context and a true equation modeling the situation, use the properties of equality of solve for an unknown value.	i. Given a context and a true equation modeling the situation, use the properties of equality of solve for an unknown value. (c) (e)	The equation provided is in the form a+b=c+_. We are wondering if this needs to be provided to distinguish the skill code from other codes.	(a) Student reasons about the unknown quantity by considering the number in the equation as consecutive numbers [code d] (b) Student reasons about the unknown quantity by adding all the numbers in an equation [code a] (c)Student reasons with the equals sign as an operation, not as a symbol of equality [code b]

		numbers (e) Two-sided number comparison - Student reason equality by comparing the corresponding numbers on either sides of the equal				
	Subcomponent: Apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. Elements that varied:	(a) I-CD-decomp: Student decomposes the amount of objects in a group to find the total amount and make a comparison when given equations representing the associative. (b) Operation: Students only reason with one operation at a time (c) Balanced	i. Recognizes the value of an unknown quantity in an equation using decomposition or at least one operation.	i. Given an equation of the form a+b-(b-1)=_, determine the unknown value by performing one operation at a time. (b) ii. Recognizes the equal sign as a balance of quantities on both sides (c) iii. Decomposes the quantities in an equation to determine an unknown value (a)	The equation could not demonstrate but the student had to demonstrate the property of equality	(c) Student tried to reason that the two sides should be balanced by trying to move values to different sides of the equation [code c]

		equation: student reasons about the unknown quantity as a value to balance an equation				