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Spatial Reasoning Cognitive Interviews: Qualitative Data Analyses

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Spatial Reasoning Cognitive Interviews: Qualitative Data Analyses

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Abstract

The purpose of this technical report is to describe the qualitative analysis of the Spatial Reasoning (SR) Cognitive Interviews (CIs) that were conducted as part of the Measuring Early Mathematics and Reasoning Skills (MMaRS) project for grades K-2. The CIs serve as one source of data for empirically recovering the hypothesized SR learning progression. This report details the methods we used to analyze cognitive interview video and audio data and the qualitative data analysis outcomes. More details about the SR cognitive interview protocol development can be found in the Spatial Reasoning Cognitive Interview Protocol Development technical report (Tech. Rep. No. 20-07). Details about the interview administration can be found in the Spatial Reasoning Cognitive Interview Administration technical report (Tech. Rep. No. 20-23).

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Spatial Reasoning Cognitive Interviews: Qualitative Data Methods and Analyses

Introduction

The purpose of this report is to describe the qualitative analyses conducted on the Spatial Reasoning (SR) Cognitive Interviews (CIs) for the Measuring Early Mathematics and Reasoning Skills (MMaRS) project. Based on the hypothesized SR Learning Progression (LP), we developed CI protocols and implemented those to inform the LP's conceptualization and empirical recovery. See the Spatial Reasoning Cognitive Interview Protocol Development (Tech. Rep. No. 20-07) and Spatial Reasoning Cognitive Interview Administration (Tech. Rep. No. 20-23) technical reports for development and administration details. This report aims to detail the analyses of qualitative data to inform our overall research questions and later LP reconciliation.

Research Questions

We designed the cognitive interviews to address four research questions related to empirically evaluating the SR learning progression. We included detailed sub-questions within each overarching research question. Questions 3 and 4 required information from the cognitive interview video and transcript data with gestures included. This report details the methods and results of analyses to address Questions 3 and 4.

RQ 1: Developmental Appropriateness

- 1.1 Do the entry and exit KSAs align with teachers' expectations of pre-requisite and target skills?
- 1.2 Does teachers' frequency of teaching KSA align with progression?
- 1.3 Does student performance and engagement indicate floor or ceiling effects that align with entry and exit KSAs?

RQ 2: Ordering

- 2.1 Are teachers' perceptions of the appropriateness aligned with the hypothesized order?
- 2.2 Do students demonstrate increasingly sophisticated reasoning aligned with the hypothesized ordering?
- 2.3 Do students appear comfortable with tasks and task elements?

RQ 3: Conceptions

- 3.1 Do students demonstrate reasoning that is consistent with the hypothesized conceptions?
- 3.2 What misconceptions and/or errors do students make? Is there a pattern leading to greater competence?

RQ 4: Interconnectedness

- 4.1 In what ways are students' KSAs interconnected?
- 4.2 In what ways do prior KSAs impact students' responding?

Table 1 describes the data used by research question.

Table 1

Data Use by Research Question

Research Question		Data Use
1		
	1.1	Teacher Survey Data
	1.2	Teacher Survey Data
	1.3	Quantitative Data; Fidelity Data
2		
	2.1	Teacher Survey Data
	2.2	Quantitative Data (c-prop, p-values)
	2.3	Fidelity Data
3		
	3.1	Quantitative and Qualitative Data
	3.2	Classification of Incorrect CI Responses and Qualitative Data
4		
	4.1	Qualitative Data
	4.2	Qualitative Data

Methods and Processing

Full protocol implementation occurred in two cycles and produced 16 complete cognitive interviews. The first cycle included 17 participants and was conducted in December 2019. We retained 16 participants' interviews for data preparation after one child's interview revealed excessive fidelity deviations. Data preparation activities revealed additional fidelity issues in the interviews of six more participants, all from the same site, causing concern over the quality of data collected. In turn, we removed all cycle one interviews from that site and interviewed six additional students from January to February 2020 as the second cycle of cognitive interviews. In total, we interviewed 23 students during data collection and retained 16 for full analysis.

Data Processing

The primary data sources for qualitative analyses were student audio and video recordings, workbooks, and observer protocols. For data handling in the field and upon secure delivery to offices at the university, see the Spatial Reasoning Cognitive Interview Administration technical report (Tech. Rep. No. 20-23).

Once data was in the Research in Mathematics Education (RME) office, a team member sorted the interview materials, including the student assent, student workbook, observer copy of the interview protocol, and fidelity observation form, before filing and locking all documents in a secure space. Video and audio files were securely uploaded to BOX, the university Institutional Review Board approved, secure, cloud-based file storage. Audio files were transcribed through Rev.com, an approved, outside transcription service, and uploaded to student-level folders in BOX. To confirm the audio transcription accuracy, an undergraduate student listened to 20% of the minutes of 20% of the interviews while reading the transcripts to verify the quality and capture of data. No pervasive inaccuracies that impacted the data were found in this process.

Following the audio file transcription, a group of internal and external team members watched the videos and inserted written descriptions of student actions and gestures into the transcripts using a process we called non-verbal transcribing. They also added any pertinent interviewer actions and subcomponent names to divide the transcripts into sections, while simultaneously removing non-mathematical conversations.

After non-verbal transcription was complete, we used the new transcript files containing non-verbals for qualitative coding and analyses. These files were uploaded into NVivo and used as the sole data source for coding, except when the non-verbals were not specific enough to determine student thinking and strategies. Those exceptions were detailed in the codebook audit trails and included viewing pictures of student work or reviewing videos when the coding team deemed it necessary. Specific methods and procedures, including training, non-verbal transcribing, first-pass pre-coding, deductive a priori coding, and open coding, are next detailed.

Non-Verbal Transcriptions

The purpose of non-verbal transcription was to allow coders to better understand the spatial reasoning cognitive interviews without consulting multiple data sources. We created transcripts of the interviews that included relevant non-verbal actions and gestures by the student and interviewer. By including these in the written transcripts, we sought to lower the coders' cognitive load while searching for themes.

Three external graduate students were onboarded to facilitate the insertion of non-verbal gestures and actions into the audio-only transcripts. They received a project overview and online training. They were connected with the Graduate Research Assistant (GRA) working on the project as an internal point of contact to complete transcript insertions over two weeks. The MMaRS GRA oversaw the flow of training, verified the accuracy of non-verbal coding on a sample video, and released data to transcribers after training. Each coder was assigned one interview type (see CI Administration technical report for interview types; Tech. Rep. No. 20-23), either W1&2, W3, or Between, and completed a sample video of the assigned protocol from our earlier try-out interviews. The GRA verified the entire try-out transcript and provided feedback to align coding practices with anticipated results.

Coders were given core concept and subcomponent specific coding instructions to write non-verbals within the transcript (see the SR CI Protocol Development technical report for details on interview structure – Tech. Rep. No. 20-07). General non-verbal transcription instructions were:

- Insert, in bold font, the skill code name exactly as it appears on the protocol (i.e., if "fold and punch," insert "fold and punch")

- Gestures on the “student” line: If a student makes a gesture during their talk turn in the transcript, you do not have to refer to the students. For example, if the student points to a triangle and the student is talking, you may insert [points to triangle].

- Gestures on the “interviewer” line: If a student makes a gesture during the interviewer's talk turn, indicate in the non-verbal insertion that the student is the one gesturing. For example, if the interviewer is talking and the student points to a triangle, you may insert [student points to triangle].

- Gestures by the interviewer: We are not generally interested in what the interviewer gestures, but if you believe something is integral to a reader’s understanding, please note [interviewer points to...] regardless of talk turn at which the gesture and insertion occur.

The GRA provided additional, individual feedback to each coder specific to their assigned protocol, as the Targeted Learning Goals (TLG) and Core Concept (CC) contexts varied and the information needed was not consistent across interview types. For example, we needed information about how many of each shape type were selected through pointing or touching in W1&2 non-verbal transcripts, but not in W3 and Between – details of multiple select responses were not needed to analyze those interviews, and therefore feedback given stated as such.

Videos were assigned to coders in groups of five. Then, as part of the analysis plan, the GRA verified 20% of 20% of the videos’ non-verbal transcriptions, or three to four subcomponents in one of each five videos. This verification process provided continuous feedback loops so that the GRA could refine coding and verification procedures. Coders benefited by becoming proficient quickly and reduced their time spent per video while inserting higher-quality non-verbals.

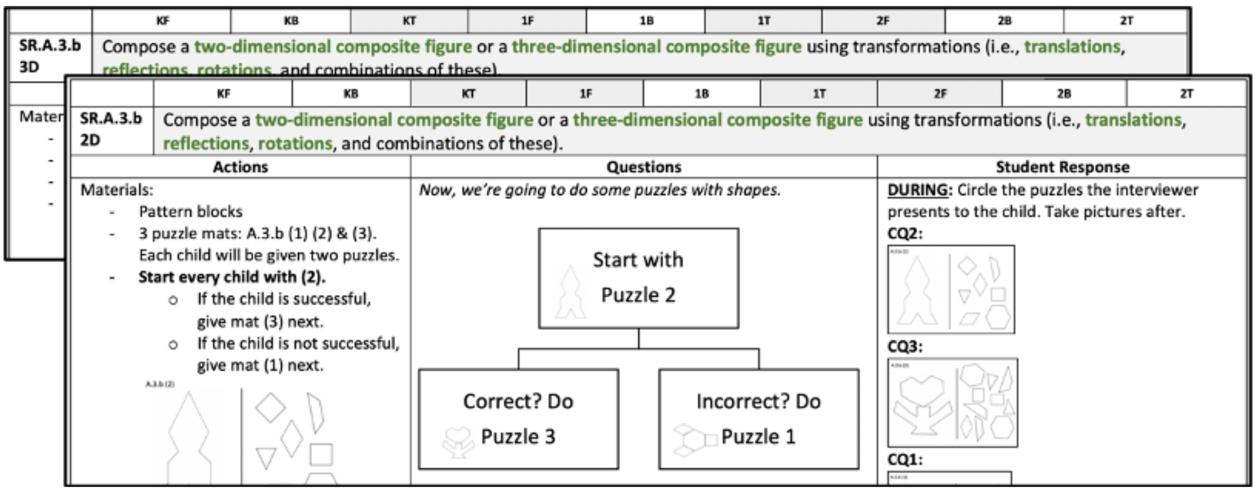
When adding non-verbals to transcripts, coders identified and marked each subcomponent in the transcripts using naming conventions to delineate the TLG, CC, subcomponent, and in some cases, the micro-conceptualization that the task assessed; subcomponent names additionally specified two- and three-dimensional (e.g., SR.A.3.b 2D or 3D; see Figure 1). There were multiple protocol tasks for some subcomponents and multiple items per task for skills that we anticipated spanning developmental appropriateness bands. For example, the subcomponent shown in Figure 1 begins at Kindergarten Foundational (KF) and extends to grade 2 Target (2T). There is a decision tree in the question column of the protocol due to that wide span of expected developmental appropriateness. Interviews began with a mid-level task from which the interviewer could adjust for the follow-up question based on student performance. Questions would all connect to the subcomponent with additional labels of Task 1, Task 2, and so forth.

Coders used a strikethrough to remove irrelevant talk turns (e.g., students talking about Santa coming between questions) as the first step in data trimming. However, they did not remove any text at this stage as researchers wanted to preserve the full transcript if needed later.

While completing non-verbal transcriptions, employees worked remotely, so specific procedures were implemented for IRB and data security purposes. Coders used a tracking spreadsheet to record transcript completion, track time spent on tasks, and communicate about verification and assignments. Videos and transcripts were accessed through BOX. All videos were viewed in preview mode, and non-verbal transcription was completed in Word Online. One coder could not complete the work and move onto the next pre-coding cycle, but the other two continued working with MMaRS staff to complete this and pre-coding, which was the next step in this data preparation process. After non-verbal transcribing was complete, coders pre-coded the same interview groups in which they had inserted non-verbals.

Figure 1

Subcomponent with Two- and Three-Dimensional Protocol Items



Note. Subcomponent SR.A.3.b, with two skill code statements separated by two- and three-dimensions plus an embedded decision tree to guide interview based on student response.

Pre-Coding

To prepare the transcripts with non-verbals—henceforth named combined transcripts—in NVivo for qualitative coding, the GRA who oversaw non-verbal transcription combined all individual student transcripts by interview type, W1&2, W3, and Between. Once in a single files by type, they removed extraneous and irrelevant talk turns (e.g., talking with a child about family, dogs, cats, etc.). If a talk turn was not mathematically relevant but provided context for the student response, it was left in the combined transcript. The lead coder/project researcher uploaded the combined transcripts into NVivo for pre-coding.

The two remaining non-verbal coders engaged in a two-part training with a MMaRS researcher to learn pre-coding procedures. The researcher led coders in reviewing the protocols before introducing them to and training them on procedures for NVivo pre-coding. They trained using a sample transcript and completed their first coding assignment on-site with the researcher's support. Once fully trained, coders were assigned groups of subcomponents, after which the researcher verified 20% of 20% of the text coded before they proceeded to the following group.

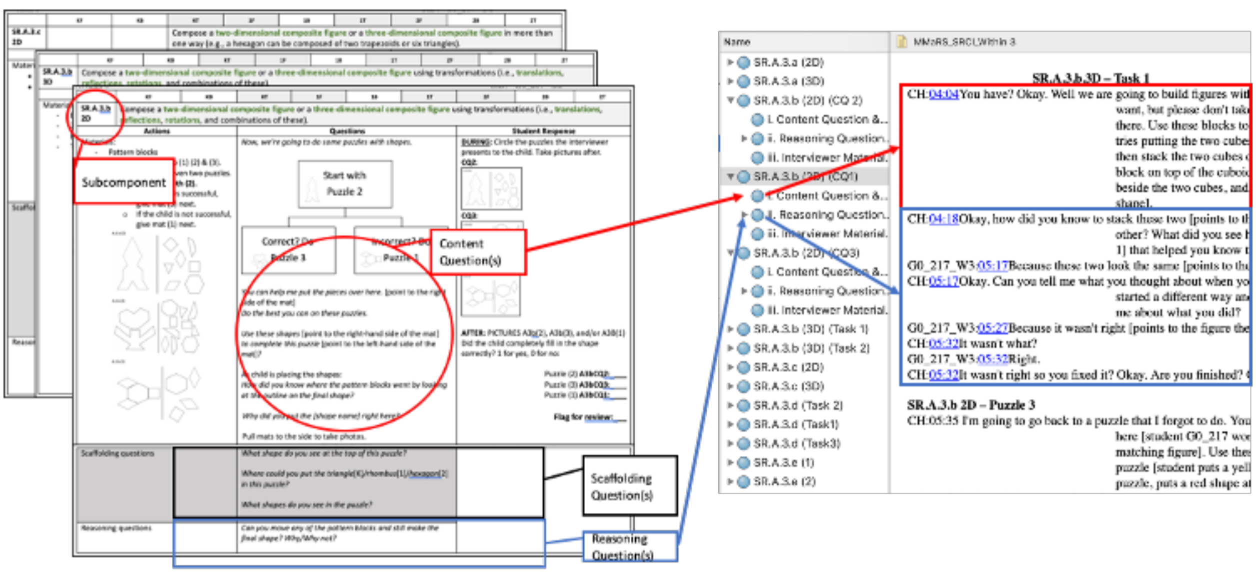
To complete pre-coding, all combined transcripts were loaded into NVivo and coders attached the text associated with each subcomponent to that subcomponent node within the NVivo file.

To clarify, codes are defined as "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study" (Miles & Huberman, 1994, p. 56), and nodes are the structures used within NVivo to organize codes. This was a structuring process – for each subcomponent, coders attached specific text to the following nodes: (a) content question and response, (b) reasoning question and response, or (c) interviewer materials. For items with multiple Content Questions (CQs), there were inherently multiple Reasoning Questions (RQs), so each had its own corresponding node, as seen in Figure 2. We found through the process that the "interview materials" node was extraneous and decided to discontinue coding at that node.

Figure 2 shows a protocol item with multiple CQs, RQs, and a decision tree. Protocol items that spanned hypothesized developmental appropriateness levels were presented in a decision tree to guide the interview appropriately from a mid-level entry point. Students were presented an item that was anticipated to require the mid-level skill of that subcomponent, and dependent upon the student's response, interviewers would move to an item with higher or lower anticipated difficulty within the same skill. If a subcomponent contained multiple CQs that were not on a decision tree or complexity continuum, interviewers could stop after the first task if the child clearly did not understand the concept or answered incorrectly.

Figure 2

Content and Reasoning Question Alignment to Nodes in NVivo



Note. Alignment from protocol questions to NVivo nodes as guide for pre-coding. Also shown is how transcript text was attached to the node.

After coders attached all associated text to the subcomponent and the relevant nodes, a researcher verified 20% of 20% of the pre-coded data. The researcher divided each NVivo file into sections with identified verification points, and coders were required to stop after completing all subcomponents within a given section, waiting for the researcher's verification before coding the next section. The researcher verified that all available transcript text was attached to the appropriate subcomponent node and that all text within subcomponents was coded to the relevant node (e.g., CQ, RQ). After verification, the researcher provided feedback to either make adjustments or continue to code the next section. Once pre-coding was complete for all interview protocols across SR LPs, the NVivo files were ready for qualitative coding and analyses.

Phase One Coding

To identify the data relevant to the study, the MMaRS team met and developed an initial coding scheme to locate and code student thinking in a way that would contribute to the refinement of the SR LP. The Principal Investigator (PI) introduced the idea of coding based on correct and incorrect thinking within conceptual, procedural, strategic, and adaptive reasoning (National Research Council [NRC], 2001). These types of thought would delineate student misconceptions and errors as inferred through the qualitative coding process to confirm or refute the content and ordering of the LP. The team discussed using *Adding it Up* (NRC, 2001) as a significant resource for defining the scheme, and a try-out of the process was proposed.

A coding team of three researchers selected two student transcripts from the try-out data and triple coded one subcomponent from each core concept using the aforementioned four types of reasoning. They developed an initial codebook (see Appendix A) using *Adding it Up* (NRC, 2001) as the guiding resource. The team processed two iterations of this coding scheme in attempt to capture the students' spatial reasoning, revising codes and adding examples and non-examples from transcript data in weekly debriefs. They determined that some codes (e.g., procedural fluency and conceptual understanding) were not evident in students' responses to spatial reasoning questions. However, students were using various strategies to reason about the spatial problems and a deeper review of extant literature was needed.

The coding team reviewed additional literature (See Appendix A), debriefed, and agreed that—within the literature consulted—there was little to no extant research on procedural and conceptual understanding of spatial reasoning tasks.

