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SCHOOL OF EDUCATION & HUMAN DEVELOPMENT

RESEARCH IN MATHEMATICS EDUCATION

Spatial Reasoning: Cognitive Interview Administration

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MATHEMATICS
EDUCATION

Spatial Reasoning Cognitive Interview Administration

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Abstract

The purpose of this report is to describe the cognitive interview administration of the Spatial Reasoning (SR) learning progression, conducted as part of the Measuring Early Mathematics Reasoning Skills (MMaRS) project. This report details methods used to prepare for and conduct the cognitive interviews, including information on participants, and the steps taken to implement the interview protocols. For more details about protocol development, see the Spatial Reasoning: Cognitive Interview Protocol Development technical report (Tech. Rep. No. 20-07).

Table of Contents

Introduction	6
Research Questions	8
Cognitive Interview Instrument	9
Interviewer Materials	9
Observer Materials	10
<i>Note.</i> Sample fidelity form with the six fidelity questions for observers with scales for scoring.	11
Site Coordinator Materials	11
Interviewer, Observer, and Site-Coordinator Training	13
Participants	14
Procedures: Protocol Implementation	15
Interviews	15
Data handling and pre-processing	16
Conclusion	16
References	17

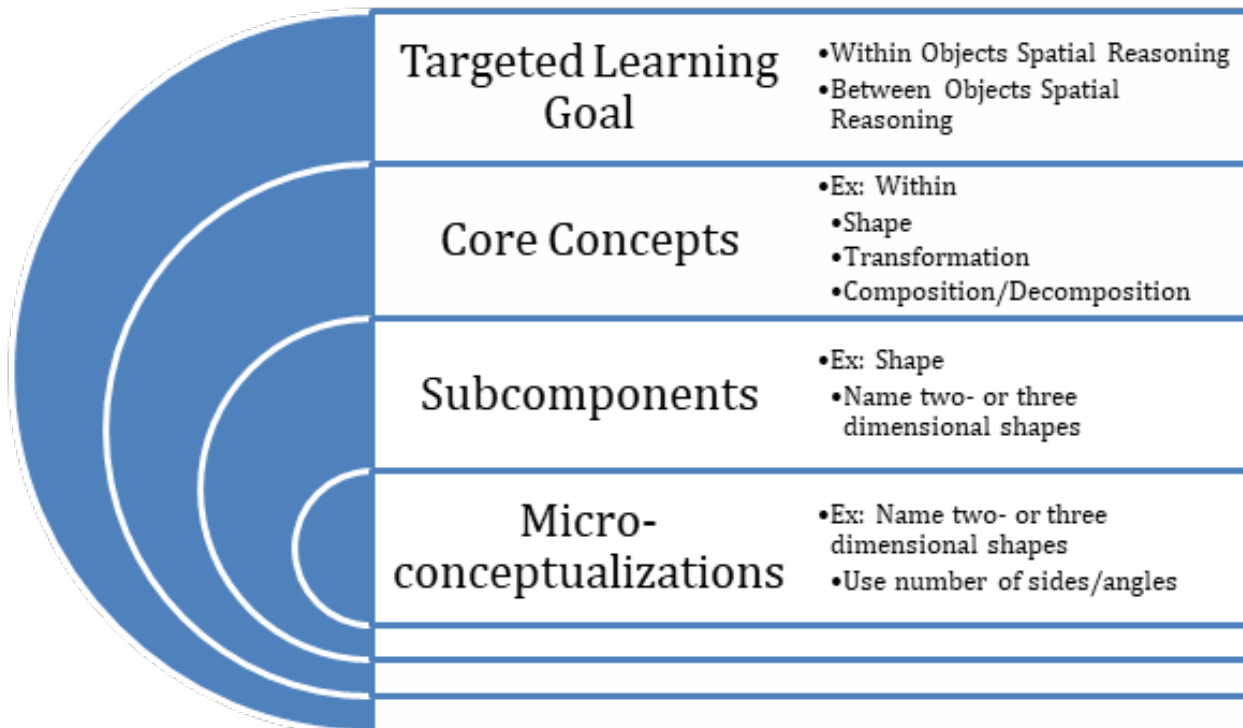
Spatial Reasoning Cognitive Interview Administration

