



SMU | ANNETTE CALDWELL SIMMONS
SCHOOL OF EDUCATION & HUMAN DEVELOPMENT

RESEARCH IN MATHEMATICS EDUCATION

Numeric Relational Reasoning: Cognitive Interview Protocol Development

RESEARCH IN
MATHEMATICS
EDUCATION

Numeric Relational Reasoning: Cognitive Interview Protocol Development

Eloise Aniaq Kuehnert • Tina Barton • Lindsey Perry • Leanne R. Ketterlin-Geller
Southern Methodist University

Spring 2020

Published by

Southern Methodist University
Department of Education Policy & Leadership
Simmons School of Education & Human Development
PO Box 750114
Dallas, TX 75275-0114
Contact information: rme@smu.edu

This research was supported by The National Science Foundation (Grant #1721100). Opinions expressed herein do not necessarily reflect those of The National Science Foundation or individuals within.

We would like to thank members of the RME team, Anthony Sparks, Robyn Pinilla, and Erica Simon, for their help finalizing this report. We also thank the students and their families who participated in this research.

Copyright © 2020. Southern Methodist University. All rights reserved. This publication, or parts thereof, may not be used or reproduced in any manner without written permission.

SMU will not discriminate in any employment practice, education program or educational activity on the basis of race, color, religion, national origin, sex, age, disability or veteran status. This document is available in alternative formats upon request.

Executive Summary

The purpose of this report is to describe the development of the interview protocols for the numeric relational reasoning (NRR) cognitive interviews. The cognitive interviews provide a key piece of evidence for empirically validating the hypothesized learning progressions for NRR of the Measuring Early Mathematical Reasoning Skills (MMaRS) project. This report details the development of the cognitive interview protocol and the supplemental fidelity of administration form. We used an iterative development process to create these interview protocols to verify that the knowledge, skills, and reasoning elicited during the interviews represented key components of the construct of numeric relational reasoning.

Table of Contents

Introduction	1
Research Questions	3
Cognitive Interview Protocol	3
Protocol Structure	4
Development Process	5
Refinement Process through Tryouts	6
Fidelity of Administration Form	7
Next Steps	8
References	9

Numeric Relational Reasoning: Cognitive Interview Protocol Development

Introduction

The Numeric Relational Reasoning (NRR) construct is defined as the ability to mentally analyze relationships between numbers or expressions, often using knowledge of properties of operations, decomposition, and known facts (Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Carpenter, Franke, & Levi, 2003; Farrington-Flint, Canobi, Wood, & Faulkner, 2007; Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Within the Measuring Early Mathematical Reasoning Skills (MMaRS) project, NRR was conceptualized to include three Targeted Learning Goals: Relations, Composition and Decomposition, and Properties of Operations. Figure 1 illustrates the structure of the NRR learning progression.