They shared these results with the PI and Project Manager (PM), specifically their difficulty in disentangling procedural and conceptual understanding. The interrelation was further evidenced by the National Academy of Sciences document saying that, "it is not always necessary, useful, or even possible to distinguish concepts from procedures because understanding and doing are interconnected in such complex ways" (p. 134). Coders felt that conceptual understandings might be evident at the core concept level, but not in individual subcomponent analysis. Procedural fluency would likely be captured in the quantitative analysis through correctness data, and determining strategic competence was not productive as students were not asked to develop mathematical problems through reasoning probes in cognitive interviews. Adaptive reasoning appeared as the only fruitful strand or proficiency presented through the initial structure, as coders could perceive the student's logic for solving the problem.

The coding team recommended coding for spatial language to determine if K-2 students can or cannot use spatial language when providing reasoning for spatial tasks. Similarly, they recommended using codes for tangential reasoning and no reasoning to provide support to determine if use of correct spatial language is a viable scheme for elementary students. The team proposed open coding the students' reasoning responses to look for themes that would emerge among the reasoning strategies. By analyzing student responses in this way, researchers planned to use thematic information to respond to research questions 3 and 4, on conceptions and interconnections. The rich interview data would illuminate reasoning during phase two coding.

Phase Two Coding

Based on consultation with an expert qualitative researcher, meetings with the PI and PM, and the coding team's recommendation, phase two coding employed open-coding of the CI data. The hypothesized learning progression and its bands of developmental appropriateness aligned with stepping stones of knowledge and understanding that occur over time, likened to a grounded theory approach of qualitative inquiry (Creswell & Poth, 2018). We developed a systematic procedure using the constant comparison between our data and codes as they were developed (Corbin & Strauss, 2015) to develop preliminary codes that led to axial codes and final themes (Saldaña, 2016).

To open code, data trimming was needed for focused analysis of mathematical reasoning in student responses to ensure that findings were grounded in meaningful student thoughts, words, and actions. This necessitated a two-cycle coding process of first a deductive, a priori schema, followed by open-coding to search for the emergent themes.

A Priori Coding

Using the project's research goals, the coding team developed a priori structural codes (DeCuir-Gunby et al., 2011). The later open-coding process involved "breaking data apart and delineating concepts to stand for blocks of raw data" (Corbin & Strauss, 2008, p.195). Open-coding was an iterative process in which coders created codes and then used axial coding to analyze them. The process led to the development of data-driven codes and involved five steps to inductively create codes for a codebook (DeCuir-Gunby et al., 2011). We:

1. Reduced the raw information in pre-coding and a priori coding (as described below)
2. Identified subsample codes
3. Compared codes across subsamples
4. Created codes
5. Determined the reliability of codes through debrief and reconciliation meetings

A coding team, consisting of one lead coder, one secondary coder, and a facilitator, engaged in this initial part of the phase-two coding process. The lead and secondary coder both had in-depth expertise in content and developmental appropriateness of strategies used. The facilitator provided training and NVivo software support during a priori coding, then gradually released responsibility for compiling individual files to obtain reliability and maintaining master files post reconciliation to the lead coder. The lead coder's file was already serving as the master, but verification and maintenance were additional steps. They applied a priori codes to all reasoning responses as “no explanation”, a “tangential explanation”, or a “mathematical explanation,” regardless of correctness (See Table 3). The coding structure was applied to both SR TLGs in NVivo (See Figure 3 for transcript source example).

Table 3

A priori codes for phase two coding

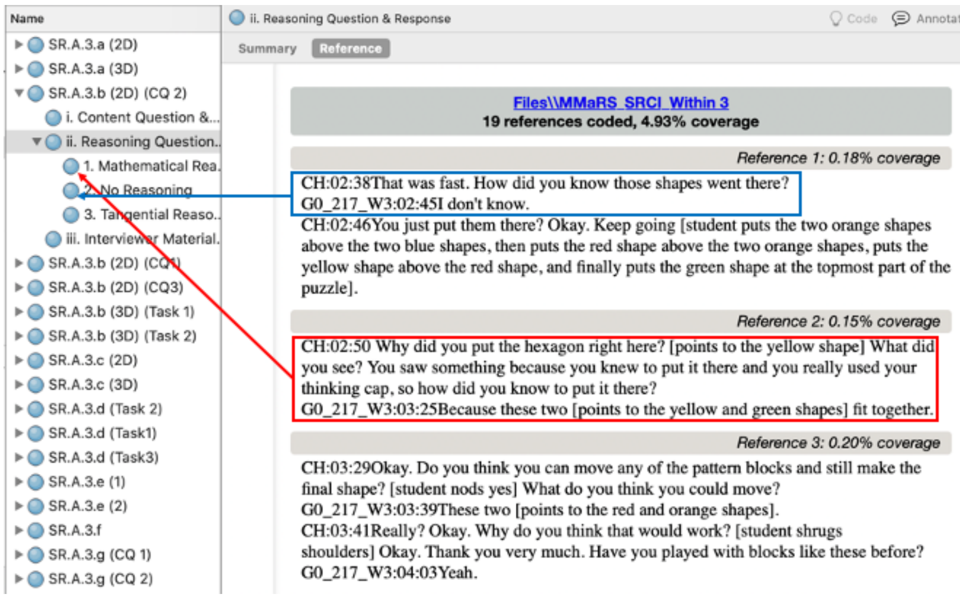
Codes	Mathematical Reasoning	No Reasoning	Tangential Reasoning
Definition	Student provided a correct or incorrect mathematical explanation to support his/her answer	Student explicitly denied to provide reasoning or stated that they do not know the answer	Student talked about irrelevant topics and did not provide or explicitly deny giving a mathematical reason
Examples	(1) “Because it looks like a barn… and these aren’t barns” (2) “This is a circle because it has no corners” (3) Sorting by any visual characteristic (e.g., color)	(1) States “I do not know” (2) Shrugs (3) States that an outside source (e.g., parent or teacher) told them	(1) “So the farmer can get [the bucket] when the pig is ready” (2) “It looks like a Bolivian flag” (3) “Because green and blue make yellow”
Non-examples	(1) Triangles are special because “you can build with it” (2) Following a different route “because” as final response (no deeper)	(1) Placement of object - “Because you put this sticker there” (repeat)	(1) “The cone looks like the orange one in Daddy’s Home 2” (connection to real life) (2) “the cat ran **out**” (spatial language)

Coders maintained an a priori codebook for each TLG, reasoning spatially within and between objects. The codebook contained spreadsheets that coders completed while coding: (a) timeline to complete the work, (b) status tracking of each task including the time to code, and (c) an audit trail to note questions or inconsistencies found in the data (See Appendix B) (Brinkmann & Kvale, 2015). The audit trail provided a space in which the team developed coding rules—these included coding to the deepest level of student thought within a single or multiple reasoning question response, chunking each unique reasoning question separately, and alerting the team if there is text that does not align with the protocol item. These rules then applied to both TLGs.

There was an additional tab for coding agreement that the coding facilitator and the lead coder used to track and record agreement between coders (See Appendix C for detailed agreement information). Twenty percent of subcomponents were double coded to verify coder agreement with a target of at least 80% coding agreement among coders. The facilitator led a coding agreement check process using NVivo coding comparison queries and reported agreement and kappa coefficients as evidence of agreement (Saldana, 2015, p. 37). After coders completed each subcomponent marked for common coding, the team debriefed to resolve any disagreements and discourage coder drift (Marston et al., 1978) from the deductive scheme. The lead coder recorded all disagreements and outcome of the debrief in the agreement tab as needed.

Figure 3

Types of Reasoning Responses



Note. A priori coding sample of "mathematical reasoning" and "no reasoning."

Coders used explicit code definitions to guide a process of elimination and arrive at the most accurate code for each child's reasoning response. If the child did not provide any reasoning about an item, that response was coded as "no reasoning". If the child reasoned using some type of logic that could be inferred as mathematical or connected to real life, the response was coded as "mathematical reasoning". If the child told a story or talked about an idea that was not connected to mathematics, the response was coded as "tangential reasoning" (See Table 3). After coding all subcomponents using a priori schemes, the reconciled file was distributed to the coders with a new codebook and folder structure in the secure BOX drive for open coding.

Open Coding

Next, a coding team engaged in open coding, looking for emergent themes using only the reduced, "mathematical reasoning" data from a priori coding. While themes are generally reporting patterns found in a data set, they also capture some information related to the research question. Given the fluid nature of interviews, an iterative process was developed to move through identifying themes, yet there is no specific rule on the number of evidential instances required to develop a theme (Braun & Clarke, 2006). The goal of open coding was to identify common themes across student reasoning when working on the same subcomponent task. By identifying themes across subcomponents and defining them through codes, the team sought to support the conceptualization of ordering, find interconnections, and identify student misconceptions and errors.

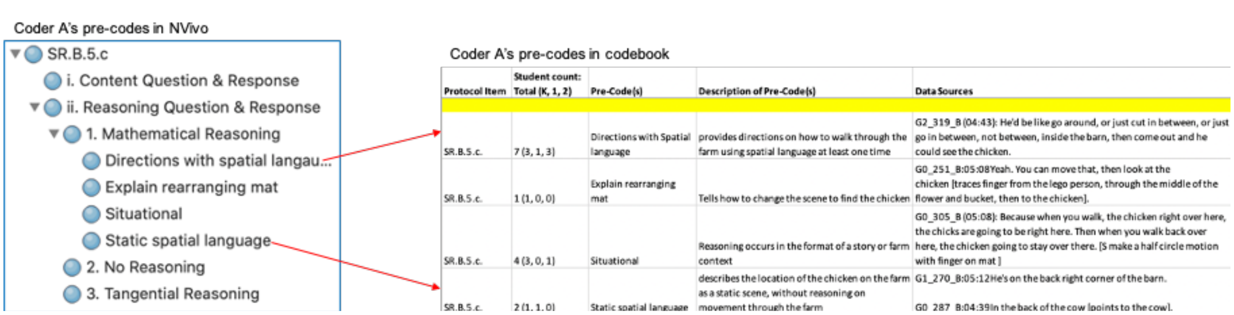
The lead and secondary coder engaged in this iterative open coding process. They independently coded each subcomponent in NVivo to each develop pre-codes, or first pass themes, based on their solitary review. Coders next compared their preliminary codes in debrief meetings and reconciled NVivo project files while determining final code names and definitions. The coding facilitator guided the development of the process and codebook procedures. The facilitator then released procedural oversight to the lead coder after processes were tested and workflows were established. Beginning with the second core concept of the within objects TLG, the two-person coding team worked collaboratively to complete open coding for the entire SR LP.

The open-coding process focused on identifying patterns of student strategies used in response to the reasoning question for specific subcomponents. Coders crafted a detailed timeline to code between one and three subcomponents, debrief, and finalize the codebook for each core concept before moving to the next. The team moved sequentially through the SR LP, first coding the Within Objects TLG, then the Between Objects TLG, with each having its own codebook. The timeline for TLG coding completion and synthesis writing was maintained in the first tab of the codebook, followed by a status spreadsheet that coders updated throughout the process. For each core concept, there was a new set of tabs in the codebook that corresponded with matching folders in BOX. This structure was to retain all NVivo files and summary statements in a single location with the codebook for quick retrieval and reference.

Independent Preliminary-Coding. Using the codebook and NVivo file simultaneously, coders independently created preliminary codes within the "mathematical reasoning" node in NVivo for each subcomponent. Preliminary codes were based on strategies or patterns of response that emerged broadly across student responses for the subcomponent protocol item. To enable later conversations, each coder developed a description based on preliminary jottings (Saldaña, 2016) about the code and identified at least one example from student transcripts. The code name, description, and sample text from an interview transcript were recorded in the codebook on coders' individual spreadsheets with the number of students by grade level who used that strategy. See Figure 4 for the alignment of preliminary codes between NVivo and the codebook.

Figure 4

Preliminary Codes in NVivo and the Codebook

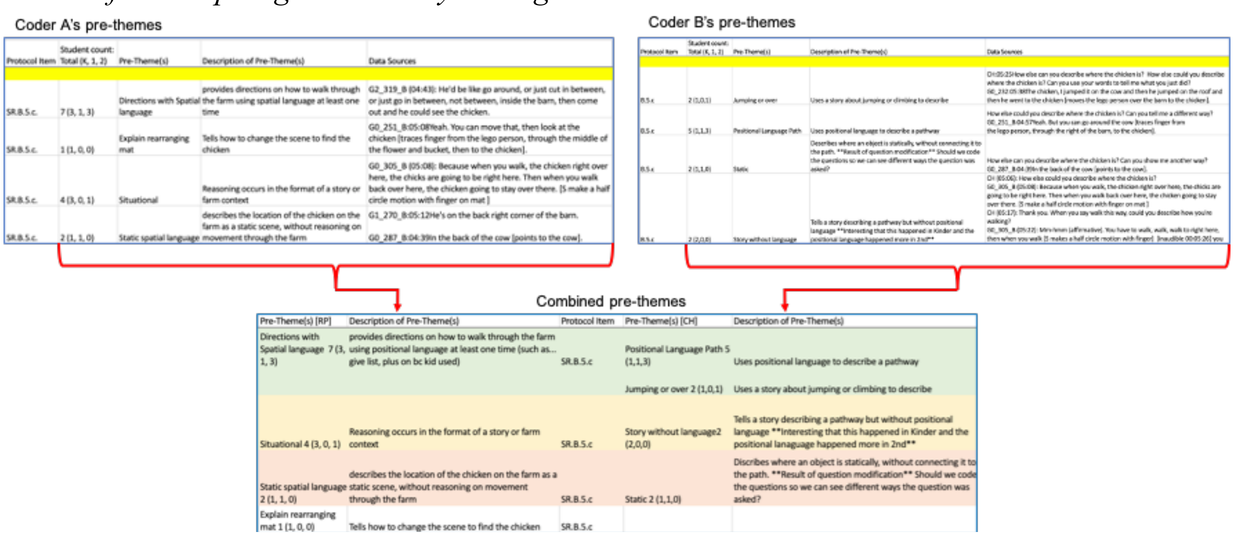


Note. As coders created preliminary codes in NVivo, they also wrote jottings to describe or define them and extracted examples from the transcripts to illustrate reasoning strategies.

Temporary Combined Codebook. After each coder independently created their preliminary codes, the lead coder retrieved the pre-code names, definitions, and student count and compiled the information in a “temporary combined codebook” to facilitate reconciliation in the debrief meeting. Before the debrief meeting, the lead coder aligned similar pre-codes in the temporary combined codebook tab within the codebook (See Figure 5 for an illustration of this process). Aligning pre-codes guided the conversation on similarities or differences between early code iterations. When one coder distinguished fine-grain codes from the data and the other used coarser codes, the latter’s big-idea codes could often encompass multiple, specific pre-codes.

Figure 5

Process for Compiling Preliminary Jottings and Codes



Note. Illustrated process of compiling the two preliminary codebooks into a single document for debrief meetings with rough alignment between separate but similar codes.

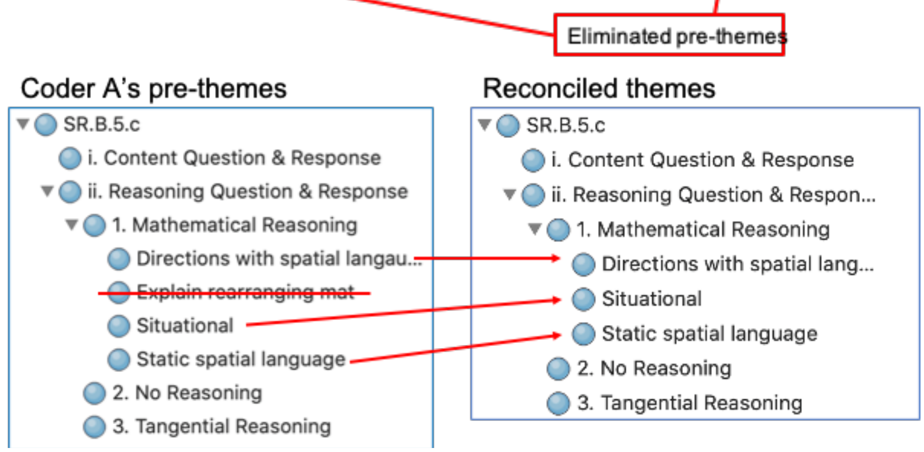
*Coder Debrief and Codebook Comparison***.** The team relied upon “dialogical intersubjectivity”, a process in which the two coders engaged an intensive discussion to achieve consensus on codes (Brinkmann & Kvale, 2015; Harry et al., 2005; Sandelowski & Barroso, 2007). These debrief meetings were held to ensure the quality of coding and come to agreement with common codes on each protocol item. Coders compared NVivo files, codes, and definitions, developing final code names and substantive definitions before concluding each meeting.

Through the meeting, coders revised code names and the text captured in their individual NVivo files as file reconciliation. When revising codes, definitions were jointly determined and examples were located in transcript data to illustrate the heart of the overall themes that codes were to represent. Further, discussion of interconnection between subcomponents emerged, which led to conversations about which skills might be interrelated and require additional analysis concerning the given subcomponent. See Figure 6 for the changes to code names from individual pre-coding to final codes.

Figure 6

Revision Process of Preliminary to Final Codes

Final Name	Pre-Theme(s) [RP]	Description of Pre-Theme(s)	Protocol Item	Pre-Theme(s) [CH]	Description of Pre-Theme(s)
Directions with spatial language	Directions with Spatial language 7 (3, 1, 3)	provides directions on how to walk through the farm using positional language at least one time (such as… give list, plus on bc kid used)	SR.B.5.c	Positional Language Path 5 (1,1,3)	Uses positional language to describe a pathway
x				~~Jumping or over 2 (1,0,1)~~	Uses a story about jumping or climbing to describe
Situational	Situational 4 (3, 0, 1)	Reasoning occurs in the format of a story or farm context	SR.B.5.c	Story without language2 (2,0,0)	Tells a story describing a pathway but without positional language **Interesting that this happened in Kinder and the positional lanaguage happened more in 2nd**
Static spatial language	Static spatial language 2 (1, 1, 0)	describes the location of the chicken on the farm as a static scene, without reasoning on movement through the farm	SR.B.5.c	Static 2 (1,1,0)	Discribes where an object is statically, without connecting it to the path. **Result of question modification** Should we code the questions so we can see different ways the question was asked?
x	~~Explain rearranging mat 1 (1, 0, 0)~~	Tells how to change the scene to find the chicken	SR.B.5.c		



Note. Comparison from pre-codes to reconciled, final codes in the codebook and NVivo node structure that are transferred to the final codebook with definitions.