Introduction

The Spatial Reasoning (SR) construct is defined within two components: spatial orientation and spatial visualization (Bishop, 1980; Burnett & Lane, 1980; Clements & Battista, 1992; Connor & Serbin, 1980; Eliot & Smith, 1983; NRC, 2009; Pellegrino et al., 1984; Sarama & Clements, 2009; Tartre, 1990). Spatial orientation involves identifying and comparing one’s position to others while also considering the perspective of others. Spatial visualization involves the creation of mental models of two- and three-dimensional figures and mentally transforming those models. The Measuring Early Mathematics Reasoning Skills (MMaRS) project divided the Spatial Reasoning construct accordingly aligned to the definition: between objects spatial reasoning (spatial orientation) and within objects spatial reasoning (spatial visualization; henceforth known as between and within, respectively). Within the MMaRS project, SR was conceptualized to include these two aspects as Targeted Learning Goals. Figure 1 illustrates the structure of the SR learning progression.

Figure 1

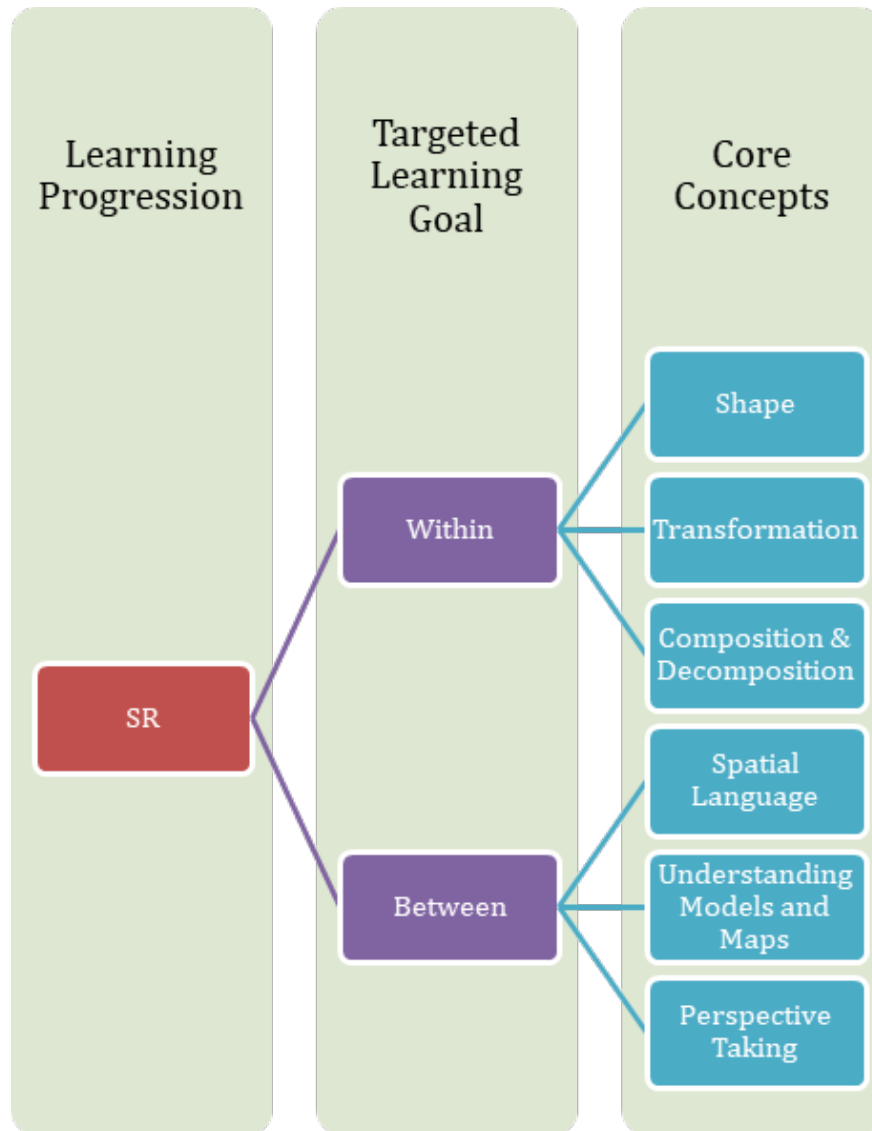
Structure of the MMaRS SR Learning Progressions



Within each of the Targeted Learning Goals, multiple Core Concepts were articulated, as displayed in Figure 2.