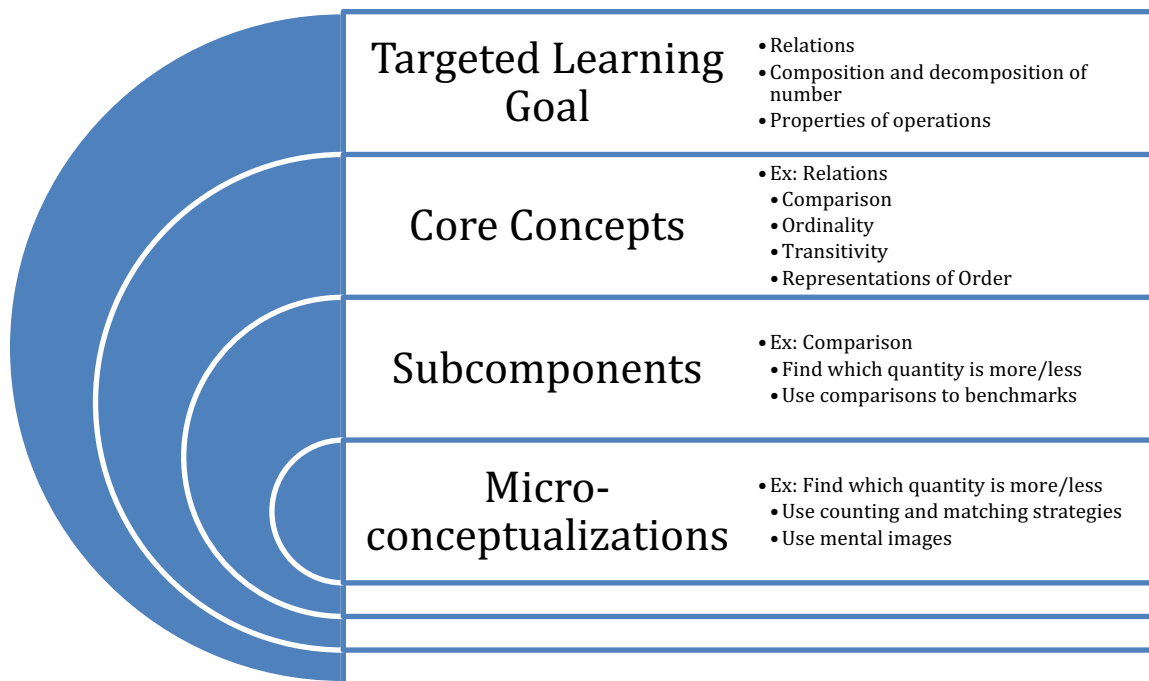


Figure 1. Structure of the MMaRS Learning Progressions.

Within each of the Targeted Learning Goals, multiple Core Concepts were articulated, as displayed in Figure 2.

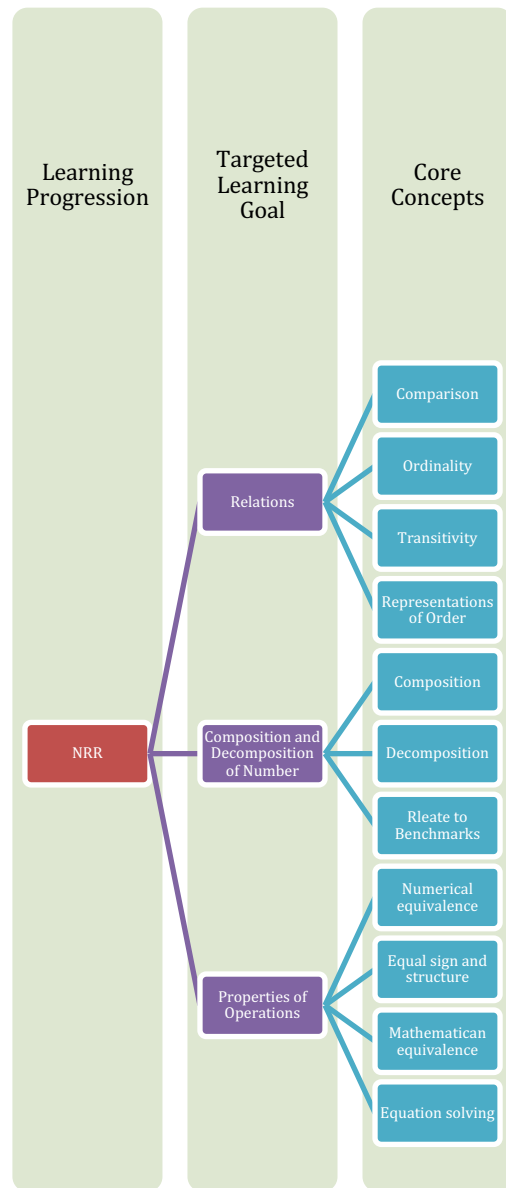


Figure 2. Core Concepts for each Targeted Learning Goal in the NRR Learning Progression.