Finalization of Codebook. At the end of each meeting, the lead coder recorded final code names, definitions, examples from the data, and any exclusion criteria in the final codebook. Student counts by code were revised, and the examples of student talk and gesture were included. The lead coder's NVivo file was updated with the new codes and uploaded to BOX as a reconciled file. These codes, were used to provide a source of evidence for the hypothesized conceptions and misconceptions in students' reasoning, as asked in research questions 3.1 and 3.2. The codes also informed our understanding of interconnectedness between students' strategies and reasoning with each subcomponent of the SR LP, which may indicate interconnections of KSAs with their responses.

Figure 7

Finalized Codebook for One Sample Code

Protocol Item	Code(s)	Description of Code(s)	Example	Non-Example/Exlusion Criteria	Data Source of Example	Total Student Count (K,1,2)
B.5.c	Directions with spatial language	provides directions on how to travel from a start to an end point using positional language at least one time (e.g., around, look in, jump on, and all others in elements that vary list)	G0_251_B:04:57Yeah. But you can go around the cow [traces finger from the lego person, through the right of the barn, to the chicken] G2_319_B (04:43): He'd be like go around, or just cut in between, or just go in between, not between, inside the barn, then come out and he could see the chicken. G0_232:05:38The chicken, I jumped it on the cow and then he jumped on the roof and then he went to the chicken [moves the lego person over the barn to the chicken].	Uses positional language as staic only or does not use positional language when describing a route.	G0_251_4:57 G1_213_4:24	8 (4, 1, 3)
B.5.c	Situational	Reasoning occurs in the format of a story or farm context	G0_305_B (05:08): Because when you walk, the chicken right over here, the chicks are going to be right here. Then when you walk back over here, the chicken going to stay over there. [S make a half circle motion with finger on mat]	Tells a story about the farm scene without using spatial or positional language about the static scene or movement in the farm	G0_305_5:08	3 (2, 0, 1)
B.5.c	Static spatial language	describes the location of the chicken on the farm as a static scene, without reasoning on movement through the farm	G0_287_B:04:39In the back of the cow [points to the cow] G1_270_B:04:48He's at the right of the barn	References movement in tandem with positional language, or uses no positional language in description	G0_287_4:39 G1_270_4:48	2 (1, 1, 0)

Note. Final codebook for subcomponent SR.B.5.c., including code names, descriptions, text examples from student transcripts with non-verbal descriptions, exclusion criteria or non-examples, and the total student count with further delineation by grade level

Results of the Qualitative Analyses

In this section, we describe the results by research question, including associated sub-questions. The results of this qualitative analysis address children's conceptions of the content, including misconceptions and errors, and interconnections of Knowledge, Skills, and Abilities (KSAs) with and across the cognitive interview protocol items.

RQ 3: Conceptions

RQ 3.1

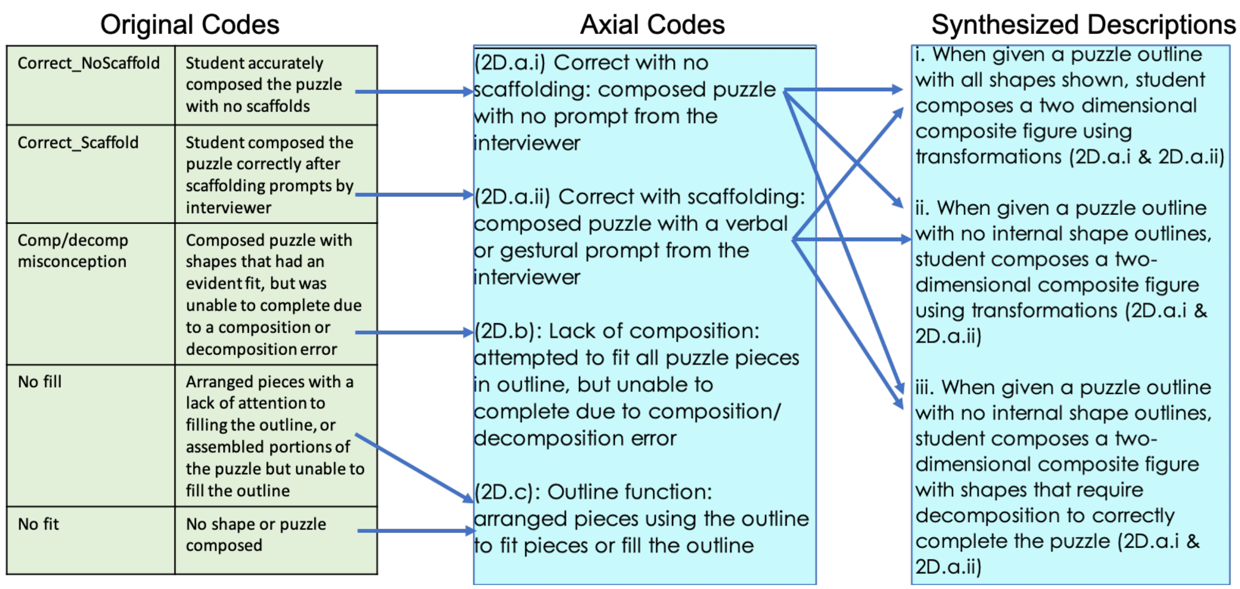
We examined the reasoning that students used through qualitative analyses as one source of evidence to inform the conceptualization of the hypothesized LP. Given the coders' knowledge of child development, educational pedagogy, recent trends in teaching practice, and content standards, we extrapolated the emergent themes into progressively sophisticated conceptions of necessary KSAs. Herein we describe reasoning patterns as a standalone source of data, not yet informed by or reconciled with correctness data. These patterns and other results are synthesized in the Bulleted Summaries found in Appendix D.

Subcomponent Synthesized Descriptions. Using a two-step process, the lead coder created synthesized descriptions of student actions that aligned with increasingly complex ways of thinking about the given construct. The first step was an independent draft of axial codes based on codebook descriptions. The second step involved a team review, or member-check for interpretative convergence (Saldaña, 2016), to ensure accurate capture of the analysis outcomes. The coders read each subcomponent, the elements that varied, the codebook codes, synthesized descriptions, and inferred student misconceptions and errors. Together, they created final statements which included the elements that varied when possible, to streamline later iterations of the TLGs.

Figure 8 details the process of creating axial codes from the original codes and synthesizing those descriptions. Figure 9 shows the *elements that vary* for the composition and decomposition core concept, and how they mapped onto a single subcomponent as they were incorporated into synthesized descriptions. These processes converged in the Bulleted Summaries, all of which can be found in Appendix D.

Figure 8

Process for Creating Synthesized Descriptions from Codes



Note. Codes from the original codebook were refined to axial codes when transferred to Bulleted Summaries (See Appendix D), then further interpreted in synthesized descriptions of student performance.

Some skills evolved linearly along a hypothesized developmental progression, while others seemingly developed non-linearly. Some patterns that emerged were not developmental and indicated errors in student thinking or misconceptions that required academic feedback to facilitate student growth; these interpretations are detailed in the findings for research question 3.2. Each synthesized statement in the progression of correct thinking aligned with the codebook's final codes, informed the statement, and supported the ordering or concurrent skill development, which can be seen in Figure 8.

Elements That Vary and Microprogressions of skill. In addition to subcomponent designations of developmental appropriateness, we identified elements that vary across those ranges that could be applied in multiple subcomponents for each TLG. Core concepts in the Within Objects TLG shared one matrix of elements, and those in the Between Objects TLG shared another. These elements were incorporated into items as they were designed but remained as a separate representation during the protocol development. See the SR CI Protocol Development Technical Report for details on retrospective decisions (Tech. Rep. No. 20-07), and Figure 9 of the Within Objects elements that vary as mapped onto a subcomponent.

Figure 9

Elements that vary mapped to a Within Objects subcomponent

Reasoning Spatially Within Objects: Elements that vary									
	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
Shapes (all can be regular or irregular)	Regular or irregular circles, squares, triangles, rectangles			Regular or irregular hexagons, rhombus, cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids				Regular or irregular quadrilaterals	
Translations	Horizontal, vertical, and diagonal translations								
Rotations	Rotations of less than 45°			Rotations of more than 45°					
Reflections				Reflections across a vertical or horizontal axis			Reflections across a diagonal axis		
Two-dimensional composite figure	Outline of all shapes are shown.	Shapes may share one side but not all outlines are shown.			Shapes may share more than one side but not all outlines are shown.				

3. Composition & Decomposition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
SR.A.3.b	Compose a two-dimensional composite figure or a three-dimensional composite figure using transformations (i.e., translations, reflections, rotations, and combinations of these).								

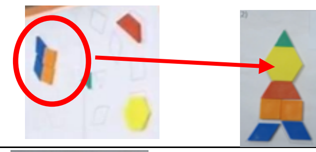
Note. Green words in subcomponent skills designate the elements that varied by grade level and developmental appropriateness designations.

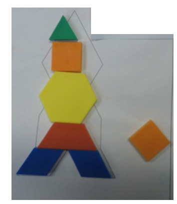
When we combined the elements that varied into the synthesized summary statements of student skills through axial coding, we noticed a trend that students progressed in sophistication of KSAs and along some elements that vary. We developed several microprogressions of students' KSAs that culminated in successful completion of the task associated with a given subcomponent. A specific microprogression is shown pictorially in Figure 10, with levels of skill described. In this example, to successfully complete a complex shape, students needed to employ composition strategies that also relied upon knowledge of shape attributes and transformations. Using KSAs elicited in earlier tasks, students who successfully composed puzzles demonstrated advanced conception of shape composition through utilizing interconnected skills.

In some instances, we designed the microprogression from the data, whereas in other instances we validated the already existing microprogressions from the elements that vary. Figure 11 further illustrates the incorporation of the elements that vary with the sophistication in students' reasoning demonstration for subcomponent SR.A.3.b (2D), shape composition.

Figure 10

Performance level descriptions in the shape composition task

Performance Level	Associated Student Response Patterns	Student Work Samples
Independent Full Composition	• Composes the full puzzle accurately on their own	
Full Composition with Scaffolding	• Attempts puzzle independently • Composes the full puzzle accurately after scaffolding questions were posed	

Partial Composition	• Creates a figure similar in overall shape to the intended composite figure, but may:
	  ◦ Fit shapes together with one another and neglect how they fit with the outline
	  ◦ leave out pieces that do not fill remaining empty spaces
	  ◦ when posed with scaffolding prompts, does not attempt to de/compose or shapes from current structure
		 

Emerging Composition	• Demonstrates "fill" or "fit" only strategies by either
	  ◦ Aligning shapes to the figure outline to fill the space
	  ◦ Joining edges or vertices of individual pieces with one another
	• Accurately places shapes to create a larger composite figure when individual shape outlines are included
		

Figure 11

Skill Microprogression Including Elements that Vary

Reasoning Spatially Within Objects: Elements that vary

	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
Shapes (all can be regular or irregular)	Regular or irregular circles, squares, triangles, rectangles		Regular or irregular hexagons, rhombus, cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids					Regular or irregular quadrilaterals	
Translations	Horizontal, vertical, and diagonal translations								
Rotations	Rotations of less than 45°		Rotations of more than 45°						
Reflections				Reflections across a vertical or horizontal axis		Reflections across a diagonal axis			
Two-dimensional composite figure	Outline of all shapes are shown.	Shapes may share one side but not all outlines are shown.			Shapes may share more than one side but not all outlines are shown.				

3. Composition & Decomposition

Code	Kindergarten				Grade 1		Grade 2		
	F	B	T	F	B	T	F	B	T
SR.A.3.b	Compose a two-dimensional composite figure or a three-dimensional composite figure using transformations (i.e., translations, reflections, rotations, and combinations of these).								

***Elements that varied not included (transformations)**

i. When given a puzzle outline with all shapes shown, student composes a two-dimensional composite figure using transformations (2D.a.i & 2D.a.ii)

ii. When given a puzzle outline with no internal shape outlines, student composes a two-dimensional composite figure using transformations (2D.a.i & 2D.a.ii)

iii. When given a puzzle outline with no internal shape outlines, student composes a two-dimensional composite figure with shapes that require decomposition to correctly complete the puzzle (2D.a.i & 2D.a.ii)

Two- and Three-Dimensional Subcomponents. Subcomponents from the Within Objects TLG contained two- or three-dimensional shapes and figures and were originally conceptualized as single subcomponents spanning dimensions. For instance, SR.A.1.a asked students to sort shapes and figures regardless of dimensionality, but SR.A.1.b, c, and d request that students recognize, name, and identify the attributes of two-dimensional shapes and three-dimensional figures. When protocols were developed for the cognitive interviews, separate items by dimension were created to assess the skills (see the SR CI Protocol Development TR for more information; Tech. Rep. No. 20-07). Through open coding, we found that some patterns of student response were similar regardless of dimensionality, but others were separate between two- and three-dimensions (See Figure 12 for example codes, including overlapping strategies). This information was critical to the ways in which the TLGs will be written and how revisions will occur during reconciliation.

Figure 12

Codes for Student Strategies Within and Between Dimensions

1. Shape									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
SR.A.1.b	Given the name of a two- or three-dimensional **shape**, recognize the **shape**.								
SR.A.1.c			Name two- and three-dimensional **shapes**.						
SR.A.1.d					Using defining attributes, classify two- and three-dimensional **shapes**.				

2D only
3D only
Both 2D and 3D

Theme(s)	Description of Theme(s)	A.1.b (2D)	A.1.c&d (2D)	A.1.b (3D)	A.1.c&d (3D)
Real life	Uses description of real-life objects or references something they know exterior to the item to describe why a shape is a shape or same/different.	1		1	1
Transformation	Transforms shape first to make a visual match for the current mental image of what that shape "should" look like	1	1		
Describes faces	Uses 2D shape name to describe one or more faces of the solid			1	1

Note. Students used some “real life” to explain their thinking in both two- and three-dimensional protocol items, but “transformation” on two-dimensional items only, and “describe faces” on three-dimensional only.

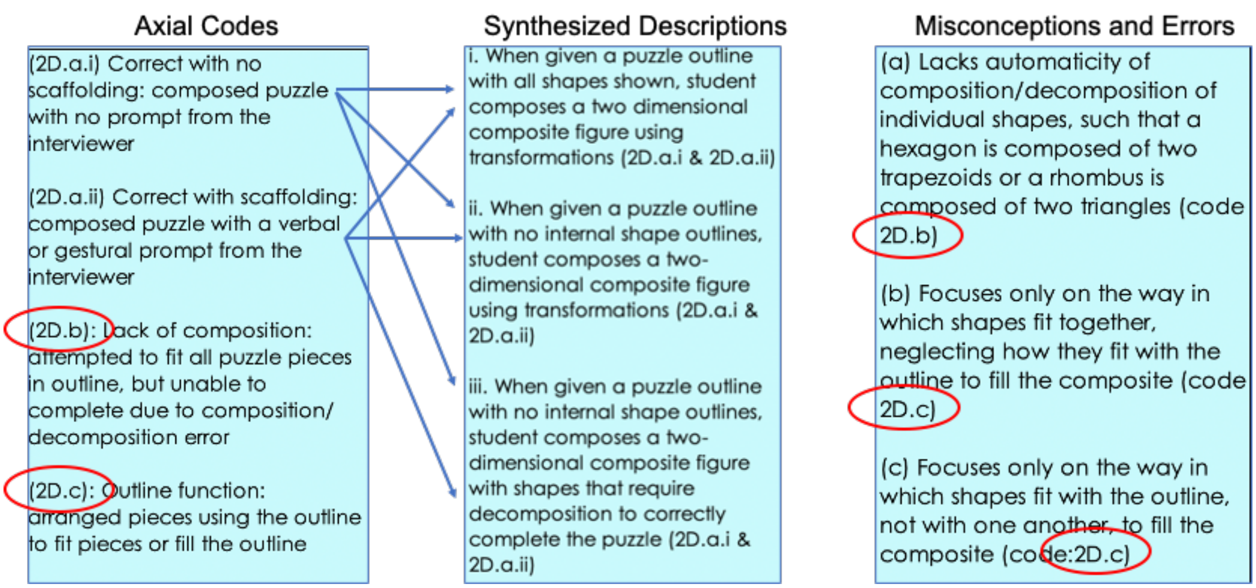
RQ 3.2

When analyzing patterns of each subcomponents’ linear development, or the skill microprogressions, we embedded the elements that varied when possible and separated codes that represented misconceptions and/or errors made by students. Errors in thinking that represented less sophistication in student reasoning evidenced patterns upon which educators could develop scaffolding for learning. In contrast, misconceptions provided a direct path to intervention to correct those conceptions. As an example, this section reports the misconceptions and errors seen on the performance task associated with the complex shape composition—SR.A.3.b. (2D)—detailed in Figure 13 and Appendix D.

Student Misconceptions or Errors. For those codes that did not represent developmental steps in a microprogression for the given subcomponent skill, we analyzed student transcripts to find misconceptions or errors in thinking. These were characterized by individual or common examples from transcripts and were further detailed in each code's descriptors. Errors and misconceptions aligned with the levels of thinking in the synthesized descriptions, which could be later used by practitioners to facilitate academic feedback or scaffolded practice. By identifying the level in which a misconception was associated, one could map to where academic feedback was needed to correctly conceptualize the skill.

Figure 13

Axial Coding Aligned to Student Levels of Conception, Misconceptions, and Errors



By examining student response patterns in codes and supplementing with photos of student work of SR.A.3.b (2D) tasks, we illustrated the process of developing a microprogression of skills that culminate with sophisticated thinking in this subcomponent. The statements and paired visuals of student reasoning can enable teachers to see a frame of students' current skills visual, then determine the next instructional moves to help them gain in sophistication of response.

RQ 4: Interconnectedness

RQ 4.1

The ways in which students' KSAs are interconnected was evidenced through their words, actions, and gestures related to reasoning responses during cognitive interviews. Each progressively sophisticated subcomponent within each core concept relied upon anticipated microprogressions of interrelated KSAs. For example, in the Between Objects TLG, subcomponent SR.B.7.d. was on recognizing a perspective other than one's own. While the subcomponent targeted this skill, it also connected back to earlier skills, including expressive use of spatial language to describe the differences, recognition of one's own perspective, and acknowledgment that an alternate perspective would not look the same.