Figure 2

Core Concepts for each Targeted Learning Goal in the SR learning progression



For the MMaRS project, the overall SR learning progression is intended to serve as a foundation for classroom assessment resources. Before relying on learning progressions to inform instrument development, empirical evidence is needed to verify the learning progression. One source of evidence comes from cognitive interviews.

The purpose of this technical report is to describe the implementation of the cognitive interview protocols for spatial reasoning as part of the Measuring Early Mathematics Reasoning Skills

(MMaRS) project. We developed the protocols to reflect the skills hypothesized to be representative of early spatial reasoning skills. The development of these protocols can be found in the Spatial Reasoning: Cognitive Interview Protocol Development technical report (Tech. Rep. No. 20-07).

Research Questions

We designed the cognitive interviews to address four research questions related to empirically evaluating the SR learning progression. We included detailed sub-questions within each overarching research question. For research questions 1.3, 2.2, and 3, correctness scores were necessary from all interviewed students. Questions 3 and 4 required information from the cognitive interview video and transcription data with gestures included. Questions 1.3 and 2.3 required the analysis of fidelity observation data. This report details the methods used to gather evidence to address all Research Questions.

RQ 1: Developmental Appropriateness

- 1.1 Do the entry and exit KSAs align with teachers' expectations of pre-requisite and target skills?
- 1.2 Does teachers' frequency of teaching KSA align with progression?
- 1.3 Does student performance and engagement indicate floor or ceiling effects that align with entry and exit KSAs?

RQ 2: Ordering

- 2.1 Are teachers' perceptions of the appropriateness aligned with the hypothesized order?
- 2.2 Do students demonstrate increasingly sophisticated reasoning aligned with the hypothesized ordering?
- 2.3 Do students appear comfortable with tasks and task elements?

RQ 3: Conceptions

- 3.1 Do students demonstrate reasoning that is consistent with the hypothesized conceptions?
- 3.2 What misconceptions and/or errors do students make? Is there a pattern leading to greater competence?

RQ 4: Interconnectedness

- 4.1 In what ways are students' KSAs interconnected?
- 4.2 In what ways do prior KSAs impact students' responding?

Data gathered during interviews provided evidence for all research questions.

Cognitive Interview Instrument

The MMaRS research team conducted cognitive interviews using iteratively developed protocols with aligned items to the learning progression. For each subcomponent, children were tasked with one or more questions through which they both interacted with materials and explained their reasoning. The research team would use their words and actions as evidence to empirically recover the learning progression by confirming or refuting the accuracy in skills and ordering. See the Spatial Reasoning: Cognitive Interview Protocol Development technical report for the development process explained (Tech. Rep. No. 20-07). In this section, we describe the interviewer, observer, and site coordinator materials and training.

Interviewer Materials

Interviewers had material responsibilities in addition to the interview itself before, during, and after the interview. Interviewers prepared materials for use, checking ordering and accessibility, before the interview. During, the interviewer ensured child comfort and completed the interview with maximum protocol fidelity while remaining cognizant of the site coordinator and observer for technology and other environmental cues. After the interview, the interviewer gave the child a token of appreciation, thanked them and reset the materials for the next interview or transport away from the site.

Assent form

For each participant whose parent provided written consent to participate, the interviewer administered an assent form at the beginning of the first interview with that child. The assent conformed to the requirements set forth by the Southern Methodist University (SMU) internal review board (IRB) for working with students aged five to eight. The assent informed the students of the limited impacts of participation and asked for their permission to be recorded. During each subsequent interview, we reminded students of the assented activities.

