## Effective Fraction Instruction

Levels 0-7


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## Effective Fraction Instruction

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| Conceptual Development of Fractions <br> (Battista, 2012, p. 10) | Texas Essential Knowledge and Skills for Mathematics (TEKS)$(\S \S 111.1-111.7)$ |  |
| :---: | :---: | :---: |
| Level 1 <br> Recognize Familiar Fraction Diagrams | TEKS 2.3 <br> The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to: |  |
|  | (B) Explain that the more fractional parts used to make a whole, the smaller the part; and the fewer the fractional parts, the larger the part. | Halves <br> Fourths <br> When a unit (or defined whole) is partitioned into a greater number of parts, each part will be smaller (i.e., partitioning one whole into 4 parts will have larger parts than that same whole partitioned into 8 parts). |

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| :--- | :--- |
| Level 3 <br> Partition Shapes into Equal <br> Parts and Select Parts | TEKS 2.3 <br> The student applies mathematical process standards to recognize and represent fractional units <br> and communicates how they are used to name parts of a whole. The student is expected to: <br> (D) Identify examples and <br> non-examples of halves, <br> fourths, and eighths. |

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| :---: | :---: | :---: |
| Level 4 <br> Partition Quantities into Equal Parts and Select Some Parts | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
|  | (A) Represent fractions greater than zero and less than or equal to one with denominators of 2,3 , 4,6 , and 8 using concrete objects and pictorial models, including strip diagrams and number lines. |  |

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| :---: | :---: | :---: |
| Level 4 <br> Partition Quantities into Equal Parts and Select Some Parts | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
|  | (B) Determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2,3 , 4,6 , and 8 given a specified point on a number line. | Describe the point on this number line in relationship to 0 . <br> This point is located 3 one-eighth units away from zero or $\frac{3}{8}$ away from zero. <br> Incorrect response: 3 units away from 0. Point out that a unit is one measure of something. These intervals are not unit intervals but one-eighth intervals. |

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| :---: | :---: | :---: |
| Level 4 <br> Partition Quantities into Equal <br> Parts and Select Some Parts | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
|  | (C) Explain that the unit fraction $1 / b$ represents the quantity formed by one part of a whole that has been partitioned into $b$ equal parts where $b$ is a nonzero whole number. | Both Model A and Model B show $\frac{1}{4}$. Explain why this statement is true. |
|  |  | Both Model A and Model B represent one unit that has been divided into 4 equal parts. Of the 4 equal parts, only one part is shaded which is represented by the fraction $\frac{1}{4}$. |

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| :---: | :---: | :---: |
| Level 4 <br> Partition Quantities into Equal Parts and Select Some Parts | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
|  | (F) Represent equivalent fractions with denominators of $2,3,4,6$, and 8 using a variety of objects and pictorial models, including number lines. | What number is represented by the shaded part of this number line? <br> The shaded part on this number line represents $\frac{1}{3}$. <br> Take your pencil and subdivide each part of the number line in half. <br> Now, what number does the shaded part on this number line represent? <br> The shaded part on this number line represents $\frac{2}{6}$. <br> Write an equation to represent the relationship between the two fractions. $\frac{1}{3}=\frac{2}{6}$ |

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| :---: | :---: | :---: | :---: | :---: |
| Level 4 | TEKS 4.3 |  |  |  |
| Partition Quantities into Equal Parts and Select Some Parts | The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to: |  |  |  |
|  | (C) Determine if two given fractions are equivalent using a variety of methods. |  |  |  |
|  |  | Fraction | Equivalent Fraction Model | Equation |
|  |  | $\frac{3}{4}$ |  | $\frac{3}{4}=\frac{6}{8}$ |

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| :--- | :--- | :--- | :--- |
| Level 4 <br> Partition Quantities into Equal <br> Parts and Select Some Parts | TEKS 4.3 <br> The student applies mathematical process standards to represent and generate fractions to solve <br> problems. The student is expected to: |
| (D) Compare two fractions <br> with different numerators and <br> different denominators and <br> represent the comparison <br> using the symbols $>,=$, or $<$. | Which statement about fractions is true? |

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| :---: | :---: | :---: |
| Level 5 <br> Use Visual Representations of Fractions to Solve Simple | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
| Fraction Arithmetic Problems | (G) Explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model. | What do you notice about these number lines? <br> The intervals (represented by the denominators) are multiples of 3 . <br> What does it mean to say that fractions are equivalent? <br> Equivalent fractions occupy the same point on a number line, represent the same distance from zero, or represent the same distance between points. |

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| :---: | :---: | :---: |
| Level 5 <br> Use Visual Representations of Fractions to Solve Simple | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
| Fraction Arithmetic Problems | (H) Compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models. | Which statement is true for comparing fractions when the numerators are the same? <br> - The greater the denominator, the larger the fraction. <br> - The greater the denominator, the smaller the fraction. <br> Provide an example to support your choice. <br> The greater the denominator, the smaller the fraction. The denominator of a fraction represents the total number of equal parts the unit has been divided into. Increasing the denominator results in smaller parts because the unit has not changed. <br> An example is $\frac{2}{3}$ and $\frac{2}{7}$. <br> The numerator represents that 2 parts of the unit are being represented. However, thirds are larger than sevenths because sevenths are divided into more equal parts. |