The codes describing strategies that emerged from the analysis of this statement overlapped heavily with those from each of the earlier skills. Students used spatial language to describe where objects in a scene were located. They focused on the direction that objects were "facing," just as they had when identifying a scene from their own perspective. Students also used one-to-one correspondence in the identification process, mapping back and forth between a physical scene and representative image to explain their reasoning. Without using KSAs targeted in previous tasks, students were unable to provide adequate reasoning for a selection. See Figure 14 for a visual depiction of the interrelation of subcomponents and student responses.

Figure 14

Interrelation of Student Strategies

single code	2+ subcomponents same context	2+ subcomponents same core concept	2+ subcomponents across core concepts

Theme(s)	Description of Theme(s)		A.1.a	A.1.b (2D)	A.1.c&d (2D)	A.1.b (3D)	A.1.c&d (3D)	A.2.a	A.2.b
Real life	Uses description of real-life objects or references something they know exterior to the item to describe why a shape is a shape or same/different.			1		1	1	1	1
Transformation	Transforms shape first to make a visual match for the current mental image of what that shape "should" look like			1	1				
Number of vertices	Uses the number of vertices to explain why the shape is the shape			1	1				

Theme(s)	Description of Theme(s)	B.6.d	B.6.e	B.6.f	B.7.a	B.7.b	B.7.c	B.7.d	B.7.e
Object Orientation	Provides reasons for facing drawn objets in the way that the student drew them, or references the orientation inside of the model	1							
Positional Language	Uses positional language to describe an objects location	1			1			1	1

Note. Student strategies used in single subcomponents, across two or more subcomponents in a single core concept, and across two or more subcomponents in more than one core concept.

RQ 4.2

After coding was complete for each core concept, we wrote narrative-style summaries to illustrate and delineate the ways in which students' prior KSAs were surmised to impact their responses. Through an initial draft of written summaries, the lead coder narrated patterns of student response from least to most sophistication in thinking, based on content and pedagogical experiences. These were saved in the coding spreadsheet as a summary written in a fluid, conversational tone that described students' inferred reasoning responses (See the SR CI: Narrative Summaries of Qualitative Analysis Technical Report for details [Tech. Rep. No. 20-28]). More granularly, the team engaged in creating the bullet-style summaries, as referenced above and seen in Appendix D. These aligned children's' demonstrated reasoning strategies to each subcomponent with levels of sophistication.

Bulleted Summaries. A bullet-style summary was created with axial codes of student reasoning and synthesized descriptions as related to the skill elicited by each subcomponent protocol item. This summary also included student misconceptions and errors that were discovered through the cognitive interview qualitative analysis to be considered for learning progression reconciliation and later assessment item writing. See Figure 15 for a single subcomponent example with details explained for each field and Appendix D for the full summary tables.

Pulling information from earlier subcomponents and similar codes led to some inferences about the interconnection of pre-requisite skills needed, that both informed the levels within microprogressions and synthesized descriptions. However, there were four subcomponents, one in the Within Objects TLG and three in Between Objects TLG that we did not fully synthesize and held for consultant support to better define the interconnections. Further investigation is needed to establish which KSAs are interconnected among these summaries definitively.

Figure 15

Bulleted Summaries for Shape Composition Task SR.A.3.b (2D)

ORIGINAL ESS & ITEMS THAT VARY	AXIAL CODES	INITIAL SYNTHESIZED DESCRIPTIONS	SYNTHESIZED DESCRIPTIONS	Questions/Rationale for changes	SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)
Compose a **two-dimensional composite shape** or a **three-dimensional composite figure** in more than one way (e.g., a hexagon can be composed of two trapezoids or six triangles). **Elements that varied:** **Two-dimensional composite figure** -Outline of all shapes are shown [KF-KB] -Shapes may share one side but not all outlines are shown [KT-1F] -Shapes may share more than one side but not all outlines are shown [1B-2T] **Three-dimensional composite figure** -All shapes can be seen [KF-1F] -There may be internal spaces or blocks that can't be seen but that are necessary to the structural integrity [1B-2T]	(2D.a) Single shape repeater within: used single shape type to compose hexagon [6 (0, 1, 5)] (2D.b) Single image repeater between: used same shape combination to compose multiple hexagons [4 (3, 1, 0)] (2D.c) All unique compositions: composed three unique hexagons [4 (1, 3, 0)]	2D.i. student composes composite shape using a single shape type and repeats the image between composites (2D.a) 2D.ii. Student composes composite shape using multiple shape types and repeats the image between composites (2D.b) 2D.iii. Student combines single shape composites and same image between composites (2D.a, 2D.b) 2D.iv. Student composes all unique composite figures (2D.c)	***Elements that varied not included (transformations)** 2D.i. Given a collection of shapes that can be composed into a given shape in multiple ways, student composes multiple identical composite shapes using a single shape type (2D.a) 2D.ii. Given a collection of shapes that can be composed into a given shape in multiple ways, student composes multiple identical composite shapes using different shape types (2D.b) 2D.iii. Given a collection of shapes that can be composed into a given shape in multiple ways, student composes multiple composite shapes (2D.c)	-Multiple students demonstrated a lack of positional language understanding when requesting to build the shapes "on" the purple hexagon; this may have compromised some student responses. -Could this not be assessed through the same item as A.3.b.; why is this after A.3.b?	(a) Lines up shapes around the shape designated to compose (no code - error?)

Note. This is the final summary for subcomponent SR.A.3.b(2D) within the composition/ decomposition core concept of the Within Objects TLG of the MMaRS SR LP. The 4th column from the left details the finalized synthesized description that incorporates elements that vary with codes based on student responses to the protocol item.

Conclusion & Next Steps

The purpose of the cognitive interviews was to provide evidence to empirically recover the spatial reasoning learning progression. We interviewed 23 students in Grades K-2 across two schools and retained 16 of the interviews for full analysis. We collected data related to the developmental appropriateness, ordering of the core concepts and subcomponents, students' conceptions, and the interconnectedness of student responses with KSAs. The data collection and analysis described in the current report are an important part of the validation process for the learning progression. In conjunction with quantitative analyses of the cognitive interviews (Tech. Rep. No. 20-08), expert reviews, and the SR teacher survey (Tech. Rep. No. 20-10), these data can inform the reconciliation of the learning progression.

Next steps include reconciling this analysis with the quantitative analysis of interviews and empirically recovering the learning progression. We will work with external reviewers for the reconciliation and to address lingering questions on the children's conceptions of spatial reasoning and interconnections of their demonstrated knowledge, skills, and abilities.

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Appendix A – Phase 1 Codebook & Literature

Codes	Definition	Example	Non-Example
Procedural Fluency	Demonstrating the ability to carry out an appropriate sequence of actions to solve problems, quickly and effectively. Procedural fluency is shown through a student constructing their response with meaning.	Correct Procedural Fluency: Student can compose a 2D puzzle. Incorrect Procedural Thinking: Student uses a guess and check method of placing and rearranging shapes, inefficiently and without deliberate procedural action.	
Conceptual Understanding	Utilizing mathematical concepts in a generalizable way and representing mathematical situations flexibly in different ways. Conceptual understandings answer how are mathematical concepts are used in novel situations.	Correct Conceptual Thinking: Student can use transformations (i.e., reflection, rotation, translation) to compose new shapes or figures. Incorrect Conceptual Thinking: Student builds figures unaligned with tasks and is unable to explain via words and/or gestures their thinking during construction.	
Strategic Competence:	The ability to maneuver between different strategies to solve complex problems. Strategic competence is shown through student products, including novel problem solutions and mathematical models.	Correct Strategic Competence: Student can build use their spatial knowledge to develop a map of the room. Incorrect Strategic Thinking: Student draws the view of the room from their current perspective.	
Adaptive Reasoning:	Implies use of some logical thinking to generalize or disprove a response as a student logically reasons about the relationships among concepts and situations. Adaptive reasoning often stems	Correct Adaptive Reasoning: Student can identify a shape as a hexagon, even though it may be irregular because it still has six sides (student is not distracted by the sides not being equal). Student is generalizing that all six-sided shapes are hexagons.	

	from careful consideration of alternatives.	Incorrect Adaptive Reasoning: Student counts the sides of a shape to determine its name, but because the shape (hexagon) is irregular, the student states that this is not a hexagon because it is irregular/doesn't fit the student's current definition.	
Language	We will be looking for correct Mathematical (Spatial, Numerical, Geometric) language.	Correct Spatial Language: To the right/left of an object. Behind/Above the Barn etc. Incorrect Spatial Language: Here, There etc.	
No Explanation	Instead of providing any reasoning, student explicitly denied to provide reasoning or admitted that he/she does not know to reason.	1. I do not know. 2. Student shrugs. 3. My teacher/parent told me.	1. So farmer can get it when pig is ready.
Tangential	Instead of providing any reasoning, student talked about non-mathematical reasoning or irrelevant topics BUT student did not explicitly denied to provide reasoning.	1. So farmer can get it when pig is ready. 2. It looks like a Bolivian flag	1. This looks like the roof tops/Mountain. 2. I do not know.

Additional literature review

	Coder 1	Coder 2
Conceptual Understanding/ Reasoning/ Thinking	• Hiebert and Lefevre (1986) defined conceptual knowledge as a "knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (pp. 3-4).	• A conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework also provides anchoring for skill acquisition. Skills can

	• Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know o A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. ▪ (The Strands of Mathematical Proficiency _ Adding it Up: Helping Children Learn Mathematics). • Use of a conceptual reasoning strategy implies an inherent understanding of numbers and their relationships (Crawford 2018)	be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understandings also supports the development of problem solving. (NCTM, 1989, p. 17) • We define conceptual knowledge as implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain. This knowledge is flexible and not tied to specific problem types and is therefore generalizable. Furthermore, it may or may not be verbalizable. (Rittle-Johnson, Siegler, & Alibali, 2001, pp. 346-347) • Conceptual understanding is recognizing and understanding the core underlying ideas of a subject such as the relationships and reasons that underlie the math problems in a certain area. It is knowledge that is generalized to a specific area and underlying core principals and does not necessarily refer to a specific set of problems. Thus, conceptual knowledge can be implicit or explicit and applied flexibly. (Burns et al., 2015, p. 52) Sparks Definition: Conceptual understanding is utilizing the mathematical concepts in a **generalizable** way. How are mathematical concept used in novel situations?

		Example: Student can use transformations (i.e., reflection, rotation, translation) to compose new shapes or figures.
Procedural Understanding/ Thinking/ Fluency	• According to Bergqvist (2005, 2006), algorithmic reasoning is based exclusively on applying a procedure or a memorized response. • Hiebert and Lefevre (1986) defined Procedural knowledge as a "Knowledge consists of rules or procedures for solving mathematical problems. It is also a familiarity with the individual symbols system and with the syntactic conventions for acceptable configurations of symbols. • Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. o Connected with procedural fluency is knowledge of ways to estimate the result of a procedure. ▪ (The Strands of Mathematical Proficiency _ Adding it Up: Helping Children Learn Mathematics)	▪ We define procedural knowledge as the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and therefore is note widely generalizable. To assess procedural knowledge researchers typically use routine tasks, such as counting a row of objects or solving standard arithmetic computations. (Rittle-Johnson, Siegler, & Alibali, 2001, p. 346) ▪ Procedural fluency is the knowledge of rules, symbols, and sequence of steps required to complete math problems, and is demonstrated by students to quickly retrieving correct answers and/or proficiently completing the algorithm to compute mathematical operations. (Burns et al., 2015, p. 52) Sparks Definition: Procedural fluency is the ability to carry out a sequence of actions to solve problems, quickly and effectively. Example: Student can compose a 2D puzzle
Strategic Competence	• Strategic competence refers to the ability to formulate mathematical problems, represent them, and solve them. ▪ (The Strands of Mathematical Proficiency _ Adding it Up: Helping Children Learn Mathematics) • Reasoning Strategies: It can also be Conceptual Strategy or Procedural Strategy	▪ Strategic competence refers to the capability of students to solve multi-step, non-routine problems and to extend this to the formulation and tackling of problems from the real world. The *products* that students may produce may therefore be designated as problem solutions and mathematical models. (Swan, 2014, p. 15).

	o Partial Strategy, Unfounded Strategy, and No Response/Reasoning Strategy ▪ (Crawford et al., 2018)	▪ Strategic competence for teachers includes the ability to (a) formulate, represent, and solve problems; (b) model mathematical ideas; and (c) demonstrate representational fluency, that is, the ability to translate and connect within and among multiple representations with accuracy, efficiency, and flexibility. (Suh & Seshaiyer, 2014, p. 78) Sparks Definition: Strategic competence refer to the ability maneuver between different strategies to solve complex problems. Example: Student can build use their spatial knowledge to develop a map of the room.
Adaptive Reasoning	• Adaptive reasoning refers to the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of *how to justify the conclusions*. • (The Strands of Mathematical Proficiency _ Adding it Up: Helping Children Learn Mathematics) • *I see Adaptive Reasoning and Conceptual Strategy (a category of Reasoning Strategies) (Crawford et al. 2018)*	• Adaptive reasoning is loosely defined as the capacity for logical thinking and the ability to reason and justify why solutions are appropriate within the context of problems. That are large in scope. (Ostler, 2011, p. 17). • Children use adaptive reasoning when they bring together facts, procedures, concepts, and solution methods to make sense of a mathematical problem. They begin to reason in this way when they find examples that satisfy generalizations and disprove conjectures through counter examples. (Herbert et al., 2015, p. 29) Sparks Definition: Adaptive reasoning implies that use of some logical thinking to generalize or disprove.

		Example: Student can identify a shape as a hexagon, even though it may be irregular because it still has six sides (student is not distracted by the sides not being equal). Student is generalizing that all six sided shapes are hexagons.

Appendix B – A Priori Coding Audit Trail

Protocol Item	Question/Comment	Response	Decision
SR.A.1.a	1: Are we going to chunk multiple question responses all together or by question?	discussed as a group	When multiple reasoning questions exist code as single chunks for each question
SR.A.1.a	3: If initial questioning yields one code, but then additional probing leads to a different code, where does it go?	discussed as a group	Look at the whole picture, code to the deepest reasoning level (e.g. if it started as no reasoning and ended at mathematical reasoning) code to mathematical reasoning. Include the no reasoning chunk in the mathematical reasoning so that we can see the full picture.
general	1: How do we ensure that the student ID is included in each chunk?	discussed as a group	We need to ensure that within every chunk the SID is included. When trimming data, ensure that you don't trim the SID out of the chunk.
SR.A.1.a	2: We talked about how colors were non defining attributes but that we were going to keep it in mathematical reasoning because we didn't know if we would have groups of kids that used non mathematical attributes, but this one "because green and blue make yellow" seems different. I coded this tangential.	1: Kindergarten Math TEKS do not detail colors as a mathematical attribute and I think mixing colors would fall into fine arts or science. If matching colors is the strategy, that is sorting, and making things the same or different in the group which would be mathematical. I coded these tangential.	Decision: In this example it's not a comparison/sorting strategy so it would be tangential we all agree. Update Codebook under example "because green and blue make yellow"
SR.A.1.a	1: This is again related to the color issue. The child puts it in the group because it's "colorful" but didn't connect it as an attribute, more so just generalized a colorful group. I coded this tangential but is it just	2: I coded as mathematical in that the child seemed to focus on matching. 3: "Little colorful one don't match" I couldn't code this statement.	Decision: If the transcript/non-verbal does not have enough information to put it in one of the codes, the information will be added to the audit trail and the item will NOT be coded. We will track these over time but not code them and we need to investigate when we would make a call to

 Note. A Priori coding audit trail for Within Objects 1.a. only. Coder names blinded to numbers 1, 2, and 3.

	incorrect reasoning but not tangential?		add additional info to the non-verbal transcript. Annotations? Implication?
SR.A.1.a	3: if the final answer is "I don't know" after additional probing, does it negate other previous reasoning the interviewer was trying to dig into? I coded as no reasoning.	2: I don't think so, because of the whole response. I coded as mathematical reasoning because of the child saying "this thing looks like that" but did put the "I don't know part with it"; 1, 2, 3: discussed	In whole group discussion we went back to line 3 in audit trail and determined that we will stick with the "deepest level of reasoning" so this was agreed as a mathematical reasoning code RP/QH changed code from No reasoning
SR.A.1.a	1: Interviewer asked another question about similarity that was not a part of the protocol and didn't relate to the last reasoning question asked, but connects back to the rectangle question within this child's overall reasoning. Should it be coded still? I coded it as a separate chunk and as mathematical reasoning	2: the child is still unable to say why they are the same - this needs to be captured, but I did not code 3: Interviewer went back to a question, how to code these?	Decision: even if it is an "extra" question. Code it.
SR.A.1.a	1: "This one is too small for this one and this one" I couldn't code this one.	discussed as a group	3: ok with coding to mathematical reasoning; did discuss that there may be times with a picture or video could help when the non-verbal isn't clear. See Row 6 about protocol for this
SR.A.1.a	In the chunk of text we all selected starting with "here's another shape" the student ID wasn't included.	discussed as a group	We decided to add extra text around it to ensure the ID was included; highlight the unnecessary text and add an annotation that says, "This text is only included for ID Number" Agreed that if we are concerned about the amount of text that is extra, we should add to the audit trail and discuss

 Note. A Priori coding audit trail for Within Objects 1.a. only. Coder names blinded to numbers 1, 2, and 3.