Interview Protocol

As described in the SR Cognitive Interview Protocol Development technical report (Tech. Rep. No. 20-07), we developed an interview protocol that guided the interviewer through the activities of assessing the students' spatial reasoning. For each protocol item, the protocol included the written subcomponent, necessary materials, interviewer script with action prompts, questions for the interview to ask, and scoring instructions to be used by the observer in the moment and scorer after the interview (See Figure 3 for a sample protocol item). Each task included content, scaffolding, and reasoning questions.

Within Objects. For each within objects cognitive interview, we organized student materials in containers with materials grouped by task. Each task had a mat and necessary manipulatives for students to demonstrate their knowledge using two- or three-dimensional shapes and figures. To document student work, interviews were video recorded, and student work was preserved on mats until photos were taken after interview completion. One task had a written component in

which students identified embedded figures; for that task, student work was retained and grouped with the interview protocol in data-handling procedures and scanned into student data files.

Between Objects. We assigned each participant a student workbook for the between objects cognitive interviews. The workbook contained all two-dimensional mats for the interview, whereas three-dimensional mats and manipulatives were held in a container similar to within objects interviews. We scanned student workbooks into secure, online student data files that also held photos of work from the three-dimensional tasks. We used the workbooks and photos for correctness scoring.

Incentives

In addition to the protocol and materials for each task, the interviewer had small incentives to reward the child throughout the interview, including stickers and erasers. At the conclusion of the last session, the child was given a university branded drawstring backpack.

Observer Materials

Before the interview, the observer ensured that all interview documents (e.g., student workbook, interviewer manual, and field notes) matched the child. During, the observer scored the interview for fidelity and student correctness on items, flagging those about which this individual is uncertain for review. After the interview, the observer delivered the child to their classroom and supported the interviewer and site coordinator as needed. Observers were directly responsible for two sensitive documents: the observer protocol and the fidelity observation form.

Observer protocol

Observers used a copy of the interview protocol to follow along during the interview (See Figure 3 for a sample protocol item). Observers ensured that the interviewer moved through the interview sequentially by subcomponent tasks, and decision trees when applicable. These decision trees were embedded with the content questions for several items in which children started on a mid-level task of the anticipated skill. Based on their correctness in response to that task, interviewers advanced the child to a more challenging second task or provided them with the opportunity to show their knowledge on a simplified task. Observers tracked correctness, when indicated, as the interview proceeded and ensured that the line of questioning matched the decision tree alignment as it was encountered. There was a “flag for review” space on all items so that observers could return and reassess the scoring during post-interview scoring.

Fidelity observation forms

Observers also tracked interviewer fidelity to the protocol (See Figure 4 for a sample fidelity form). For each task in the protocol, observers scored six dimensions of the questioning sequence interaction: (a) repeating the content question, (b) repeating the content question, (c) rewording the reasoning question, (d) repeating the reasoning question, (e) perceived student comfort with materials, and (f) perceived student comfort with the task. Items were dichotomously scored except for perceived child comfort, which was recorded on a Likert type scale.