For each Core Concept, detailed subcomponents were specified to provide greater specificity to the knowledge, skills, and reasoning underlying NRR. The overall NRR learning progression is intended to serve as a foundation for classroom assessment resources. Before relying on the learning progressions to inform instrument development, empirical evidence is needed to verify the learning progression. One source of evidence comes from cognitive interviews.

In conjunction with other sources of evidence including the NRR Teacher Survey (Sparks et al., 2020), the purpose of the cognitive interviews was to support or refute the conceptualization of content and ordering of the hypothesized learning progressions for NRR. By administering the cognitive interviews in a one-on-one setting with children in kindergarten through third grade, the MMaRS research team was able to gather evidence on prevalent student strategies used

within the NRR construct and evidence to evaluate the ordering and conceptualization of the hypothesized learning progression.

The purpose of this technical report is to describe the development of the interview protocols for the NRR cognitive interviews. In addition, we describe the development of the supplemental fidelity of administration form.

Research Questions

The cognitive interviews were designed to address four research questions related to gathering validity evidence for the NRR learning progression. These research questions rely on data collected directly from the cognitive interviews and from the fidelity of administration form. The research questions include:

- RQ 1: What level of evidence exists to confirm or disconfirm the ordering, content, and developmental appropriateness of the learning progressions?
- RQ 2: What are characteristics of kindergarten, first- and second-grade students' numeric relational reasoning within one-on-one cognitive interviews?
- RQ 3: What was the level of fidelity of implementation (fidelity by interviewer/observer)?
- RQ 4: What was the level of accessibility and comfort of students on all tasks within every learning progression?

Table 1 describes the data use by research question.

Table 1

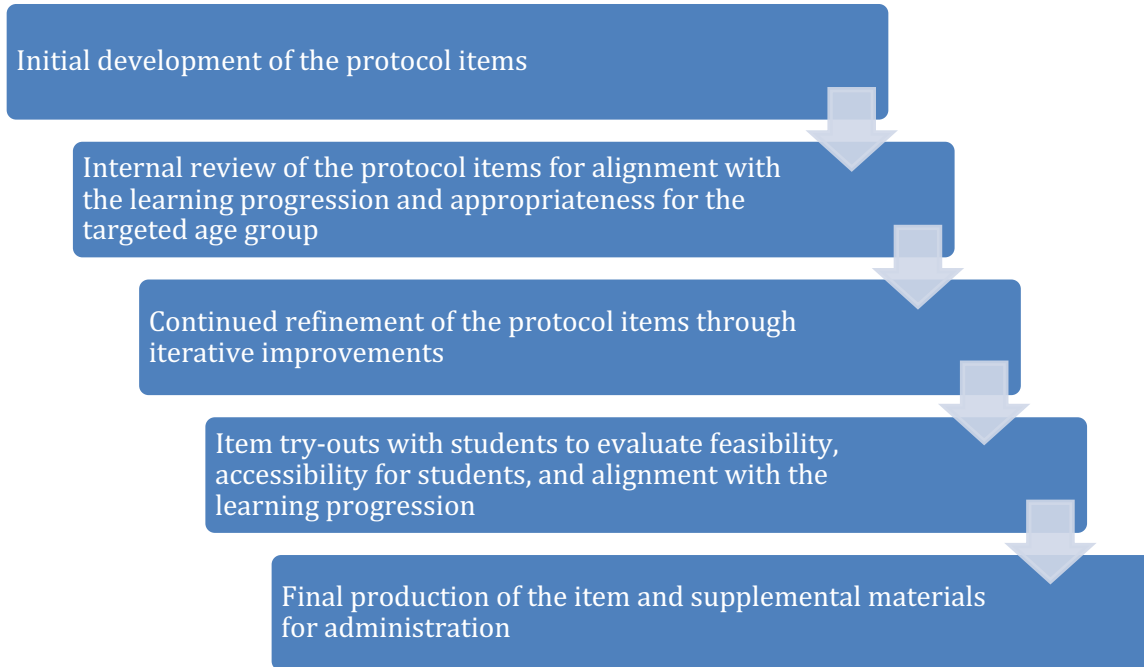
Data use by research question

Research Question	Data Use
1	Cognitive Interview Data
2	Cognitive Interview Data
3	Fidelity Data
4	Fidelity Data