Appendix C – Reliability Data for a priori coding

Within

Protocol Item	Code (Reasoning)	Kappa	NVivo Agreement	M & H Agreement Needed (0=no; 1 = yes)	Number of Agreements (A)	Number of Disagreements (dA)	M & H Agreement =A/(A+dA)	Debrief Required (0=no; 1 = yes)
A.1.a	Mathematical Reasoning	0.95	99.66	0				0
	No Reasoning	0.93	99.99	0				0
	Tangential Reasoning	0.85	99.94	0				0
A.1.b (2D)	Mathematical Reasoning	0.84	99.58	0				0
	No Reasoning	0.68	99.86	1	4	3	57%	1
	Tangential Reasoning	0.11	99.72	1	1	1	50%	1
A.1.b (3D)	Mathematical Reasoning	0.85	99.63	0				0
	No Reasoning	0	99.97	1	1	0	100%	0
	Tangential Reasoning	0	99.84	1	0	2	0%	1
A.1.c (3D)	Mathematical Reasoning	0.93	99.79	0				0
	No Reasoning	0	99.98	1	0	1	0%	1
	Tangential Reasoning	0	99.94	1	0	2	0%	1
A.2.d (3D Figure) Task 1	Mathematical Reasoning	0.96	99.81	0				
	No Reasoning	1	100	0				
	Tangential Reasoning	0	99.98	1				

Protocol Item	Code (Reasoning)	Kappa	NVivo Agreement	M & H Agreement Needed (0=no; 1 = yes)	Number of Agreements (A)	Number of Disagreements (dA)	M & H Agreement =A/(A+dA)	Debrief Required (0=no; 1 = yes)
A.2.d (3D Figure) Task 2	Mathematical Reasoning	0.21	99.63	1	7	0	100%	0
	No Reasoning	1	100	0				
	Tangential Reasoning	1	100	0				
A.2.d (3D Figure) Task 3	Mathematical Reasoning	0.97	99.93	0				
	No Reasoning	0.42	99.97	1	1	0	100%	0
	Tangential Reasoning	1	100	0				
A.2.d (3D Figure) Task 4	Mathematical Reasoning	0.96	99.88	0				
	No Reasoning	1	100	0				
	Tangential Reasoning	1	100	0				
A.2.d (3D Figure) Task 5	Mathematical Reasoning	0.99	99.97	0				
	No Reasoning	1	100	0				
	Tangential Reasoning	1	100	0				
A.2.e (Fold & Punch) 3D	Mathematical Reasoning	0.95	99.84	0				
	No Reasoning	0	99.95	1	0	1	0%	1
	Tangential Reasoning	1	100	0				
A.3.a (3D)	Mathematical Reasoning	0.95	99.71	0				
	No Reasoning	0.77	99.91	1	1	0	100%	0

Protocol Item	**Code (Reasoning)**	**Kappa**	**NVivo Agreement**	**M & H Agreement Needed (0=no; 1 = yes)**	**Number of Agreements (A)**	**Number of Disagreements (dA)**	**M & H Agreement =A/(A+dA)**	**Debrief Required (0=no; 1 = yes)**
	Tangential Reasoning	1	100	0				
A.3.c (2D)	Mathematical Reasoning	0.99	99.88	0				
	No Reasoning	1	100	0				
	Tangential Reasoning	1	100	0				

Between

Protocol Item	Code (Reasoning)	Kappa	NVivo Agreement	M & H Agreement Needed (0=no; 1 = yes)	Number of Agreements (A)	Number of Disagreements (dA)	M & H Agreement =A/(A+dA)	Debrief Required (0=no; 1 = yes)
SR.B.5.a	Mathematical Reasoning	0.85	99.54	0				
	No Reasoning	0.61	99.89	1	3	2	60%	1
	Tangential Reasoning	0	99.98	0	0	1	0%	1
SR.B.6.c	Mathematical Reasoning	0.94	99.83	0				
	No Reasoning	0.96	99.98	0				
	Tangential Reasoning	0.99	100	0				
SR.B.6.f	Mathematical Reasoning	0.94	99.94	0				
	No Reasoning	0	99.97	1	0	1	0%	1
	Tangential Reasoning	1	100	0				
SR.B.7.d	Mathematical Reasoning	0.97	99.92	0				
	No Reasoning	0.83	99.98	0				
	Tangential Reasoning	1	100	0				

Appendix D – Bulleted Summaries

A bulleted style summary was created with axial codes of student reasoning and synthesized descriptions as related to the skill. This summary also included student misconceptions and errors that were discovered through the cognitive interview qualitative analysis to be considered for learning progression reconciliation and later assessment item writing. See Figure X for visual example, with details explained below for each field.

Naming Conventions. All naming conventions for the core concept name and number were included for continuity across documents, with an added field for a short description of the subcomponent. In this example, the subcomponent descriptor became “sorting”.

Original subcomponent Statement and Elements That Vary. The subcomponent statement of student actions was included from the detailed learning progression descriptions, including the target grade band and developmental level. All elements that varied within the subcomponent based on hypothesized developmental appropriateness were included, delineated by grade band and level for further granularity in description.

Open Coding Themes. Synthesized themes captured in open coding were detailed with refined names. Some direct student examples were included if necessary to illustrate what students had done as reasoning. These codes served as the axial codes that aligned with all other steps in the bullet summaries.

Subcomponent Synthesized Description. In two steps, the lead coder first created synthesized descriptions of student actions that aligned with increasingly complex or mature ways of thinking about the given construct. Some skills evolved more linearly in progressions while others may develop simultaneously, while still others were not developmental in nature and were errors in student thinking or misconceptions that required academic feedback to facilitate student growth. For each synthesized statement, final codes from the codebook were aligned and included to inform the statement and support the ordering or concurrent skill development.

Student Misconceptions or Errors. For those codes that did not represent developmental steps in a microprogression of the subcomponent, coders analyzed student transcripts to find misconceptions or errors in thinking. These were characterized by individual or common examples from transcripts and were further detailed in the descriptors of each. They often aligned with the levels of thinking that were listed in the synthesized descriptions, which could be later used by practitioners to facilitate academic feedback or scaffolded practice.

Learning Progression SR: Spatial Reasoning

	Within Objects					
	ORIGINAL SUBCOMPONENT STATEMENT & ELEMENTS THAT VARY	**AXIAL CODES**	**INITIAL SYNTHESIZED DESCRIPTIONS**	**SYNTHESIZED DESCRIPTIONS**	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
Shape	Sort similar **two-dimensional shapes** and **three-dimensional figures** regardless of size, orientation, and dimensionality. [KF-KB] **Elements that varied:** **Shapes:** -Circles, squares, triangles, and rectangles (regular or irregular when applicable) [KF-KB] -Regular or irregular hexagons, rhombus, [KT-2F] -Regular or irregular quadrilaterals [2B-2T] **Figures**: -Cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids [KT-2F]	(a) Match shapes: Students matched shapes, without naming them, to create groups - "Looked like that one" or "This one is the same as the other" [8 (3, 2, 3) (b) Identify 2D shapes: Used formal or informal shape names to define groups, (e.g., diamond for rhombus) - Used 2D shape name, such as a rectangle, to explain grouping [12 (4, 4, 4)] (c) Apply 2D shape name to 3D solid: Categorized 3D solids with 2D shapes, but applied the name of a single face's 2D shape to the 3D solid as a whole - Triangular prism called triangle due to triangular-shaped face [10 (3, 4, 3)] (d) Recognize Dimensionality: Attended to 2D versus 3D explicitly. Reasoned about sorting on the basis that a solid is 3D and a shape is 2D [1 (0, 0, 1)] (e) Compare mathematical or non-mathematical visible attributes: Compared color or size of shapes, grouped blue shapes together based on color, or compared shapes using words such as "big", "little", or "middle" *All students who used size used at least one other theme* [8 (3, 2, 3)]	i.a. The student sorts shapes as two-dimensional versus three-dimensional. i.b. The student sorts two-dimensional shapes into groups by matching a shape to one or more others with the same or similar attributes ii. The student is able to sort similar two- and three-dimensional shapes regardless of size, orientation, and dimensionality.	**Elements that vary not included:** i. Given a collection of shapes and figures, the student sorts two-dimensional shapes into groups by matching a shape to one or more others with the same or similar attributes (a and e) ii.a. Given a collection of shapes and figures, the student sorts similar two-dimensional shapes and three-dimensional figures regardless of size, orientation, and dimensionality (e.g., places circle with cylinder). (b and c) ii.b. Given a collection of shapes and figures, the student sorts shapes as two-dimensional versus three-dimensional figures (d)	-Do we want ii.b to be a goal? If so, is that a separate sub component? It is currently listed as a strategy and misconception -How does that support spatial reasoning, and could this show area vs volume? -Rationale: In the correctness data we looked at all three of these statements. The qual data showed that 1 second grader was able to distinguish between 2D and 3D. -Concern: If teachers present 2D and 3D items together, will they understand that dimensionality is NOT the goal if we take it out. Is the "regardless of dimensionality" clear enough -After this item we start asking questions on 2D and 3D separately, so they can do that if they don't do ii.b	(1) Sorts based on dimensionality without naming and as deepest level of grouping (code: d) (2) Groups shapes on commonality rather than sorting based on attribute (code: a) (3) Attends to color (non-mathematical attribute) when sorting rather than focusing on a mathematical attribute. (code: e) (4) Sorts based on size, or reason using size comparatives within groups (code: e)

	Given the name of a t**wo-dimensional shape** or t**hree-dimensional figure**, recognize the **shape or figure.** **Elements that varied:** **Shapes:** -Circles, squares, triangles, and rectangles (regular or irregular when applicable) [KF-KB] -Regular or irregular hexagons, rhombus, [KT-2F] -Regular or irregular quadrilaterals [2B-2T] **Figures:** -Cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids [KT-2F]	(2D.a) Real life: compared shape to a real life object, or common image exterior to the item - Compared a triangle to a roof or slide [1 (0. 1, 0)] (2D.b) Transform shapes to enable recognition: Oriented shape to familiar mental image of what that shape is - Turned the mat if a triangle was not equilateral with a horizonal base [2 (1, 0, 1)] (2D.c) Use attributes: gave number of vertices or sides to describe why a shape is that shape - Triangles have three sides [7 (1, 2, 4)] (2D.d) Lack of attributes: Matched shapes without specifying reasons, and recognition lacked automaticity (is this a misconception or error?) - Triangles have points, they were the same, It's not round [9 (3, 4, 2)]	2D.i. Given a two-dimensional shape name, the student recognizes shapes without mathematical reasoning (2D.a, 2D.d) 2D.ii.a Given a two-dimensional shape name, the student recognizes shapes based on mathematical processes (2D.b) 2D.ii.b Given a two-dimensional shape name, the student recognizes shapes based on mathematical reasoning (2D.c)	**Elements that vary not included:** 2D.i.a Given a two-dimensional shape name, the student recognizes shapes (2D.a, 2D.d) 2D.i.b Given a two-dimensional shape name, the student recognizes shapes including those that are not in a familiar orientation (e.g., an equilateral triangle with the point down, or the square at a 45 degree angle) (2D.b) 2D.ii. Given a two-dimensional shape name, the student recognizes shapes and describes them using mathematical attributes (e.g., sides and vertices) (2D.c)	-Our current categories focus on the reasoning - how do we embed the elements that vary? Is this appropriate for within/shapes?	(1) Transforms shape to orient with current mental image of that shape (does not recognize regardless of orientation) (code: 2D.b) (2) Lacks automaticity in shape recognition (code: 2D.d)
		(3D.a) Real life: compared shape to a real life object, or common image exterior to the item - Compared a cylinder to a wheel and a cube to a house [6 (1, 2, 3)] (3D.b) Exterior forces for function: described what an exterior force could do to/with the shape, such as stacking, rolling, or building [4 (1, 1, 2)] (3D.c) Lack of dimension: named the 2D shape of a face or applied 2D shape name to 3D solid based on the shape of the face or holistic shape - cylinder as circle, cone as triangle [8 (2, 4, 2)]	3D.i.a Given a three-dimensional shape name, the student recognizes shapes without using mathematical reasoning (3D.a, 2D.e) 3D.i.b Given a three-dimensional shape name, the student recognizes shapes using extrinsic and/or intrinsic physical function of the shape (2D.b) 3D.ii. Given a two-dimensional shape name, the student recognizes shapes based on mathematical reasoning (2D.c, 2D.d)	**Elements that vary not included:** 3D.i. Given the name of three-dimensional figure, the student recognizes figures (3D.a, 3D.e) 3D.ii. Given the name of three-dimensional figure, the student recognizes figures and describes them using defining attributes (3D.d,)	-Is real life a developmental strategy or a misconception?	(a) Does not yet recognize dimensionality (code: 3D.c) (b) Applies 2D shape name to 3D figures (code 3D.c) (c) Lacks attribute use to describe why a 3D figure has a certain name (e.g., it's not round, it has points) (code 3d.e) (d) Uses extrinsic forces to describe intrinsic characteristics of the figure (code 3D.b)

	(3D.d) Use attributes: gave number of faces, edges, or vertices to describe why a shape is that shape - two flat sides (cylinder), like a triangle and has four sides (pyramid) [6 (3, 1, 2)] (3D.e) Lack of attributes: Matched shapes without specifying reasons, and recognition lacked automaticity (is this a misconception or error?) - Triangles have points, they were the same, It's not round				
SR.A.1.c. Name two-dimensional **shapes** and three-dimensional figures. [KT-2F] SR.A.1.d. Classify two-dimensional **shapes** and three-dimensional figures and describe their defining attributes. [1B-2T] **Elements that varied:** **Shapes:** -Circles, squares, triangles, and rectangles (regular or irregular when applicable) [KF-KB] -Regular or irregular hexagons, rhombus, [KT-2F] -Regular or irregular quadrilaterals [2B-2T]	(a) Use attributes: gave number of vertices or sides to describe why a shape is that shape - Triangles have three sides [5 (1, 2, 2)] (b) Lack of attributes: Matched shapes without specifying reasons, and recognition lacked automaticity - Triangles have points, they were the same, It's not round [7 (4, 1, 2)] (c) Transforms shape first to make a visual match for the current mental image of what that shape "should" look like [1 (1, 0, 0)]	i.a. Student names two-dimensional shapes without specific reasoning stated, based on matching physically or to a mental image (b) i.b. Student transforms the shape (e.g., turning the paper) to make it match their current mental image of what that shape is (c) ii. Student uses mathematical attributes to name and describe a shape based on its defining attributes (a)	**Elements that vary not included:** i. Given a two-dimensional shape, student names the shape (b) ii. Given a two-dimensional shape, student names and describes a shape based on its defining attributes (a)	- A.1.d was the reasoning question for A.1.c, so they were bundled together.	(a) Relies on non-defining attributes to classify (code: b) (b) Uses size or scale to classify (misaligned from subcomponent) (code: b) (c) Applies incorrect names to shapes (error only) (d) Transforms the shape to make it match their current mental image (e.g. turns the paper with a triangle to a typical orientation) (code: c)
SR.A.1.c. Name two-dimensional shapes and three-dimensional **figures.** [KT-2F] SR.A.1.d. Classify	a) Real life: compared shape to a real life object, or common image exterior to the item - Compared a cylinder to a wheel and a cube to a house [5 (2, 2, 1)]	i. Student matches shapes without providing mathematical reasoning or focuses on what exterior forces could do when acting upon objects without naming and providing classification	**Elements that vary not included:** I. Given a three-dimensional figure, student names the figure (a, b, e) ii. Given a three-dimensional	'- A.1.d was the reasoning question for A.1.c, so they were bundled together.	(a) Applies 2D shape name to 3D solids (code: c) (b) Uses extrinsic forces to describe intrinsic characteristics of the

	two-dimensional shapes and three-dimensional **figures** and describe their defining attributes. [1B-2T] **Elements that varied:** **Figures:** -Cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids [KT-2F]	(b) Exterior forces for function: described what an exterior force could do to/with the shape, such as stacking, rolling, or building [5 (4, 0, 1)] (c) 2D attributes: Applies 2D shape names holistically to 3D solid and/or Uses 2D shape name to describe one or more faces of the solid [5 (1, 2, 2)] (d) 3D attributes: Gives a count of the faces, edges or vertices of the solid, regardless of correctness or formal/informal language, or explicitly states that the solid is three-dimensional [4 (1, 0, 3)] (e) Lack of attributes: matched shapes without specifying reasons, and recognition lacked automaticity Triangles have points, they were the same, It's not round [12 (5, 3, 4)]	details (b, e) ii. Student names the 3D shape by the name of a single face (e.g., circle for cylinder) or holistic shape (e.g., triangle for cone) (c) iii. Student names a shape and defines its attributes through use of a real life example (a) iv. Student names 3D shapes and/or identifies the shape as three-dimensional, using attributes of the solid to classify it (d)	figure, student identifies the figure as three-dimensional, states its name, and describes it using defining attributes (d)		figure (code: b) (c) Does not express attributes for three-dimensional figures (code: e)

	Within Objects - Transformation					
	ORIGINAL ESS & ITEMS THAT VARY	**AXIAL CODES**	**INITIAL SYNTHESIZED DESCRIPTIONS**	**SYNTHESIZED DESCRIPTIONS**	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
Transformation	Recognize a two-dimensional shape that has been **translated**. [KF-1F] **Elements that varied:** **Translation** -Horizontal, vertical, and diagonal translations [KF-KB] **verb changed to demonstrate in leveled statements**	(a) Real life: described real life objects to explain similarity/ difference between shapes [4 (2, 2, 0)] (b) Action: demonstrated a translation using cut out [5 (0, 4, 10} (c) Other transformation: described a transformation other than a translation [4 (1, 2, 1)] (d) Translation: explicitly described a translation or slide [3 (1, 1, 1)] (e) Visual comparison: used visual matching or discrimination to describe similarity or difference between the shapes [23 (7, 11, 5)] ***higher counts due to MCQs**	i. Student explains similarity or difference between shapes through comparison to real life objects or through visual comparison or discrimination between the shapes (a, e) ii.a. Student acts out a translation with real objects, but does not explain via words or representational gesture (b) ii.b. Student describes transformations other than translations, using formal or informal language, to state the similarity or difference between the shapes (c) iii. Student explicitly states use of ta slide or translation, using formal or informal language or uses explanatory gestures, to indicate the transformation shown is a translation.	***Elements that varied not included** i. When asked to demonstrate a translation, student acts out a translation with real objects (b) ii. When asked to demonstrate a translation, student explicitly states use of ta slide or translation, using formal or informal language or uses explanatory gestures, to indicate the transformation shown is a translation. (d)	-Concerned about 50/50 chance to respond "correctly" due to original ESS; no cutouts were given during the incorrect translations -Is it appropriate to change the verb to demonstrate an understanding of a translation? - We did not include code (a) because it was just how children were comparing:K4 i. When shown two shapes that do or do not '(W1) Shape'!H9 a transformation, student explains similarity or difference between shapes through comparison to real life objects or through visual comparison or discrimination between the shapes (a, e) [students stated that the shapes looked like an L or a T; this was only stating "looks like"]	(a) relies on visual comparison to determine similarity or difference between the shapes, including size, rather than on the transformation to put the shape in the second position on the mat (code e) (b) applies alternative transformation to explain the translation or non-translation (code c)
	Recognize a two-dimensional shape that has been **rotated**. [KF-2T] **Elements that varied:** **Rotation** -Rotations of less than 45 degrees [KF-KB] -Rotations of more than 45 degrees [KT-2T] **verb changed to demonstrate in leveled statements**	(a) Real life: described real life objects to explain similarity/ difference between shapes [7 (3, 4, 0)] (b) Action - Rotation: demonstrated a rotation using cut out [14 (3, 7, 4)] (c) Action - Other transformation: demonstrated one or more transformations other than rotation using a cut out [2 (0, 0, 2)] (d) Other transformation: described one or more transformations other than a rotation [5 (2, 0, 3)]	i. Student explains similarity or difference between shapes through comparison to real life objects or through visual comparison or discrimination between the shapes (a, f) ii. Student acts out other transformation(s) to show the difference3 between the shapes (c) iii.a. Student acts out a rotation with real objects, but does not explain via words or representational gesture (b) iii.b. Student describes transformation(s) other than rotations, using formal or informal language, to state	***Elements that varied not included** i. When asked to demonstrate a rotation, student acts out a rotation with real objects (b) ii. When asked to demonstrate a rotation, student explicitly states use of a turn or rotation, using formal or informal language or explanatory gestures, to indicate the transformation shown is a rotation (e)	-Concerned about 50/50 chance to respond "correctly" due to original ESS; no cutouts were given during the incorrect translations -Is it appropriate to change the verb to demonstrate an understanding of a translation?	(a) Applies alternative transformations verbally or through action to explain the rotation or non-rotation (code: c, d) (b) Does not recognize a shape as actually different rather than rotated (e.g., a shape L versus T would not be recognized as different) (code: f) (c) Explains multiple transformations rather than a single when reasoning through the recognition (code: d)

