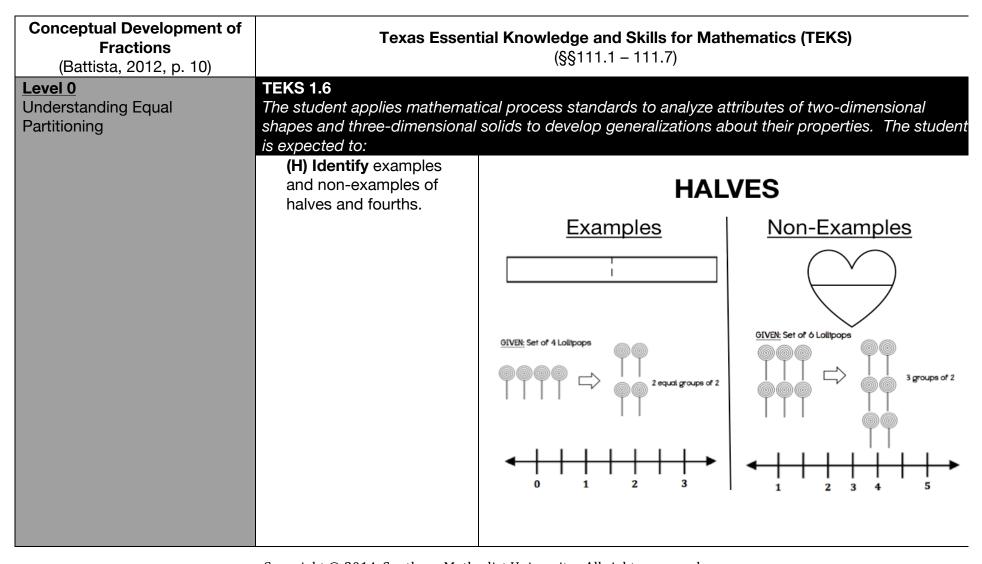
Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
Level 0 Understanding Equal Partitioning		which shape is divided into 4 equal parts? A. B. C. D. Answer: B

Levels 0-7



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Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 0 Understanding Equal Partitioning	TEKS 2.3 The student applies mathematical process standard and communicates how they are used to name part (A) Partition objects into equal parts and name eighths, using words. Note: Students can partition objects and sets into equal parts.	e the parts, including halves, fourths, and	
	Which word describes an object divided into 4 equal parts? A. Fourths B. Forty C. Fourteen D. Fourteenths Answer: A	Which shape is divided into fourths? A. B. D. C. D. Answer: A	

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Kno	wledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)
(Battista, 2012, p. 10) Level 1 Recognize Familiar Fraction Diagrams	TEKS 2.3 The student applies mathematical prod	

Levels 0-7

Conceptual Development of Fractions		

Texas Essential Knowledge and Skills for Mathematics (TEKS)

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Level 2

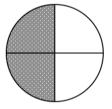
Understand Fractions as Counting All Parts and Shaded Parts

TEKS 2.3

The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:

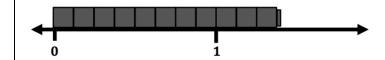
(C) Use concrete models to count fractional parts beyond one whole using words and recognize how many parts it takes to equal one whole.

Why does this shape show two-fourths shaded?



This circle is divided into 4 equal parts. This means that each part of this circle represents one-fourth. Since 2 of the 4 equal parts are shaded, this circle represents two-fourths.

Why do the linking cubes on this number line represent eleven-eighths?



There are 8 linking cubes that compose one unit interval. This means that each cube represents one-eighth. There are a total of 11 one-eighth cubes or eleven-eighths.

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 3 Partition Shapes into Equal Parts and Select Parts	TEKS 2.3 The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:		
	(D) Identify examples and non-examples of halves, fourths, and eighths.	EIGH	HTHS
		<u>Examples</u>	Non-Examples
			GIVEN: Set of 8 Lollipops 4 groups of 2
		0 1	1 8

Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
TEKS 2.6 The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:	
(B) Model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets.	Laura shares 6 lollipops with 2 friends. Use a model to show a way Laura could evenly divide the lollipops between herself and her friends.
	3 groups of 2
r	The student applies mathematical procenultiplication and division situations that o: (B) Model, create, and describe contextual division situations in which a set of concrete objects