Cognitive Interview Protocol

The purpose of the cognitive interviews was to evaluate the alignment between students' actual reasoning skills associated with NRR and the hypothesized reasoning outlined in the NRR learning progressions. Students' actual reasoning skills were elicited through the NRR cognitive interview protocol. Because of the importance of this protocol as a tool for gathering evidence, care was taken to verify that it was appropriately designed to measure the intended knowledge and skills.

In this section, we describe the development of the cognitive interview protocol including the protocol structure, development processes, and the refinement steps through student tryouts. For more information on the development of the NRR learning progression, refer to the NRR Learning Progression Development technical report (Tech. Rep. No. 20-02).



Protocol Structure

We based the NRR interview protocol on the structure of the NRR learning progression as depicted in Figure 1. The interview protocol was designed to assess each subcomponent for the core concepts in the NRR learning progression. The interview protocol was structured in a linear manner to progress through each core concept sequentially. Also, the subcomponents were assessed sequentially to progress from least to most sophisticated.

According to the hypothesized NRR learning progressions, some aspects of the subcomponents may vary depending on the grade level. For example, variations in the size of the number ranges were designed to account for the developmental appropriateness of students across grades K-2. The varied number ranges afforded students access to the content of associated activities regardless of student support level and grade level.

In Figure 3, Green cells [cells marked with an X] indicate number ranges hypothesized to a particular grade level (National Governors Association Center for Best Practices, Council of State School Officers, 2010). Interviewers intended to adapt to the child in the interview by going one number range above or below the hypothesized number range for the grade level indicated in the Figure. The only exception was the upper boundary of 199. The protocols were not designed to accommodate numbers above 199.

	0-5	0-10	0-19	0-50	0-99	0-199
K	X	X	X			
1			X	X	X	

2				X	X	X
---	--	--	--	---	---	---

Figure 3. Number ranges assigned to each grade level.

Development Process

The MMaRS research team developed the NRR cognitive interview protocols. We created tasks to coincide with each skill statement within the subcomponents of the NRR learning progressions. Each task included student materials and assessor materials.

The purpose of the tasks in the interview protocols was to assess student thinking on a particular skill and to develop an understanding of strategies, thinking processes, and reasoning that students use when solving problems related to these skills. Each task on the interview protocol consisted of (1) *content question(s)*, which elicited the student’s understanding of a specific subcomponent, (2) *scaffolding question(s)* to assist a student that might be challenged by processing the content question, and (3) *reasoning question(s)* to better understand the student’s thinking and reasoning that underlie their response to the content question. Each task was called a protocol item. Figure 3 shows one protocol item for skill NRR.B.6.d.

Time:		Content Questions		SID #:		6. Decomposition						
Skill NRR.B.6.d. (three parts: equipartitioning up to 25)	Actions	# Range	Question	Student Responses								
				# Range	Number	Number of Barns	Initially Correct	Self Corrects	Fair share	Other Strategy		
	<ul style="list-style-type: none"> Give child the first set of barns and the number of animals (individually laminated cows) based on their number range. Document child’s verbal response and/or actions. Regardless if child is correct, repeat with increased number of barns (if possible). For range 0-19, if 12 is difficult, then try 9 animals with 3 barns. Move through initial thinking and follow-up questions based on child’s response. Circle/highlight prompts and questions asked. 	0-5	Here are 3 farm animals that need to be placed into barns. There should be the same number of animals in each barn. Before you move the animals, how many animals do you think will be in each barn?	0-5	3	3						
		0-10	Here are 6 farm animals that need to be placed into barns. There should be the same number of animals in each barn. Before you move the animals, how many animals do you think will be in each barn?	0-10	6	3						
		0-19	Here are 12 farm animals that need to be placed into barns. There should be the same number of animals in each barn. Before you move the animals, how many animals do you think will be in each barn?	0-19	12	3						
		0-25	Here are 24 farm animals that need to be placed into barns. There should be the same number of animals in each barn. Before you move the animals, how many animals do you think will be in each barn?	0-25	24	3						
				Describe child’s verbal response and/or actions:								
				Describe child’s verbal response and/or actions:								
Initial thinking & scaffolding	<ul style="list-style-type: none"> If child is not responding, try asking questions to elicit a response or try rephrasing the initial question. Highlight question(s) asked. Skip gray rows if child begins to respond without prompting. 		How are you starting to think about this question?									
			What are you seeing in your head as you try to figure this out?									
			Please show me using pictures, words, or numbers what you are thinking in your head.									
			What questions do you have that would help you figure out how many animals might go in each barn?									