		(e) Rotation: explicitly described a rotation or turn [15 (0, 11, 4)] (f) Visual comparison: used visual matching or discrimination to describe similarity or difference between the shapes [24 (11, 7, 6)] ***higher counts due to MCQs**	the similarity or difference between the shapes (d) iii. Student explicitly states use of a turn or rotation, using formal or informal language or explanatory gestures, to indicate the transformation shown is a rotation (e)			
	Recognize a two-dimensional shape that has been **reflected**. [KT-2T] **Elements that varied:** **Reflection** -Reflections across a vertical or horizontal axis [KT-1B] -Reflections across a diagonal axis [1T-2T] **verb changed to demonstrate in leveled statements**	(a) Action - Reflection: demonstrated reflection using cut out [6 (2, 1, 3)] (b) Action - Other transformation: demonstrated one or more transformations that included at least one transformation that was not a reflection [12 (2 , 8, 2)] (c) Other transformation: described one or more transformations other than reflection [10 (1, 6, 3)] (d) Reflection: explicitly described a reflection or flip [5 (1, 2, 2)] (e) Visual comparison: used visual matching or discrimination to describe similarity or difference between the shapes [26 (10, 7, 9)] ***higher counts due to MCQs**	i. Student explains similarity or difference between shapes by making a visual comparison of the shapes, including their orientation, without explaining the motion required to make the change between the shapes € ii. Student acts out other transformation(s) to show the difference between the shapes (b) iii.a. Student acts out a reflection with real objects, but does not explain via words or representational gesture (a) iii.b. Student describes transformation(s) other than reflections, using formal or informal language, to state the similarity or difference between the shapes (c)	***Elements that varied not included** i. When asked to demonstrate a reflection, student acts out a reflection with real objects, but does not explain via words or representational gesture (a) ii. When asked to demonstrate a reflection, student explicitly states use of a turn or reflection, using formal or informal language or explanatory gestures, to indicate the transformation shown is a reflection (d)	-Concerned about 50/50 chance to respond "correctly" due to original ESS; no cutouts were given during the incorrect translations -Is it appropriate to change the verb to demonstrate an understanding of a translation?	(a) Uses shape orientation rather than transformations to describe differences between shapes; (code: b, e) (b) Describes reflection across a diagonal axis as a rotation, translation, or combination of the two; (code: b, c) (c) Recognizes internal lines of symmetry or reflection but not external lines of reflection (code: d)
	Recognize **three-dimensional figures** that have been **rotated.** [1B-2T] **Elements that varied:** **Figures:** -Cubes, cones, cylinders, spheres,	(F.a) Size or visual comparison: used comparison of figure sizes or other visual discrimination, including color and figure name, to describe similarity or difference (F.b) Attributes: counted faces, edges, or other defining attributes when	i. Student explains similarity or difference between shapes by making a visual comparison of the shapes, including size, color or shape name (Shape.a) ii. Student compares shape attributes to determine if a shape has undergone a rotation or is a different shape	***Elements that varied not included** i. When asked to demonstrate a rotation of a three-dimensional figure, student compares attributes to determine if a figure has undergone a rotation or is a different figure than the first (F.b)	-Is there a difference between a three-dimensional rotation and reflection in standard geometrical figures? Code shape.d: students stated that the result was a "flip", but could they have just rotated around a different axis? With geometric shapes, it is not evident	(a) Attends to non-mathematical attributes of the shape to describe similarity or difference (e.g., color) (code: F.a) (b) Attends to non-defining attributes to describe similarity or difference (e.g., size, defining attributes)

	pyramids, prisms, trapezoids [KT-2F] **Rotation:** -Rotations of less than 45 degrees [KF-KB] -Rotations of more than 45 degrees [KT-2T]	comparing (F.c) Inferred rotation: used informal language and/or gesture to indicate an understanding of rotation without explicit explanation (F.d) Flip: Said "flip" to describe the change between figures (F.e) Rotation: explicitly stated rotate/turn or used representational gesture	than the first (Shape.b) iii. Student expresses the leading edge of understanding about 3D rotation through non-specific and/or informal language and/or gesture (Shape.c) iv. Student recognizes the rotation and describes the change using rotate, turn, or flip to describe the difference, with or without a representational gesture (Shape.d, Shape.e)	ii. When asked to demonstrate the rotation of a figure, student expresses the leading edge of understanding about three-dimensional rotation through non-specific and/or informal language and/or gesture (F.c) iv. When asked to demonstrate the rotation of a figure, student recognizes the rotation and describes the change using rotate, turn, or flip to describe the difference, with or without a representational gesture (F.d, F.e)	-Request consultant support -This item tended toward figure recognition, as a rotate or flip - did we need a different way to assess this? -Page 11 gives some specificity around why this is rotate only for three-dimensional: https://smu.app.box.com/file/675093043904 **-By only using geometric shapes, we confused the statement; perhaps use only real life objects?**	(code: F.a) (c) Uses language to indicate alternate transformations (code: F.c, F.d)
		(CF.a) Size or visual comparison: used comparison of shape sizes or other visual discrimination, including color and shape name, to describe similarity or difference (CF.b) Sleep: stated that one figure is laying down to sleep or being lazy (CF.c) Action: demonstrates a rotation around any axis, with or without a verbal description (CF.d) Inferred rotation: used informal language and/or gesture to indicate an understanding of rotation without explicit explanation (CF.e) Flip: Said "flip", turn, or rotate to describe the change between shapes or used representational gesture (CF.f) Structural change: verbally or physically manipulated a figure to state what change needed	i.a. Student explains similarity or difference between the figures by making a visual comparison of the figures (Figure.a) i.b. Student compares 3D orientation of the figures to one another, stating that one is laying down to sleep or standing (Figure.b) ii. Student expresses the leading edge of understanding about 3D rotation through non-specific and/or informal language and/or gesture (Figure.d) iii. Student acts out a rotation with a third figure to confirm or deny the second figure as a rotation of the first (Figure.c) iv.a.. Student recognizes the rotation and describes the change using rotate, turn, or flip to describe the difference, with or without a representational gesture (Figure.e) iv.b. Student describes and/or physically manipulates a structure to demonstrate a	***Elements that varied not included** i. When asked to demonstrate a rotation of a three-dimensional composite figure, student compares visually to determine if a composite figure has undergone a rotation or is a different figure than the first (CF.b) ii. When asked to demonstrate the rotation of a composite figure, student expresses the leading edge of understanding about three-dimensional rotation through non-specific and/or informal language and/or gesture (CF.c) iv. When asked to demonstrate the rotation of a composite figure, student recognizes the rotation and describes the change using rotate, turn, or flip to describe the difference, with or without a representational gesture (CF.d, CF.e)		(a) Applies alternate transformations to describe the change between the figure (code: CF.a) (b) Uses connection to real life to explain directionality of figure (e.g., stating that figures are sleeping, lazy, or watching TV) (code: CF.b) (c) Restructures figures non-congruently with the stimulus or describes changes that would not accurately iterate the initial figure (CF.f)

		to be made to show a rotation of the stimulus shape.	rotation from the stimulus figure to the response figure (Figure.f)			
	Recognize the three-dimensional result of folding a two-dimensional shape. [2F-2T]	(F.a) Angle: explains response based on the angle of the fold in comparison to the dotted line on the stimulus and/or the outline of the shape [4 (1, 1, 2)] (F.b) Area: explains response based on the area of the shape pre- and post-fold [3 (0, 1, 2)] (F.c) Corners & Edges: references where a corner or edge of the pre-fold shape falls on the post-fold shape [3 (0. 2. 1)] (F.d) Situational: relates the response to the situational story told (e.g., the dogs are playing together) [2 (1, 1, 0)]	i. Student uses a situational explanation without mathematical reasoning when recognizing the result of a shape folded (F.d) ii.a. Student attends to the angle of the resultant fold compared to the outline of the shape and response option with the dotted line on the original (F.a) ii.b. Student attends to the area of the shape pre- and post-fold to determine the correct response shape (F.b) ii.c. Student uses corners and/or edges of the original and/ or response shape to describe why one response option is correct (F.c)	***Elements that varied not included** i. When asked to recognize the result of a fold, student describes the angle of the fold compared to the outline of the shape (F.a) i.b. When asked to recognize the result of a fold, student describes the area of the shape pre- and post-fold (F.b) i.c. When asked to recognize the result of a fold, student describes corners and/or edges of the pre- and post-fold shape (F.c)	-For fold only, we asked students to determine what it looks like after a fold -Curious if this is indeed a three-dimensional representation; notes from LP development research: Folding abilities appear around the age of 5 and improve as students age. (Harris, Newcombe, Hirsh-Pasek, 2013) https://smu.app.box.com/file/263456953999	a) Uses non-mathematical rationale for selected response (e.g., the situation, dogs playing- could be due to item writing) (code F.d.) (b) Attends to features of responses that do not accurately show a fold (e.g., colors)
		(F&P.a) One hole: reasoned that a single punch made only one hole, despite folded paper [4 (2, 1, 1)] (F&P.b) Spacing: focused on the spacing between holes or holes and edges on response options [5 (2, 2, 1)] (F&P.c) Punch position: focuses on the location of the punch on the folded paper (e.g., middle, top, etc.) [8 (1, 4, 3)] (F&P.d) Two hole: reasoned that a single punch would make two [8 (0, 4, 4)]	i. Student uses a situational explanation without mathematical reasoning when recognizing the result of a shape folded (F.d) ii.a. Student attends to the angle of the resultant fold compared to the outline of the shape and response option with the dotted line on the original (F.a) ii.b. Student attends to the area of the shape pre- and post-fold to determine the correct response shape (F.b) ii.c. Student uses corners and/or edges of the original and/ or response shape to describe why one response option is correct (F.c)	Ask for consultant support because the codes are really related to assessment item, not the skill construct	-This was originally one item assessed two different ways; one wound up being the inverse of one another -In this second task for the subcomponent, we asked what the folded paper would look like when unfolded to a single layer	(a) Lacks recognition that a fold results in a double layer of paper and in turn two holes for each one punch; (b) Attributes punch position exclusively to its position on the folded paper, not accounting for reversing the fold (F&P.a)

Composition & Decomposition	Within Objects – Composition & Decomposition					
	ORIGINAL ESS & ITEMS THAT VARY	**AXIAL CODES**	**INITIAL SYNTHESIZED DESCRIPTIONS**	**SYNTHESIZED DESCRIPTIONS**	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
	Recognize the result of mentally **translating** two-dimensional shapes or three-dimensional figures together. [KF-1F] **Elements that varied: Translations** -Horizontal, vertical, and diagonal translations [KF-KB]	(2D.a) Corresponding shapes: compared corresponding parts of stimulus and response shapes [5 (2, 1, 2)] (2D.b) Orientation comparison: compared the orientation of the entire or parts of the response shape to the stimulus shape [11 (3, 3, 5)] (2D.c) Size comparison: compared stimulus and response shapes on size or length [8 (4, 2, 2)] (2D.d) Visual comparison: did not specify transformation or mathematical comparison between the shapes [5 (1, 3, 1)]	2D.i. Student provides non-specific rationale when recognizing the result based on an inferred visual comparison, often using language of "the same" (2D.d) 2D.ii. Student compares the orientation or size of corresponding parts between the stimulus and response (2D.a, 2D.b, 2D.c)	***Elements that varied not included** 2D.i. Student recognizes the result of translating two two-dimensional shapes together (2D.d) 2D.ii. Student recognizes the result of translating two two-dimensional shapes together and reasons by comparing orientation or size of corresponding parts (2D.a, 2D.b, 2D.c)	-Size comparison may have not really been a code, but instead the result of the item. -The item did not assess the elements that vary; it was a horizontal translation only.	(a) Lack of visual discrimination based on corresponding part sizes (code: 2D.c) (b) Selects non-congruent parts of corresponding shapes when making comparison (Code: 2D.a)
		(3D.a) Corresponding parts: compared corresponding parts of stimulus and response figures [10 (5, 1, 4)] (3D.b) Orientation comparison: described differences between the stimulus and response figures using directionality of parts or the whole response figure [10 (2, 3, 5)] (3D.c) Looks like: compared figure to a real life or exterior to the protocol item object [5 (1, 2, 2)] (3D.d) Visual comparison: did not specify	3D.i. Student uses a visual comparison only between the stimulus and response figures without specifying mathematical comparisons (3D.d) 3D.ii. Student compares figures, either before or after the translation, to real life objects (3D.c) 3D.iii.a. Student compares corresponding parts of the stimulus and response to recognize the result (3D.a) 3D.ii.b. Student compares the orientation of parts of the stimulus and response to recognize responses that do not reflect the translations (3D.b)	***Elements that varied not included** 3D.i. Student recognizes the result of translating two three-dimensional figures together (3D.d) 3D.ii. Student recognizes the result of translating two three-dimensional figures together and reasons by comparing orientation of corresponding parts (3D.a, 3D.b)	-The item did not assess the elements that vary; it was a horizontal translation only.	(a) Does not recognize when one part is in a different orientation between the stimulus and response (code: 3D.b) (b) Corresponds parts incorrectly or regardless of orientation (Code: 3D.a) (c) Identifies the response based on one solid being "the same" when the other is not (Code: 3D.c) (d) Attends to what a figure looks like or a real life comparison rather than focusing on the true result (code 3D.c)

		transformation or mathematical comparison between the shapes [4 (1, 2, 1)]				
	Compose a **two-dimensional composite shape** or a **three-dimensional composite figure** using **transformations** (i.e., translations, reflections, rotations, and combinations of these). [KF-2T] **Elements that varied:** **Two-dimensional composite shape:** -Outline of all shapes are shown [KF-KB] -Shapes may share one side but not all outlines are shown [KT-1F] -Shapes may share more than one side but not all outlines are shown [1B-2T] **Translations** -Horizontal, vertical, and diagonal translations [KF-KB] **Rotations** -Rotations of less than 45° [KF-KB]; Rotations of more than 45° [KT-2T] **Reflections** -Reflections across a vertical or horizontal axis [KT-1B]; Reflections across a diagonal axis [1T-2T] **Three-dimensional composite figure:** -All shapes can be seen [KF-1F] -There may be internal	(2D.a.i) Correct with no scaffolding: composed puzzle with no prompt from the interviewer (2D.a.ii) Correct with scaffolding: composed puzzle with a verbal or gestural prompt from the interviewer (2D.b): Lack of composition: attempted to fit all puzzle pieces in outline, but unable to complete due to composition/ decomposition error (2D.c): Outline function: arranged pieces using the outline to fit pieces or fill the outline ***Did not include numbers because counts varied by item complexity**	2D.i. Given the outline of a composite figure and shapes, student uses individual shapes trying to fill the outline (2D.c) 2D.ii.a. Student fits pieces together with one another when filling the outline (2D.c) 2D.ii.b. Student fits pieces with one another to fill the outline, but is unsuccessful due to composition error (2D.b) 2D.iii. Student composes puzzle accurately when given a prompt to attend to an individual shape or portion of the outline (2D.a.ii.) 2D.iv. Student composes puzzle accurately independently (2D.a.i)	***Elements that varied not included (transformations)** i. When given a puzzle outline with all shapes shown, student composes a two dimensional composite figure using transformations (2D.a.i & 2D.a.ii) ii. When given a puzzle outline with no internal shape outlines, student composes a two-dimensional composite figure using transformations (2D.a.i & 2D.a.ii) iii. When given a puzzle outline with no internal shape outlines, student composes a two-dimensional composite figure with shapes that require decomposition to correctly complete the puzzle (2D.a.i & 2D.a.ii)	-There is a large developmental gap between puzzles 1 and 2 in comparison to the gap between puzzle 2 and 3; could there be an Intermediary between puzzle 1 and puzzle 2 or do the puzzles need revision? We found a floor effect. -We wrote a microprogression in an AERA proposal that creates levels of completeness within each puzzle level: https://smu.app.box.com/folder/118097522901 -H7 aligns with the codebook and ideas that informed the microprogression; I7 aligns with the puzzle types	(a) Lacks automaticity of composition/decomposition of individual shapes, such that a hexagon is composed of two trapezoids or a rhombus is composed of two triangles (code 2D.b) (b) Focuses only on the way in which shapes fit together, neglecting how they fit with the outline to fill the composite (code 2D.c) (c) Focuses only on the way in which shapes fit with the outline, not with one another, to fill the composite (code:2D.c)
		(3D.a.i): Correct with no scaffolding: composed puzzle with no prompt from the interviewer (3D.a.ii): Correct with scaffolding: composed puzzle with a verbal or gestural prompt from the interviewer (3D.b): Lack of base level construction/alignment: Level 1 composition in response figure did not match stimulus (3D.c): Lack of vertical congruence: Response figure built with more vertical levels than stimulus	3D.i. Students constructs a figure taller or shorter than the stimulus figure (3D.c) 3D.ii. Student constructs a figure with base level misaligned to the stimulus that may or may not be attached (3D.b, 3D.d) 3D.iii. Student accurately constructs three-dimensional composite with scaffolding prompts (3D.a.ii) *this did not occur in sample 3D.iv. Student accurately constructs three-dimensional composite with no prompting (3D.a.i.)	***Elements that varied not included (transformations)** i. Given a stimulus figure with all figures visible, student constructs a three-dimensional composite figure (3D.a) ii. Given a stimulus figure with internal spaces or blocks that can't be seen, student constructs a three-dimensional composite figure (3D.a)	-Should we consider revisiting TOSA or request support from consultants to make better sense of levels? -Item writing: (3D.d): Detached: Response contained more than one figure - the figure could be made in different ways	(b) Aligns base as a mirror image to stimulus base (code: 3D.b, 3D.d) 3D.i. Students constructs a figure taller or shorter than the stimulus figure (code: 3D.c)