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| :---: | :---: | :---: |
| Level 5 <br> Use Visual Representations of Fractions to Solve Simple | TEKS 4.3 <br> The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to: |  |
| Fraction Arithmetic Problems | (A) Represent a fraction $a / b$ as a sum of fractions $1 / b$, where $a$ and $b$ are whole numbers and $b>0$, including when $\mathrm{a}>\mathrm{b}$. | Model hops on top of the number line to show the joining of 2 intervals of $\frac{1}{3}$. Write an expression representing the sum. |

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| :---: | :---: | :---: | :---: | :---: |
| Level 5 <br> Use Visual Representations of Fractions to Solve Simple | The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to: |  |  |  |
| Fraction Arithmetic Problems | (G) Represent fractions and decimals to the tenths or hundredths as distances from zero on a number line. |  |  |  |
|  | Fraction | Decimal | Number Line Representation | Equation |
|  | $\frac{7}{10}$ | 0.7 |  | $\frac{7}{10}=0.7$ |

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| :---: | :---: | :---: |
| Level 6 <br> Understands Symbolic Computation of Fractions | TEKS 3.3 <br> The student applies mathematical process standards to represent and explain fractional units. The student is expected to: |  |
|  | (D) Compose and decompose a fraction $a / b$ with a numerator greater than zero and less than or equal to $b$ as a sum of parts $1 / b$. | Identifying the fractional intervals within one unit (whole) helps to describe the distance from zero or the distance between two points on a number line. <br> Note: There is more than one way to model the expression. Accept any model that represents the sum of 2 one-third intervals. |

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| :---: | :---: | :---: |
| Level 6 <br> Understands Symbolic Computation of Fractions | TEKS 4.3 <br> The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to: |  |
|  | (B) Decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations. | Identify 3 different ways to write $\frac{5}{3}$ as an expression. $\begin{array}{\|l\|l\|l\|l\|l\|l\|l} \longleftrightarrow & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$ $\begin{gathered} \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} \\ \frac{2}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} \\ \frac{4}{3}+\frac{1}{3} \end{gathered}$ |

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| :--- | :--- |
| Level 6 <br> Understands Symbolic <br> Computation of Fractions | TEKS 4.3 <br> The student applies mathematical process standards to represent and generate fractions to solve <br> problems. The student is expected to: |
| (E) Represent and solve <br> addition and subtraction of <br> fractions with equal <br> denominators using objects <br> and pictorial models that build <br> to the number line and <br> properties of operations. | Draw a model to represent this expression. |

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## Conceptual Development of Fractions

 (Battista, 2012, p. 10)

A group of 4 students are given a pizza to share evenly between them.

- There is 1 pizza.
- There are 4 people in the group.
- 1 pizza divided between 4 people can be represented as $1 \div 4$.
- If everyone receives the same amount of pizza, each person wil receive of the pizza.
- $1 \div 4=\frac{1}{4}$

Ms. Jones has a pencil box and divides the pencils evenly among 8 students in her class. How much of the pencil box will each student receive?

Notice that this situation does not provide us with the number of pencils that are actually in the box. We can still determine how much of the pencil box each student received, even though we do not know the number of pencils in the box by dividing the box by 8 people.

What equation could we write to represent this situation?

$$
1 \div 8=\frac{1}{8}
$$

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| :---: | :---: | :---: |
| Level 7 <br> Solve Fraction Arithmetic Problems and Explain Why | TEKS 4.3 <br> The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to: |  |
|  | (F) Evaluate the reasonableness of sums and differences of fractions using benchmark fractions $0,1 / 4$, $1 / 2,3 / 4$, and 1 , referring to the same whole. | Are the fractions $\frac{4}{4}$ and $\frac{4}{1}$ located at the same point on the number line? Explain why or why not. <br> The fractions $\frac{4}{4}$ and $\frac{4}{1}$ represent different distances from zero on a number line. The fraction $\frac{4}{4}$ is equal to 1 . Therefore, this fraction is actually 1 unit (or 4 one-fourth units) away from zero. The fraction $\frac{4}{1}$ is 4 units (or 4 one units) away from zero. |

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| :---: | :---: | :---: |
| Level 7 <br> Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works | TEKS 5.3 <br> The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to: |  |
|  | (H) Represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations. | Construct a number line to solve this equation. $\frac{1}{4}+\frac{3}{8}=$ $\square$ $\frac{1}{4}+\frac{3}{8}=\frac{5}{8}$ |

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| :---: | :---: | :---: | :---: |
| Level 7 <br> Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works | TEKS 5.3 <br> The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to: |  |  |
|  | (I) Represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models. | Construct a model to solve this equation. |  |

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| :---: | :---: | :---: |
| Level 7 <br> Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works | TEKS 5.3 <br> The student applies mathematical pro positive rational number computation student is expected to: <br> (J) Represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1 / 3 \div$ 7 and $7 \div 1 / 3$ using objects and pictorial models, including area models. | ess standards to develop and use strategies and methods for in order to solve problems with efficiency and accuracy. The <br> Construct a number line to solve this equation. $\frac{1}{2} \div 3=$ $\square$ $\frac{1}{2} \div 3=\frac{1}{6}$ |

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Battista, M.T. (2012). Cognition-based assessment \& teaching of fractions: Building on students' reasoning. Portsmouth, N.H.: Heinemann

Texas Essential Knowledge and Skills for Mathematics, 19 Texas Administrative Code. §§ 111-111.1-111.7 (2012).