All Grades - Fall 2018

MMaRS – Numeric Relational Reasoning – Composition & Decomposition

18

Time:		SID #:		6. Decomposition								
Skill (Probing thinking/reasoning)	Actions	# Range	Question	Student Responses								
				# Range	Number	Number of Barns	Initially Correct	Self Corrects	Fair share	Other Strategy		
	<ul style="list-style-type: none"> If child is still not responding, allow them to move the farm animals. Circle/highlight questions asked. Regardless if child is correct, ask: 		You can move the animals if it helps. How many animals would you put in each barn?	0-5	4	2						
				0-10	6	2						
				0-19	12	2						
						3						
						4						
						6						
					9	3						
				0-25	24	2						
						3						
						4						
						6						
			How did figure out that there are __ animals in each barn?	Describe child’s verbal response and/or actions:								
			Can you prove that there would be __ animals in each barn by using these things [point to the barns and animal cards]?	Describe child’s verbal response and/or actions:								

The protocol items were created by two members of the MMaRS research team. Content, scaffolding, and reasoning questions were carefully designed to align with each subcomponent of the NRR learning progression.

Following the initial development, two other members of the MMaRS team completed a series of independent reviews of the protocol items to verify the alignment of the content, scaffolding, and reasoning questions with the hypothesized learning progressions and appropriateness for the targeted age group. All student materials needed to administer the protocol items were evaluated for usability. The assessor materials were evaluated for feasibility of scoring. After each review, the items were further refined until the team was confident that they adequately represented the targeted subcomponents of the learning progressions.

Refinement Process through Tryouts

To further evaluate the cognitive interview protocols, we conducted tryouts with children in the targeted age ranges. Four students in Grades K-3 (one student per grade level; three female and one male) participated in a tryout version of the cognitive interviews. One core concept from the NRR learning progressions was assigned to each student, with the exception of Relations, which was assigned to two students. Tryouts were video recorded for future use with training of interviewers and observers.

The initial developers of the protocol items served as the assessors for the tryouts. Observers were also present, and used a copy of the interview protocol to document timestamps, interviewer questions, and student responses. Timestamps were recorded to facilitate triangulation of student responses from observer notes and transcripts. If the interviewer rephrased a question, the observer noted the change in phrasing. Figure 4 also shows that the observer circled the second reasoning question and indicated that the second question listed was the first question asked.