spaces or blocks that can't be seen but that are necessary to the structural integrity [1B-2T]	(3D.d): Detached: Response contained more than one figure ***Did not include numbers because counts varied by item complexity**				
Compose a **two-dimensional composite shape** or a **three-dimensional composite figure** in more than one way (e.g., a hexagon can be composed of two trapezoids or six triangles). **Elements that varied:** **Two-dimensional composite figure** -Outline of all shapes are shown [KF-KB] -Shapes may share one side but not all outlines are shown [KT-1F] -Shapes may share more than one side but not all outlines are shown [1B-2T] **Three-dimensional composite figure** -All shapes can be seen [KF-1F] -There may be internal spaces or blocks that can't be seen but that are necessary to the structural integrity [1B-2T]	(2D.a) Single shape repeater within: used single shape type to compose hexagon [6 (0, 1, 5)] (2D.b) Single image repeater between: used same shape combination to compose multiple hexagons [4 (3, 1, 0)] (2D.c) All unique compositions: composed three unique hexagons [4 (1, 3, 0)]	2D.i. student composes composite shape using a single shape type and repeats the image between composites (2D.a) 2D.ii. Student composes composite shape using multiple shape types and repeats the image between composites (2D.b) 2D.iii. Student combines single shape composites and same image between composites (2D.a, 2D.b) 2D.iv. Student composes all unique composite figures (2D.c)	***Elements that varied not included (transformations)** 2D.i. Given a collection of shapes that can be composed into a given shape in multiple ways, student composes multiple identical composite shapes using a single shape type (2D.a) 2D.ii. Given a collection of shapes that can be composed into a given shape in multiple ways, student composes multiple identical composite shapes using different shape types (2D.b) 2D.iii. Given a collection of shapes that can be composed into a given shape in multiple ways, student composes multiple composite shapes (2D.c)	-Multiple students demonstrated a lack of positional language understanding when requesting to build the shapes "on" the purple hexagon; this may have compromised some student responses. -Could this not be assessed through the same item as A.3.b.; why is this after A.3.b?	(a) Lines up shapes around the shape designated to compose (no code - error?)
	(3D.a) Base height: Lack of attention to the height of level 1 between stimulus and responses [2 (1, 0, 1)] (3D.B) Base split: Composition of two 2x2s as one 4x2 not recognized in response [2 (0, 2, 0)] (3D.c) All unique compositions: composed three unique figures [7 (2, 1, 4)]	3D.i. Student composes single iteration using two blocks of direct proportion to the stimulus (3D.a, 3D.b) 3D.ii.a. Student composes iteration with direct width relation of the base to the stimulus, recognizing that height of the 4x2 matters (3D.a) 3D.ii.b. Student composes iterations with direct height relation to the stimulus, recognizing that two 2x2 Duplo blocks is the same as one 4x2 (3D.b)	***Elements that varied not included (transformations)** 3D.i. Given a collection of figures that can be composed into a composite figure in multiple ways, student composes multiple identical composite figures using a single figure type 3.D.ii. Given a collection of figures that can be composed into a composite figure in multiple ways, student composes multiple identical composite figures using different figure types	-Codes based on picture evidence only and are potentially misconceptions -Would TOSA help to understand levels in construction?	(a) Constructs composite figures that do not replicate a stimulus figure (no code - error?) (b) Requires blocks of correct aerial view dimensions (e.g., 4x2) to build composite figures (Code 3D.a) (c) Requires blocks of correct height dimension to build composite figures (e.g., tall) (code: 3D.b)

		3D.iii. Student composes three unique composite figures, utilizing composition to equate for base height and width with the stimulus through use of base height and split compositions (3D.c)	3D.iii. Given a collection of figures that can be composed into a composite figure in multiple ways,, student composes multiple composite figures		
Find embedded figures within larger figures **Elements that varied: Embedded Figures** -The larger figure is a whole shape (e.g., rectangle) without any other line segments crossing through the shape [KT-1F] -The larger figure is a figure made up of line segments (e.g., triangle) with the possibility of other line segments crossing through the figure. [1B-2F] -The larger figure is a line segment group (e.g., lines formed as an "L") with the possibility of other line segments crossing through the figure [2B-2T]	(a) Matching: visually compared and matched parts of the embedded shape to the stimulus (included size, shape, and inferred visual matching) [20 (5, 7, 8)] (b) Attention to attributes: described or used the sides and/or vertices of the stimulus to explain the embedded shape [6 (2, 2, 2)] (c) Orientation of embedded shape: used language related to orientation on the page when reasoning about which lines to trace [10 (1, 4, 7)]	i. Student locates figure based on visual match of lines, sometimes stating the shape name or size (a) ii. Student locates figure through attention to shape attributes of stimulus found in response figure (b) c) Student uses positional or orientation related language to describe their process for determining which lines make the embedded feature	i. Given a shape with a single embedded shape and no connecting line segments student finds the shape ii. Given a shape with an embedded shape with connecting line segments, student finds the shape iii. Given an irregular shape with an embedded shape and connecting line segments, student finds the shape and identifies additional line segments	-Classification accuracy could be compromised in this analysis due to item writing and presentation; no students responded incorrectly to the entry level question -If the element that varies is the embedded figure, we have no data on task 1 - no child was presented it. -these were without photo evidence, revised based on elements that vary and currently sound more task oriented without the reasoning embedded from the codes. –Based on Task 2, students traced as expected. On Task 3, several traced all lines	(a) Only recognizes the embedded shape holistically when in familiar orientation (code b, c) (b) Attends to intersecting lines through the shape (level ii or iii only) (c) Only recognizes closed figures as embedded shapes (iii)
Recognize the two-dimensional cross section created by cutting a three-dimensional shape into two parts. **Elements that varied: Cross sections** -Congruent cross-sections [KT-1F] -Incongruent cross-sections [1B-2T]	(a) Congruent with base: referenced the shape of a base as the cross section (correct when congruent) [10 (3, 4, 3)] (b) Cross section: (b.i) Independent: used a cutting action to describe reasoning for the response shape [3 (0, 1, 2)] (b.ii) With scaffold: used a cutting action to describe reasoning for the response shape [3 (0, 1, 2)]	i. Student recognizes that a cross-section parallel to a base of a three-dimensional figure will be the same shape as that base (a) ii.a. Student lacks specificity in reasoning about incongruent cross-section recognition when provided a visual scaffold. (c) ii.b. Student uses a cutting action or verbalization to describe recognition of a cross section when provided a visual scaffold (b) iii.a. Student lacks specificity in	i. Recognizes the two-dimensional result of a cross section slice when congruent to a base (e.g., horizontal across cylinder to create circle) (a) ii. Recognizes the two-dimensional result of a cross section slice when incongruent with a base (e.g., diagonal across cone to create oval) (b)	is it an expectation that students can do this without a visual scaffold, or does that skill represent an additional level	(a) Names the resultant cross-section as the given three-dimensional figure (code: b) (b) Gestures or describes the cutting action but does not accurately identify the shape formed by the cut (code c)

	(c) Lack of specificity: (c.i) Independent: unable to verbalize mathematical or logical reasoning [2 (2, 0, 0] (c.ii) With scaffold: unable to verbalize mathematical or logical reasoning [1 (0, 1, 0] *used with and without visual scaffold	reasoning about incongruent cross-section recognition (c) iii.b. Student uses a cutting action or verbalization to describe recognition of a cross section (b)			
Decompose a **two-dimensional composite shape** in such a way that the parts can be used to create another given **composite shape**. **Elements that varied: Two-dimensional composite shape** -Outline of all shapes are shown [KF-KB] -Shapes may share one side but not all outlines are shown [KT-1F] -Shapes may share more than one side but not all outlines are shown [1B-2T] ***Change subcomponent to "decompose into new composite shape"**	(a) Fill: used shapes individually within the outline to fill the puzzle (b) Fit: relied on fitting shapes together make the puzzle, treating each shape individually (c) Build a picture: put shapes together to create a situational context or picture [1 (0, 0, 1)] (d) Decompose shape: decomposed one or more shapes into smaller shapes to use the same shapes from one picture to create another puzzle [4 (1, 2, 1)]	i. Given one composite figure, students place shapes inside the outline of a second composition (a) ii.a. Given one composite figure, students fit shapes together inside the outline of a second composition, but do not fill it (b) ii.b. Given one composite figure, students create a situation to justify the placement of shapes in the second composition (c) iii. Given a whole shape composed of two smaller shapes as part of a composite figure, students decompose the whole shape to fit given shapes into a new figure (d) i. Given a composite figure, student decomposes and uses shapes to compose a second figure with all outlines provided using transformations	***Elements that varied not included** ii. Given a composite figure, student decomposes and uses shapes to compose a second figure without internal outlines using transformations	-Because this protocol item was marked as 1T-2T, we only designed the task to meet the upper level of the elements that varied, therefore we do not have evidence of lower levels - the i statements are hypothesized based on previous composition *this is a direct continuation of 3B; critical aspect was the decomposition (e.g., if the child did not decompose a hexagon into two trapezoids and use them to compose the new puzzle as separate shapes) -Students were not able to "trade" in decomposition - we gave the two trapezoids as a hexagon, thus allowing the decomposition inherent to the protocol item. -Our codes fit the misconceptions, not necessarily the levels.	(a) Attempts to fit shapes onto the puzzle when building it through trial and error (code a, b) (b) Switches shapes when asked to decompose that did not accurately reflect a decomposition of the given shape (code d) (c) Does not break whole shapes into two smaller shapes/ does not decompose (code d) (d) Decomposes, but does not use decomposed shapes to create second composite shape (code a) (e) Relies on contextual situation when composing second figure over mathematical reasoning (code c)
Compose a **two-dimensional composite shape** and iterate it to compose another composite shape. **Elements that varied: Two-dimensional**	(a) Fit: relied on fitting shapes together to make the puzzle, treating each shape individually [11 (1, 5, 5)] (b) Composite shape: described the shape made by composition of	i.a. Students fit individual shapes together to fill the puzzle, transforming individual shapes as necessary (a, c) i.b. Students fit individual shapes together to fill the initial puzzle and recognize those pieces as a composite shape	i. When asked to compose a composite figure, student fits individual shapes together to fill the initial puzzle (b) ii. When asked to iterate a composite figure, student iterates the composite to fill the puzzle, transforming the	-Because this protocol item was marked as 2B-2T, we only designed the task to meet the upper level of the elements that varied,	(a) Does not recognize that the initial composition should be iterated to create the second composite shape (code a) (b) Transformed individual puzzle pieces

	composite shape -Outline of all shapes are shown [KF-KB] -Shapes may share one side but not all outlines are shown [KT-1F] -Shapes may share more than one side but not all outlines are shown [1B-2T]	puzzle pieces (parallelogram) (c) Other transformation: described use of transformations when completing the puzzle [6 (0, 2, 4)] (d) Iteration: copied and described use of iteration to complete the puzzle [8 (2, 1, 5)]	(b) ii. Students create a composite shape by fitting pieces together and then iterate the composite to fill the puzzle, transforming the composite as necessary (c, d)	composite as necessary (c, d)		to fill the puzzle rather than iterate composites (code a)

Learning Progression SR: Spatial Reasoning

Spatial Language	Between Objects – Spatial Language					
	ORIGINAL ESS & ITEMS THAT VARY	**AXIAL CODES**	**INITIAL SYNTHESIZED DESCRIPTIONS**	**SYNTHESIZED DESCRIPTIONS**	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
	Identify an object's spatial position in relation to other objects. [KF-1F]	(a) paper as reference: used the paper rather than objects on the paper to determine sticker placement and replicated [3 (1, 1, 1)] (b) image directionality reference: recognized "top" of paper based on the directionality of objects, but not placement in relation to the objects themselves [4 (1, 1, 2)] (c) correct placement: aligned with image directionality and object reference in sticker placement position [9 (4, 3, 2)] (d) spatial language: described sticker location using positional language related to other objects [9 (3, 5, 1)] (e) no spatial language: used non-position specific language [11 (5, 3, 3)] (f) mapping: language related to travel or a route to the sticker [3 (2, 1, 0)]	i. Student uses alternate reference, such as the paper as a whole or partial alignment via directionality of other objects, when identifying the placement of an object, and lacks specificity in positional descriptions (a, b, e) ii.a. Student uses other objects as reference when identifying the position of a given object, and describes using positional language (c, d) ii.b. Student uses other objects as reference when identifying position of a given object, and may use a connection to mapping as directions from one object to another (d, f)	i.a. Student uses other objects as reference when identifying the position of a given object, and describes using positional language (c, d) i.b. Student uses other objects as reference when identifying position of a given object, and may use a connection to mapping as directions from one object to another (d, f)		(a) Uses placement without relation to other objects (e.g., in CI item if assessor put it at the bottom right corner of the page, child also put on right corner of the page, and didn't use objects as support) (code: a, b) (b) States directionality of objects as top and bottom, but not left to right (code: b) (c) Does use positional language and instead uses words like here or there (code: e)
	Place an object when given **positional language**. [KF-1F] **Elements that varied: Positional Language** -Under, up, down, beside, between, in front of, behind, over, near, far, around, across, toward [KF-KB] -All of the previous	(a) alternate spatial language: used positional words not included in protocol language to reason about placement [3 (0, 1, 2)] (b) protocol spatial language: used positional words stated by the interviewer to describe placement [6 (1, 2, 3)]	i. Student uses non-descript positional language and may tell a story to describe the placement to describe placement (d, e) ii. Student describes placement of an object using either a limited or expanded set of specific	i.a. When given positional language and multiple objects, student places an object, reasoning about placement through context or using non-positional language (e.g., here, there) (d, e) i.b. When given positional language and multiple objects, student places an object, reasoning about	-This skill code in particular pulls in reasoning as part of the progression rather than focusing solely on the subcomponent. -There were 9 students who did not use spatial language to describe object placement for one or more objects - given the subcomponent, this should	(a) Places an object because there is "space" within the scene code: c, e) (b) Confuses right and left (code: b) (c) Demonstrates lack of receptive language through placement (e.g., child does not

	terms + left and right [KT-1B] **ADD IN OTHER LANGUAGE: ON**	(c) other objects: reasoned about placement of other objects on the mat, with or without spatial language [3 (0, 2, 1)] (d) situational: told a story about the farm [2 (1, 1, 0)] (e) no positional language: spatial language and position of objects was unclear after reasoning response [9 (5, 2, 2)]	positional language about the given object or other objects present (a, b, c)	placement using provided positional language (b) ii. When given positional language and multiple objects, student places an object, reasoning about placement using alternate positional language that was not given (a)	not be an M/E. -This functions much like A.1.c&d in that the reasoning question for B5b taps into B5c - it makes sense to double-barrel this content into reasoning into reasoning. -Make B5bi about placing an object, B5bii is about the description? How could we get to a child's reasoning of placement **without** asking them a question related to B5c?	place object in the given positional language spot) (code: b, e)
	Describe an object's location in relation to other objects using **positional language**. [KF-1B] **Elements that varied: Positional Language** Under, up, down, beside, between, in front of, behind, over, near, far, around, across, toward [KF-KB] All of the previous terms + left and right [KT-1B]	(a) directions with spatial language: described traveling using positional language [8 (4, 1, 3)] (b) static spatial language: described location of object as a static scene, no travel [2 (1, 1, 0)] (c) situational: told a story about the farm [3 (2, 0, 1)]	i. Student tells a story lacking specific positional language that would rely on visual cues in addition to verbal or written description of location (c) ii. Student provides a specific location in relation to one or more objects in the scene, either using static cues or describing how to travel to the given location (a, b)	i.a. When asked to describe the location of an object in relation to others, the student states the location using positional language with contextual reference (e.g., directions to the chicken, G2_319: "he'd... go around... or inside the barn then come out and he would see the chicken") (a) i.b. When asked to describe the location of an object in relation to others, the student states the static location using positional language (e.g., specifically where the object is located, G1_270: He's at the right of the barn") (b)	-Does the interconnection with mapping (e.g., travel with positional language) matter for this subcomponent? Or does it just point to interrelation that needs to be further explored? -The protocol item pushed toward directions rather than a static location of the chicken - this introduced skills not necessitated by the subcomponent	(a) Tells a story without using spatial language (e.g., when you walk over here, the chicken will stay there) (code: c) (b) Uses positional language incorrectly and describes an alternate location (code: a, b)