Figure 3

Protocol item with decision tree and scoring rules

Decision Tree

SR.A.2.d 3D shape	KF	KB	KT	1F	2B	1T	2F	2B	2T															
Recognize three-dimensional shapes or figures that have been rotated.																								
Actions		Questions				Student Response																		
Materials: <ul style="list-style-type: none"> Stimulus and comparison shape mats 1-5 Go through each comparison shape one at a time based on decision tree 		ASK REASONING QUESTIONS FOR EACH MAT PRESENTED. <div style="border: 1px solid black; padding: 5px; margin: 5px;"> Start with task 1 blue </div> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; text-align: center;"> Correct? Do both tasks 4 & 5 green </div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> Incorrect? Do both tasks 2 & 3 red </div> </div> [Cover shape on right with box] Look at this shape... [point to shape on the left of the mat] Without touching the shape, imagine rotating this shape in your head in different ways. CQ: [point to original shape] Are these shapes the same? [point to new shape] RQ: How do you know the shape is the same/not the same? OR Why did you say these are the same/not the same?				DURING: Circle the child's response in the chart below. Put N/A if the question was not asked. Task 1: (rectangular prisms) <table border="1" style="width: 100%;"> <tr><td>Same</td></tr> <tr><td>Not the same</td></tr> <tr><td>A2d3DCQ1: _____</td></tr> </table> Incorrect – RED TASKS Task 2: (sphere /semi-sphere) <table border="1" style="width: 100%;"> <tr><td>Same</td></tr> <tr><td>Not the same</td></tr> <tr><td>A2d3DC2: _____</td></tr> </table> Task 3: (square pyramids) <table border="1" style="width: 100%;"> <tr><td>Same</td></tr> <tr><td>Not the same</td></tr> <tr><td>A2d3DC3: _____</td></tr> </table> Correct – GREEN TASKS Task 3: (Triangular pyramid/square pyramid) <table border="1" style="width: 100%;"> <tr><td>Same</td></tr> <tr><td>Not the same</td></tr> <tr><td>A2d3DCQ1: _____</td></tr> </table> Task 4: (Cones) <table border="1" style="width: 100%;"> <tr><td>Same</td></tr> <tr><td>Not the same</td></tr> <tr><td>A2d3DCQ1: _____</td></tr> </table>				Same	Not the same	A2d3DCQ1: _____	Same	Not the same	A2d3DC2: _____	Same	Not the same	A2d3DC3: _____	Same	Not the same	A2d3DCQ1: _____	Same	Not the same	A2d3DCQ1: _____
Same																								
Not the same																								
A2d3DCQ1: _____																								
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A2d3DC3: _____																								
Same																								
Not the same																								
A2d3DCQ1: _____																								
Same																								
Not the same																								
A2d3DCQ1: _____																								
Scaffolding questions:		How are you starting to think about this question? If child tries to touch the shape: Can you answer without touching the shape?																						
Reasoning questions		Embedded above as RQ.																						
AFTER: TRANSCRIPT (if needed: 1 = correct/0 = incorrect/NA = Not Asked. Correct answer is bolded. Flag for review.																								

Note. Interview protocol for the within objects targeted learning goal, transformations core concept, figure rotation subcomponent. The gray section contains the scaffolding questions.

Figure 4

Fidelity Form

LP: Spatial Reasoning – Within 3

Skill Code	Did the interviewer reword the content question?	Did the interviewer repeat the content question?	Did the interviewer reword the reasoning question?	Did the interviewer repeat the reasoning question?	Did the student seem comfortable with the materials?			How comfortable did the student appear with the task?			
	0 - No 1 - Yes	0 - No 1 - Yes	0 - No 1 - Yes	0 - No 1 - Yes	NA	0 - No	1 - Yes	0	1	2	3
SR.A.3.a 2D	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3
SR.A.3.a 3D	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3
SR.A.3.b 2D	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3
SR.A.3.b 3D	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3
SR.A.3.c 2D	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3
SR.A.3.c 3D	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3
SR.A.3.d	0 1	0 1	0 1	0 1	NA	0	1	0	1	2	3

Note. Sample fidelity form with the six fidelity questions for observers with scales for scoring.

Site Coordinator Materials

The site coordinator's primary responsibility was to organize data and maintain confidentiality. Site coordinators ensured video and audio recorders were properly functioning throughout

interviews and handled any technology-related tasks or issues. One Secure Digital (SD) card was used per interview for video and still photos to streamline data processing. Because site coordinators were all internal staff, they were also responsible for data handling, including daily pick-up and delivery to secure storage at the university, and providing daily updates to the project manager and specialist.

Interview checklist

In addition to communication and organizational activities, site coordinators used a checklist to ensure the collection of all data (See Figure 5 for a sample). Checklists identified the student (by identification number), date and time of the interview, corresponding protocol, and the interviewer’s and observer’s names, with additional details as seen in Figure 5. Before securing the file post-interview, the site coordinator checked each item as being in the materials box, and each photo was taken before disassembling student work samples. The checklist differed by interview type (i.e., Within 1 & 2, Within 3, or Between) targeted learning goals and core concepts.