Time: 4:38



5. Composition																																	
Skill	Actions	Questions	Student Responses																														
NRR.B.5.a. (single object)	<ul style="list-style-type: none"> Give child a pile of the same color counters. Document child's response (verbal and/or actions) 	<table border="1"> <thead> <tr> <th># Range</th> <th>Number in pile</th> </tr> </thead> <tbody> <tr> <td>0-5 ▲</td> <td>5</td> </tr> <tr> <td>0-10 ◆</td> <td>10</td> </tr> <tr> <td>0-19 ♥</td> <td>15</td> </tr> <tr> <td>0-50 ♣</td> <td>20</td> </tr> </tbody> </table> <p>What different numbers could you make using these counters? ✓</p> <p>You can make a lot of different values! Let's do the same thing with numbers. <i>You could ... however</i></p>	# Range	Number in pile	0-5 ▲	5	0-10 ◆	10	0-19 ♥	15	0-50 ♣	20	<table border="1"> <thead> <tr> <th># Range</th> <th>Child's Numbers</th> <th>Initially Correct</th> <th>Self Corrects</th> </tr> </thead> <tbody> <tr> <td>0-5 ▲</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0-10 ◆</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0-19 ♥</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0-50 ♣</td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Describe child's verbal response and/or actions: <i>C. asked - 10s</i> <i>10 fingers</i> <i>5:14</i> <i>6, 7, 8</i>  <i>5:43</i> <i>2+8 = 10</i></p>	# Range	Child's Numbers	Initially Correct	Self Corrects	0-5 ▲				0-10 ◆				0-19 ♥				0-50 ♣			
# Range	Number in pile																																
0-5 ▲	5																																
0-10 ◆	10																																
0-19 ♥	15																																
0-50 ♣	20																																
# Range	Child's Numbers	Initially Correct	Self Corrects																														
0-5 ▲																																	
0-10 ◆																																	
0-19 ♥																																	
0-50 ♣																																	
[Initial thinking & scaffolding]	<ul style="list-style-type: none"> Wait 7-10 seconds. If child is not responding, check if a tool would be helpful or show an example. If child is still unresponsive, then ask child to create a specific number. Highlight question(s) asked. Skip this section if child begins to respond without prompting. 	<p>Can you show me what you are doing in your head?</p> <table border="1"> <thead> <tr> <th># Range</th> <th>Question</th> </tr> </thead> <tbody> <tr> <td>0-5 ▲</td> <td>Can you show me 3 using these counters?</td> </tr> <tr> <td>0-10 ◆</td> <td>Can you show me 8 using these counters?</td> </tr> <tr> <td>0-19 ♥</td> <td>Can you show me 11 using these counters?</td> </tr> <tr> <td>0-50 ♣</td> <td>Can you show me 15 using these counters?</td> </tr> </tbody> </table>	# Range	Question	0-5 ▲	Can you show me 3 using these counters?	0-10 ◆	Can you show me 8 using these counters?	0-19 ♥	Can you show me 11 using these counters?	0-50 ♣	Can you show me 15 using these counters?	<table border="1"> <thead> <tr> <th># Range</th> <th>Child's Number</th> <th>Initially Correct</th> <th>Self Corrects</th> </tr> </thead> <tbody> <tr> <td>0-5 ▲</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0-10 ◆</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0-19 ♥</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0-50 ♣</td> <td></td> <td></td> <td></td> </tr> </tbody> </table>	# Range	Child's Number	Initially Correct	Self Corrects	0-5 ▲				0-10 ◆				0-19 ♥				0-50 ♣			
# Range	Question																																
0-5 ▲	Can you show me 3 using these counters?																																
0-10 ◆	Can you show me 8 using these counters?																																
0-19 ♥	Can you show me 11 using these counters?																																
0-50 ♣	Can you show me 15 using these counters?																																
# Range	Child's Number	Initially Correct	Self Corrects																														
0-5 ▲																																	
0-10 ◆																																	
0-19 ♥																																	
0-50 ♣																																	
[Probing thinking/ reasoning]	<ul style="list-style-type: none"> Regardless if child is correct, ask: <i>6:06</i> ② Are there other numbers you can make using these counters? ③ How do you know that there are 10 counters? Record unscripted questions here. 	<p>Describe child's verbal response and/or actions: <i>10 fingers</i></p> <p>Describe child's verbal response and/or actions: <i>3+7</i></p>	<p>Describe child's verbal response and/or actions: <i>10 fingers</i></p> <p>Describe child's verbal response and/or actions: <i>3+7</i></p> <p><i>what ... add ... less ... one to next gap</i></p>																														

Figure 4. Sample observer notes on a page from the composition and decomposition interview protocol.