Understanding Models and Maps	Between Objects – Understanding Maps & Models					
	ORIGINAL ESS & ITEMS THAT VARY	**AXIAL CODES**	**INITIAL SYNTHESIZED DESCRIPTIONS**	**SYNTHESIZED DESCRIPTIONS**	**Questions/Rationale for changes**	**SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)**
	Recognize a three-dimensional representation (e.g., model) of a three-dimensional space. [KF-KB]	(a) scale: focuses on model scale or difference in size [10 (5, 3, 2)] (b) correspondence: uses one to one correspondence to compare models [6 (2, 2, 2)]	i. Student recognizes models as representative of a space, or not, based on the size of the model or objects it contains as same or different (a) ii. Student demonstrates one to one correspondence of objects and the space to correspond models as same or different (b)	Hold for consultant support - did we assess this subcomponent?	-These codes are based on two dioramas that are not larger than oneself. -There is a protocol issue found as a third code in the codebook (e.g., the rat in the smaller diorama was made of paper, the Lego man had missing legs) -Literature: https://smu.app.box.com/file/256009713364	(a) Assigns size as a characteristic of the model that makes one different from another and does not connect the smaller model as representative (b) Focuses on physical differences between the space and its model (c) Lacks receptive language knowledge necessary for understanding "represents"
	Scale distances and figures based on the size of the representation (e.g., place an object on a line based on the relative placement of an object on a smaller line). [KF-1B]	(a) compares to smaller stimulus: references the small scale stimulus to describe the placement of an object as the same on the larger scale response [11 (4, 5, 2)] (b) positional language: (b.1.) middle: uses positional language to describe the midpoint between two objects [9 (2, 3, 4)] (b.2.) other: uses positional language to describe a location other than the midpoint between two objects [2 (1, 1, 0)] (c) distance: uses language related to distance or spacing between objects (e.g., closer, further) [16 (6, 4, 6)]	i.a. student references smaller stimulus when placing an object on a larger scale response prompt without using positional or spatial language to describe the location (a) (this became a misconception) i.b. Student attends to space available for placement of an object on the response prompt, and may reference the smaller stimulus when choosing or describing placement location (a, d) (this is the same misconception) ii.a. student recognizes the midpoint between two items and replicates the placement, calling the placement "middle" (a, b.1.) ii.b. Student recognizes differential distances when an object is located in an alternate position from the midpoint between two items and uses positional language	i. Student scales distances and figures by recognizing the midpoint between two items and replicates the placement (a, b.i, c, d) ii. Student scales distances and figures by recognizing differential distances when an object is closer or further from given objects and replicates the placement (a, b.2, c)	-Are there other ways to assess this: The original statement without the (e.g.), did not provide end points; could this be done with just one static point (e.g., a boy and a ball) with one response choice scaled accurately; would this also assess the same skill, or somehow assess the subcomponent differently? -This is about a skill; do we need to put in language about positional language, distance, and space? We have evidence that children's use of each, but are unsure if our focus is on the skill or reasoning in this summary.	(a) Uses space without relative distance through scaling (code: c) (b) Positional language and distance language misaligned (e.g., if the object is closer or further from another object, it is also not in the middle) (code: b) (c) Not comparing two distances or figures when describing scaling (error only)

		other than "middle" or calls the location further from or closer to another object (a, b.2, c)			
Recognize a two-dimensional representation (e.g., model or map) of a three-dimensional space. [KF-1B]	(a) compares response to stimulus: direct comparison of response map to the scene, either static or describing changes [10 (3, 3, 4)] (b) distance: uses language related to distance or spacing between objects (e.g., closer, further) [7 (3, 2, 2)] (c) physical change description: describes changes needed to create an alternate map [8 (2, 3, 3)] (d) same: recognizes a map as representing the space but unable to state why [3 (1, 2, 0)]	i. student selects a map without stating reasoning why it represents the space (d) ii.a. student compares the stimulus to the response when selecting the representative map, and describes what would need to move to create an alternate map (a, c) ii.b. student uses language about the distance or spacing between objects in the space when selecting the map (b)	i. Student identifies a two-dimensional representation of a three-dimensional space without stating reasoning why it is representative (d) ii. Student describes a two-dimensional representation of a three-dimensional space using distance or comparison (a, b) iii. Student identifies and describes what would need to move in the three-dimensional space to create a given two-dimensional representation of a three-dimensional space (c)	-The reasoning question required expressive language, which led to codes a &b, but that is not the subcomponent; code c was elicited through a second reasoning question and not necessary for the skill -iii goes beyond the original subcomponent - is this developmentally appropriate? (e.g., do we need to analyze qual codebook data with correctness data to confirm that students who described did so correctly - if it was not correct, it was likely developmentally inappropriate)	(a) Selects a map without stating reasoning why it represents the space (code: d)
Creates a map to represent a three-dimensional space, larger than oneself, such as a classroom. [KB-2F]	(a) Map drawing skills: (a.1.) pre-mapping: includes items as floating without clear locations in the map [4 (4, 0, 0)] (a.2.) side view: draws items in model using a side rather than aerial view without correspondence to locations on the map [3 (1, 2, 0)] (a.3.) Partial aerial: demonstrates an aerial view for freestanding objects, but draws items from the wall as extending into the room [9 (1, 3, 5)] (b) Reasoning: (b.1) positional language: uses positional language (e.g., on, by, at) to describe placement of items on the map [2 (2, 0, 0)] (b.2) Object orientation:	i. Using pre-mapping skills, student reasons about object placement with positional language or space around individual items (a.1; b.1., b.3.) ii. Using a side view of the model for mapping, the student reasons space in the map or in the model to explain object placement (a.2.; b.3.) iii. Using partial aerial mapping skills, the student reasons about object placement using a combination of map to model correspondence with spacing between objects and positional language and object orientation of individual objects (a.3.; b.1, b.2, b.3, b.4)	i. Student demonstrates pre-mapping skills when creating a map to represent a 3D space, such as a classroom (e.g., items floating without a clear location) (a.1) ii. Student demonstrates the use of a side view when creating a map to represent a 3D space, such as a classroom (a.2) iii. Student demonstrates partial aerial map drawing skills when creating a map to represent a 3D space, such as a classroom (e.g., some, not all, objects drawn in static location) (a.3.)	-Aerial map drawing skill not included because we had no students demonstrate the skill -Items were on the floor and walls in the diorama. This may have impacted students' ability to draw the map using an aerial view; what is developmentally appropriate for students to map? -H7 embeds the reasoning, where-as I7 does not; what is our goal? -Original skill statement is about larger than oneself, this is not something we could do in our assessment setting and we used diorama. Our new descriptions match the diorama.	*(a) Representation of a 2D object from a vertical surface (e.g., door, board on wall) as extending from the wall onto middle of map *this may be an item problem, not a misconception* (code: a) *(b) objects not "grounded" in a location for pre-mapping skills observed (code a.1, b.iii) *(c) side view of objects as seen from on level in environment (code: a.2) *If we leave the i, ii, iii statements in column I, these are NOT M/E

	reasons about the direction objects are facing either on the map or in the model [3 (1, 1, 1)] (b.3) Space: attended to space available or not within the model or map [8 (5, 1, 2)] (b.4) Map to model: corresponded map to the model via verbal, action, or gestural reasoning (included making "the same") [9 (4, 1, 4)]				
Use a map to find locations of objects, including one's own location. [KT-2T]	(a) Space: attended to space available or not within the model or map [1 (0, 0, 1)] (b) Map to model: corresponded map to the model via verbal, action, or gestural reasoning (included making "the same") [11 (6, 2, 3)]	i.a. Student demonstrates use of spatial reasoning about the distance between objects when placing "self" on map. (a) i.b. Student states or gestures direct relation between objects on map with those in corresponding model when placing "self" on map. (b)	Hold for consultant support - did we assess this subcomponent?	-The item we gave students required that they locate an object in a 3D space and mark it on a self-created map; the task did not address or assess the subcomponent	(a) Relates individual characteristics of Lego person as different from self (code: b) (b) Places self on top of table rather than beside it (spatial language understanding or representation error)
Identify the grid reference system coordinates of an object on a grid. [1B-2T]	(a) no coordinates: uses positional language in comparison to the visual field or outline rather than using grid locations [2 (1, 1, 0)] (b) counts: counts squares or axes labels to explain an object's location [4 (1, 1, 2)] (c) coordinates: attends to the coordinate system labels on the axes when verbally or gesturally reasoning [7 (3, 2, 2)] (d) traces: uses fingers to trace from the object to the axes, without prompting [5 (3, 1, 1)] (d.1) traces when prompted [3 (1, 2, 0)]	i. student describes the location of an object without attending to the grid coordinates, using positional language in relation to the field (a) ii. Student counts squares or labels from the or origin (b) iii. student attends to and uses the coordinate grid system and its labels when naming the location, tracing fingers to the location, and/or counting to describe the space (b, c, d(d.1.))	Hold for consultant support	-The reasoning pulled in strategies for HOW they knew all codes; the subcomponent was originally identify and we questioned them to explain and make the reasoning visible through expressive language, but that is not the subcomponent; -H9 is pulling the reasoning, whereas the subcomponent did not ask for reasoning in F9 - which is the better/more appropriate way to order this?	(a) Counts grid squares from alternate edges rather than the marked origin (misconception around how a coordinate system works) (code: b, c) (b) Attends to a single axis only (e.g., either x or y axis) (code: c)
Describe and follow routes on maps. [1T-2T]	(a)Route drawing: (a.1) Straight path [1 (1, 0,	i. student describes a route of their choosing, with or	i. When given a map showing a route, student replicates and	-Replicating the route is one skill, describing the	(a) Does not attend to/understand the grid

	0)] (a.2) Stairstep [3 (2, 0, 1)] (a.3) Pre-route replicator [5 (1, 2, 2)] (a.4) Route replicator [7 (2, 3, 2)] (b) Reasoning: (b.1) context: puts traversing a route into a story to explain using real life comparison [3 (2, 0, 1)] (b.2) traces: traces along route with finger as additional action-based reasoning [13 (4, 5, 4)] (b.3) positional language directions: describes navigating the route using spatial language [6 (3, 1, 2)] (b.4) compares response to stimulus: direct comparison of response map to the scene, either static or describing changes [6 (1, 3, 2)] (b.5) compass: uses cardinal directions to explain the route, with or without reference to the printed compass [2 (0, 1, 1)]	without attention to the grid system, within the context of a story (a.1, a.2, a.3, b.1) ii. When provided a map showing a route, student replicates and describes the route, and uses verbal and/or action oriented reasoning when explaining the path (a.2, a.3, b.2, b.3) iii.Student replicates a stimulus route on a response map and uses verbal and/or action reasoning when explaining the path (a.4, b.2, b.3, b.4) iii.b. student uses all above reasoning and includes cardinal directions (b.5)	describes a route within the context of a story (a.1, a.2, a.3, b.1) ii.a. When given a map showing a route, student replicates and describes the route using positional language (a.2, a.3, a.4, b.3) ii.b. When given a map showing a route, student replicates and describes the route referencing back to the given map and tracing the route (a.2, a.3, a.4, b.2, b.4) iii.. When given a map showing a route, student replicates and describes the route using cardinal directions (a. 4, b.5)	route is another, and reasoning is a third; this became a triple-barreled reasoning response that assessed the subcomponent but added additional - Tracing might be a description of students' reasoning through non-verbal expression; does this align with gesture or action?	system upon which the route is drawn (code: a.1) (b) Does not replicate the route given (code: a.2., a. 3)
Identify the location of an object on a grid when given map coordinates. [2B-2T]	(a) traces: uses fingers to trace from the object to the axes, without prompting [6 (2, 2, 2)] (b) counts: counts squares or axes labels to explain an object's location [3 (0, 1, 2)] (c) coordinates: (c.1) single axis only [5 (3, 2, 0)] (c.2) use of full coordinate system: attends to the coordinate system labels on the axes when verbally or gesturally reasoning [10 (2, 3, 5)]	i. student counts squares on the grid separately from reading the cartesian coordinates (b) ii.a. student traces from axes to the location without specifically and/or correctly identifying the location using the coordinate system (a) iii. student uses the coordinate system, naming a letter and number from the x and y axes, and may pair this label with tracing and/or counting (a, b, c.2)	i. When given map coordinates in a grid coordinate system, student identifies the map coordinate location. (a, c.2)	-Codes a and c2 are two different strategies we did not feel the necessity to parse apart; all others are just errors..	(a) Uses coordinate system incorrectly, including not starting from the origin or creating diagonals as references (code: c) (b) Only references a single axis when identifying locations and reasoning about said location (code: c.1) (c) Identifies a location by counting squares (b)

Perspective Taking	Between Objects – Perspective Taking					
	ORIGINAL ESS & ITEMS THAT VARY	AXIAL CODES	INITIAL SYNTHESIZED DESCRIPTIONS	SYNTHESIZED DESCRIPTIONS	Questions/Rationale for changes	SUBCOMPONENT MISCONCEPTIONS (M) or STUDENT ERROR (E)
	Recognize the view from one's own perspective. [KF-KB]	(a) context: described relations of objects based on the way they were "facing" or what they were "looking at" [6 (3, 2, 1)] (b) positional language: described where objects were in relation to one another [6 (0, 3, 3)] (c) static: focused on only the scene or the image in reasoning [11 (5, 3, 3)] (d) correspondence: compared the image to the scene when describing positions [5 (2, 2, 1)] (e) Distance/space: focus of distance between objects or space around objects [2 (0, 1, 1)]	i.a. students used contextual clues of the scene and/or paired images to recognize and compare their current view (a) i.b. students used positional language related to an image or the scene apart from the other (b, c) i.c. students described the distance between objects in the scene and/or images (e) ii. student corresponds the scene and selected image, potentially using context and/or distance/ space (a, d. e)	i.a. When asked to recognize the view from their own perspective, student uses contextual clues of the scene and/or paired images to recognize and compare their current view (a) i.b. When asked to recognize the view from their own perspective, student uses positional language related to an image or the scene apart from the other (b, c) i.c. When asked to recognize the view from their own perspective, student describes the distance between objects in the scene and/or images (e)	-May need to consider comparing codes across items when checking for interconnections as these are not unique -Is correspondence an inferred skill through the selection?	(a) Lacks positional language or any other specificity about why an image is correct in description (code: b) (b) Does not locate own perspective in an image (code: c, d, e) (c) Provides no evidence of correspondence between scene and image in reasoning response (code: d)
	Understand that changes in perspective changes the view. [KF-KB]	(a) external perspective: recognizes the perspective as from the alternate location, without embodiment [5 (0, 3, 2)] (b) adapted internal perspective: uses positions of objects in the scene to describe the difference between current perspective, as though taking that alternate perspective [3 (1, 1, 1)]	i. When asked to take an alternate perspective from their own, student reasons about the similarities and differences without taking on the position of the alternate location (e.g., student identifies that the photo was being taken from an alternate location, but doesn't talk about the features of the scene) (a) ii. When asked to take an alternate perspective from their own, student mentally assumes that alternate perspective and reasons about how objects might look differently or their location in the image might change, based on the alternate perspective (b)	i. When asked to take an alternate perspective from their own, student reasons about the similarities and differences without taking on the position of the alternate location (e.g., student identifies that the photo was being taken from an alternate location, but doesn't talk about the features of the scene) (a) ii. When asked to take an alternate perspective from their own, student mentally assumes that alternate perspective and reasons about how objects might look differently or their location in the image might change, based on the alternate perspective (b)	-The reasoning for this question demonstrates another double-barrel reasoning=content for 7b to 7d; the reasoning question for b was the content for d.	(a) states that the perspective is the same due to inability to dissociate from current perspective (error only)
	Describe relative spatial positions of	(a) alignment comparisons: references the differences	i. student draws lines in the air to demonstrate relative	i. When reasoning about relative spatial positions of	-Protocol issue: there was a "grid" on the carpet in	(a) Compares distance of objects to self instead of

objects from different perspectives (e.g., "the chair would be closest to me if I stood over there"). [KB-1B]	between the alignment of objects as distance measures (e.g., "If I make a line right here [traces finger pointing from the ball towards the box], like imagination line, it won't connect") [5 (1, 2, 2)] (b) imaginary lines: draws lines from objects to the alternate perspective in the air [8 (4, 2, 2)] (c) distance: (c.1)uses spatial distance language (e.g., closer, further) when reasoning [7 (3, 3, 1)] (c.2) counts imaginary or real references to compare distance [2 (0, 0, 2)]	distances from the alternate perspective (b) i.a. student performs actions above and uses spatial distance language to describe the relative position (c.1) i.b. student performs actions and/or language above and counts references on floor (c.2) ii. student compares the alignment of objects to the alternate perspective, and may use imaginary lines and spatial language (a) (b & c)	objects from different perspectives, student draws lines in the air to demonstrate distances from the alternate perspective (b) ii.a. When reasoning about relative spatial positions of objects from different perspectives, student draws lines in the air to demonstrate distances from the alternate perspective and uses spatial language to describe the relative position (c.1) iii. When reasoning about relative spatial positions of objects from different perspectives, student infers alignment and may use or draw imaginary lines and spatial language to describe how objects are related to one another spatially (a, b & c)	one site; children used squares to count distances with rather than estimating distance differences; do we need to design and account for this random issue from which we found interesting data? The grid system connects to B.6.f and B.7.e. (e.g., student performs actions and/or language above and counts references on floor) -Is it a misconception or protocol problem that children did not take on the perspective of the stuffed animal?	other perspective (error only) (b) Attempts to create measurement system to quantify the relative distance (code: C2)
Recognize views from different perspectives (e.g., identifies what photo could be taken from a specific viewpoint of a concrete or pictorial representation of a three-dimensional space or object). [KT-2T]	(a) context: described relations of objects based on the way they were "facing" or what they were "looking at " (b) positional language: described where objects were in relation to one another (c) static: focused on only the scene or the image in reasoning (d) correspondence: compared the image to the scene when describing positions	i.a. students used contextual clues of the scene and/or paired images to recognize and compare their current view (a) i.b. students used positional language related to an image or the scene apart from the other (b, c) i.c. students described the distance between objects in the scene and/or images (c) ii. student corresponds the scene and selected image, potentially using context and/or positional language (a, c)	i.a. When asked to recognize the view from a perspective other than their own, student uses contextual clues of the scene and/or paired images to recognize and compare the proposed view (a) i.b. When asked to recognize the view from a perspective other than their own, student uses positional language related to an image or the scene apart from the other (b, c) i.c. When asked to recognize the view from a perspective other than their own, student describes the distance between objects in the scene and/or images (e)	-Codes for 7d replicated the findings from 7a, even though they were from different perspectives. B.7.a code e was not replicated as no students explicitly talked about distance or space. -7d had an (e.g.), but 7a did not - for consistency, either both or neither do.	(a) Recognizes view as different, but unable to identify differences (code: b, c) (b) Uses images only without reference to scene when selecting alternate view (code: d)
Construct a three-dimensional object or space given at least two images of	(a) Image/view (a.1) no image/no view: created scene without information from views (a.2) one view: noted use of	i. student composes scene with given objects without referencing images, but may use positional or spatial language to describe	i. When given three images from the top, side, and front of a three-dimensional space, student accurately constructs a scene by indicating use of	-The Lego mat had dots/knobs that students counted when determining space, which ties to a grid reference	(a) Believed each view was a different space rather than multiple perspectives (expressed desire to build three

	top, front, or side views. [1T-2T]	only one image to inform construction (a.3) two views: noted use of two images to inform construction (a.4) three views: used all three images to inform construction of the space (b) Positional language: described where objects were in relation to one another (c) correspondence: compared the image(s) to the scene when describing (d) distance/space: used spatial language to compare distances between objects	placements (a.1, b, d) ii. Student uses a single view to inform and reason about composition, and may use positional, correspondence, or spatial language to describe placements (a.2, b, c, d) III.a. Student uses two or three views to inform and reason about composition, and may use positional, correspondence, or spatial language to describe placements (a.3, a.4, b, c, d) iii.b. Student uses third view when introduced to review and revise composition (a.4)	all three views (a.4, c) ii. When given two images from the top, side, and/or front of a three-dimensional space, student accurately constructs a scene (code: a.3, c)	system - 6f and accidental 7c -What would happen if we put all of the construction items in a different place versus the recognizing items? -There were 2 tasks with different levels of difficulty, 2 versus 3 views as necessary; how does this impact our levels? -Codes b and d related to expressive language, not the subcomponent. Most of W3 was composing rather than recognizing -How does B7e correspond with W3, comp/decomp?	scenes) (code: a) (b) Used a single or no images in the construction, indicating a misconception about viewpoints showing the same object (code: a)