Figure 5

Sample Site Coordinator Interview Checklist

Check off that the data for this SID is in this packet

- ___ SD Card
- ___ Fidelity Form
- ___ Observer Protocol
- ___ Student work pages in folder - SRA3d (3 pages)

Picture List: (N/A for any that the child was not presented)

- o ___ A3b2D(2)
- o ___ A3b2D(3)
- o ___ A3b2D(1)
- o A3b3D task 1: ___Front ___Back ___Side 1 ___Side 2 ___Top
- o A3b3D task 2: ___Front ___Back ___Side 1 ___Side 2 ___Top
- o ___ A3c2D
- o A3c3D (All together): ___Front ___Back ___Side 1 ___Side 2 ___Top
- o ___ A3f(1)
- o ___ A3f(2)
- o ___ A3g(1)
- o ___ A3g(2)

Post-Interview Information:

Facts	Perceptions

Note. Customized checklist for interviews of the W3 protocol, including technology and organizational components site coordinators followed to ensure data-handling compliance.

Interviewer, Observer, and Site-Coordinator Training

Two external interviewers and two external observers were jointly trained during the week prior to cognitive interview administration by the research Project Manager (PM), who was also the third interviewer. The four took part in a two-day training and were provided additional practice opportunities after completing the formal training. RME staff members served as site coordinators and were on each site during interviews to facilitate technology and organizational responsibilities, including data handling.

Interviewer and Observer Training

The goals of the training sessions were to: (a) introduce interviewers and observers to the purpose of cognitive interviews; (b) explain interviewer and observer responsibilities; (c) describe interview protocols; (d) describe interviewer and observer materials; and (e) provide example observer notes. The stated purpose of the cognitive interviews was shared as observing ways in which students demonstrate reasoning within the SR construct. The interviewers were chiefly responsible for conducting the interviews with assented children, and observers' primary responsibilities were tracking fidelity and scoring the interview on specified protocol items.

Interviewers and observers were first introduced to the SR core concepts and protocols. They next practiced observing a video recorded try-out interview. After each protocol item in the interview, the group debriefed, and research staff answered questions or clarified processes as needed. The PM introduced materials during each item's debriefing chat and gave feedback on notes taken by each participant from the try-out video.

Both external interviewers practiced administering mock interviews to further familiarize themselves with protocols and materials. Observers were also trained to follow the interview on their own protocol to ensure that no questions were missed and that decision trees were used properly based on student responses. After mock interviews, interviewers were prepared to administer assigned cognitive interviews. Likewise, observers were equipped to document specific student actions and responses while facilitating smooth interview completion.

Site Coordinator Training

Three SMU staff served as site coordinators for these interviews. They participated in one half-day training to learn the data handling procedures and technical responsibilities for the interviews. Site coordinators were introduced to the checklist they would fill out for each interview that detailed secure materials and photos to be taken post-interview and stored. Site coordinators were instructed on the audio and video-recorder operation and security protocols for student data. At the conclusion of the training, site coordinators were prepared to ensure the secure handling of data from the time it was collected until deposited in a secure location at the RME office through their structured training.

Participants

Twenty-three students in grades K-2 were interviewed; 16 remained as participants in the study. Seven students were removed from the study due to fidelity issues in the protocol administration. Students were selected from one private and one public charter school in a metropolitan area. Table 2 summarizes the sample of students by school, grade, and support level.

Every child completed three interviews that covered both Targeted Learning Goals of the SR LP. Students were classified by support levels: (1) Students who require frequent support in order to use and understand math concepts; (2) Students who require some support in order to use and understand math concepts. (3) Students who independently use and understand math concepts. Where available, we used empirical data to justify student support levels. Data from one school included reading results for kindergarten on the Texas Primary Reading Inventory (TPRI) Early Reading Assessment (Children’s Learning Institute, 2014), and the reading and mathematics scores on the TerraNova 2 (CTB/McGraw-Hill, 2001) for grades 1 and 2. At the other school, empirical data were not available for grade K. Therefore, we relied on teacher judgement of necessary support. For grades 1 and 2, we based support decisions on reading and mathematics performance on the Iowa Assessments (Dunbar & Welch, 2015).