As a result of the tryouts, minor revisions were made to the protocol items and the number and ordering of the reasoning questions. Initially, two reasoning questions were included for each content questions. Because of the length of time taken for the students to respond to both reasoning questions, we decided to reduce the number of reasoning questions to one per content question. Reducing the number of required reasoning questions aided in shortening the length of the interviews.

Additional revisions included logistical adjustments to reduce distractions. For example, during one of the tryout videos, a child's marker consistently rolled off of the table so a specific location for the child's marker was included during data collection.

Fidelity of Administration Form

To verify that the cognitive interview proceeded as expected, we created a fidelity of administration form. The purpose of this form was to gather evidence to address Research Questions 3 and 4:

- RQ 3: What was the level of fidelity of implementation (fidelity by interviewer/observer)?
- RQ 4: What was the level of accessibility and comfort of students on all tasks within every learning progression?

To inform these research questions, two members of the MMaRS research team drafted the Fidelity of Administration Form (see Figure 5). To address RQ3, questions prompted the observers to note if the assessor re-worded the protocol items or repeated the questions. To address RQ4, questions prompted the observers to evaluate whether the student seemed comfortable with the material and the specific components of the task.

This form was intended to be completed during the cognitive interviews by a trained observer. The observer was expected to respond to all four prompts for each protocol item during the cognitive interview.

Did the interviewer reword the question?	Did the interviewer repeat the question?	Did the student seem comfortable with the materials?	How comfortable did the student appear with the task?
0 - No 1 - Yes	0 - No 1 - Yes	NA 0 - No 1 - Yes	0 1 2 3 Not comfortable to Very comfortable
<p>This column focuses on whether the interviewer re-worded the first part of the task, which is the first row within each new skill statement. You do not need to track whether the interviewer re-worded the reasoning question.</p> <p>Re-wording is considered changing significant language or structure. For example, changing “compose the number” to “make the number” would be considered re-wording. However, adding in “and” or “ok, so...” would not be considered re-wording.</p>	<p>This column focuses on whether the interviewer repeated the first part of the task, which is the first row within each new skill statement. You do not need to track whether the interviewer repeated the reasoning question.</p> <p>Mark “Yes” when the interviewer repeats the question (on their own or if the child asks) because the child seems to not understand or not remember the question.</p>	<p>This column focuses on whether the student seemed comfortable or at ease with the materials. You might want to consider these questions when determining how to respond in this column:</p> <ul style="list-style-type: none"> • Did the student seem to know how to use the materials? • Did the student act like they had seen the materials before? 	<p>This column focuses on whether the student seemed comfortable with the task. This response should reflect students’ interaction with the task.</p> <ul style="list-style-type: none"> • Did they seem to understand what the task was asking them to do? • Was the student able to easily access the task and demonstrate their knowledge (regardless of whether they were correct or not)?

Figure 5. Detailed description for each of the four fidelity questions for observers.

Next Steps

Final adjustments to the protocol items were made following the tryouts. Once these revisions were made and approved, the cognitive interview protocols were finalized. Final production of the protocol items included developing all of the student materials and assessor documents for administration of the cognitive interviews.

References

- Baroody, A. J., Purpura, D. J., Eiland, M. D., Reid, E. E., & Paliwal, V. (2016). Does fostering reasoning strategies for relatively difficult basic combinations promote transfer by K-3 students? *Journal of Educational Psychology*, 108(4), 576-591.
- Carpenter, T. P., Franke, M. L., Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.
- Farrington-Flint, L., Canobi, K. H., Wood, C., & Faulkner, D. (2007). The role of relational reasoning in children's addition concepts. *British Journal of Developmental Psychology*, 25, 227-246.
- Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education*, 38(3), 258-288.
- Sparks, A., Perry, L., Geller, J., Haider, M. Q., & Ketterlin-Geller, L. R. (2020). *Numeric relational reasoning (NRR): Teacher survey administration* (Tech. Rep. No. 20-01). Dallas, TX: Southern Methodist University, Research in Mathematics Education.