Table 2

Student demographics

Characteristic	Count (%)
Grade	
K	6 (38%)
1	5 (31%)
2	5 (31%)
Race	
Black	6 (38%)
White	10 (62%)
Ethnicity	
Hispanic	5 (31%)
Non-Hispanic	11 (69%)
Gender	
Female	6 (38%)
Male	10 (62%)
Support Level	
1	5 (31%)
2	8 (50%)
3	3 (19%)

Regardless of assessment utilized, we chose support levels based upon the national percentile rank reported on the assessment for each student. Generally, if the student scored below the 50th percentile in mathematics, they were considered needing frequent support. Students who scored between the 50th and 70th percentile in mathematics were considered needing some support, while students scoring above the 70th percentile were considered needing no support. In the instance where TPRI scores were used, we used a score of four on the listening test as needing some support and a score of six as needing no support because the test does not report a national percentile.

Procedures: Protocol Implementation

Full protocol implementation occurred in two cycles and produced 16 complete cognitive interviews. We undertook the first cycle in December 2019 and the second in January and February of 2020. During this time, we interviewed 23 students in total. Cycle one included 17 participants, from which we retained 16 for first round data preparation after one participant's interview revealed excessive fidelity deviations. The first round of data preparation revealed additional fidelity issues causing the removal of all interviews from one site, or seven total. To mitigate, we interviewed six additional students at that site across the three protocols during the second cycle.

Interviews

For each day in the field, the site coordinator picked up interview materials from the university and met the interviewer and observer at the assigned site. Working in spaces as assigned by schools, they interviewed students at assigned times based on a schedule prepared by the project specialist that was created to align with the school schedule. Interviews were spaced to allow for deep student thought and interviewer questioning to maximize outcomes of the cognitive interview process.

Field Notes

At the conclusion of each interview before securing all data collected, the interviewer, observer, and site coordinator reflected on any environmental concerns and student actions or perceived dispositions. All facts and perceptions of the interview were recorded on the site coordinator's checklist for the interview. Environmental facts might be a fire alarm going off or an individual not involved in the interview entering the room. Student nervousness, distractibility, or dis/comfort with tasks would be entered on perceptions. These field notes informed the validity of the interview at post-interview data processing.

Lessons Learned

A live document was stored in Box on which internal team members collected "lessons learned" through the process. The stated purpose was to track and capture ways in which we might improve in future data collection opportunities, and ensure challenges would be discussed in later meetings. Notes included the directionality of items when presented to children for fidelity (e. g., the door of the diorama should be closest to the child), and that team members should review the next day's schedule before departure to ensure on-time arrivals. Notes that were pertinent to data,

including the formatting of SD cards and capture of photos including student identification numbers, were reviewed daily by the project manager to mitigate any future technology challenges.

Data handling and pre-processing

At the end of each day in the field, site coordinators returned all data to a secure location on the university campus, as required by the IRB. They communicated with the MMaRS project manager and project specialist to update the team on progress, celebrations, and any potential concerns that occurred in the field.

On the day following interviews, a research team member processed audio video sources following a playbook of data handling procedures in accordance with the IRB. Student level folders were established in the secure Box drive that the team member transferred all picture, video, and audio files to from the assigned SD card. Each source (i.e., audio recordings, video recordings, still photos) had a subfolder per student and explicit naming conventions per file type. All video and photo files were backed up to an external hard drive and deleted from the RME machine used for processing, per IRB requirements.

The MMaRS project specialist downloaded the video files from the RME secure folder on Box and uploaded the videos to the Rev.com secure website for transcription. All information was transmitted with username and password protection, as stipulated in the IRB. Once transcripts were completed, the project specialist logged into Rev.com, downloaded the transcripts, and then recoded from generic “speaker” labels to the interviewer’s initials and the student’s identification number. Once in a standardized format, transcripts were uploaded to the secure Box file with other audio/visual materials.

Conclusion

Once all pieces of data were securely stored in the RME office, research staff prepared the sources for quantitative and qualitative analysis. See the Spatial Reasoning: Cognitive Interview Methods and Quantitative Data Analysis (Tech. Rep. No. 20-08) for details on sources used and processes for correctness scoring. Refer also to the Spatial Reasoning: Cognitive Interview Methods and Qualitative Data Analysis (Tech. Rep. No. 20-21) for procedures to prepare and analyze data through qualitative inquiry.

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