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
Level 4 Partition Quantities into Equal Parts and Select Some Parts	TEKS 3.3 The student applies mathematical proc student is expected to: (A) Represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines.	AS A POINT (OR LOCATION) AS A DISTANCE FROM ZERO AS A DISTANCE BETWEEN TWO POINTS

Levels 0-7

Conceptual Development of Fractions

(Battista, 2012, p. 10)

Texas Essential Knowledge and Skills for Mathematics (TEKS)

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Level 4

Partition Quantities into Equal Parts and Select Some Parts

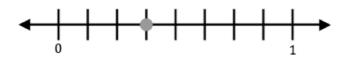
TEKS 3.3

The student applies mathematical process standards to <u>represent and explain fractional units</u>. The student is expected to:

(B) Determine the

corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line.

Describe the point on this number line in relationship to 0.



This point is located 3 one-eighth units away from zero or $\frac{3}{8}$ away from zero.

<u>Incorrect response:</u> 3 units away from 0. Point out that a unit is one measure of something. These intervals are not unit intervals but one-eighth intervals.

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
Level 4 Partition Quantities into Equal Parts and Select Some Parts	TEKS 3.3 The student applies mathematical process standards to <u>represent and explain fractional units</u> . The student is expected to:	
	(C) Explain that the unit fraction 1/b represents the quantity formed by one part of a whole that has been partitioned into b equal parts where b is a nonzero whole number.	Both Model A and Model B show $\frac{1}{4}$. Explain why this statement is true. MODEL A Both Model A and Model B represent one unit that has been divided into 4 equal parts. Of the 4 equal parts, only one part is shaded which is represented by the fraction $\frac{1}{4}$.

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
Level 4 Partition Quantities into Equal Parts and Select Some Parts	TEKS 3.3 The student applies mathematical process standards to <u>represent and explain fractional units</u> . The student is expected to:	
	of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines. The shaded partial this number lines.	art on this number line represents $\frac{1}{3}$. In this number line represents $\frac{2}{6}$. It on this number line relationship between

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
Level 4 Partition Quantities into Equal Parts and Select Some Parts	TEKS 3.7 The student applies mathematical process standards to <u>select appropriate units</u> , <u>strategies</u> , <u>and tools to solve problems involving customary and metric measurement</u> . The student is expected to:	
	(A) Represent fractions of halves, fourths, and eighths as distances from zero on a number line.	Which number line shows $\frac{5}{4}$ as a distance from zero?
		A. B. 0 1 1 1 2
		C. D.
		Answer: B

Levels 0-7

Conceptual Development of Fractions

(Battista, 2012, p. 10)

Texas Essential Knowledge and Skills for Mathematics (TEKS)

(§§111.1 – 111.7)

Level 4

Partition Quantities into Equal Parts and Select Some Parts

TEKS 4.3

The student applies mathematical process standards to <u>represent and generate fractions to solve</u> problems. The student is expected to:

(C) Determine if two given fractions are equivalent using a variety of methods.

Fraction	Equivalent Fraction Model	Equation
3 4		$\frac{3}{4} = \frac{6}{8}$

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)	
Level 4 Partition Quantities into Equal Parts and Select Some Parts	TEKS 4.3 The student applies mathematical problems. The student is expected to (D) Compare two fractions with different numerators and different denominators and represent the comparison using the symbols >, =, or <.	Which statement about fractions is true? A. $\frac{3}{4} > \frac{1}{2}$ B. $\frac{1}{3} > \frac{3}{5}$ C. D. $\frac{1}{4} > \frac{2}{3}$ Answer: A

Levels 0-7

Conceptual Development of				
Fractions				
(Battista 2012 n. 10)				

Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)

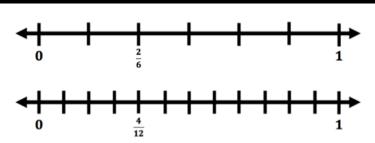
Level 5

Use Visual Representations of Fractions to Solve Simple Fraction Arithmetic Problems

TEKS 3.3

The student applies mathematical process standards to <u>represent and explain fractional units</u>. The student is expected to:

(G) Explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model.



What do you notice about these number lines?

The intervals (represented by the denominators) are multiples of 3.

What does it mean to say that fractions are equivalent?

Equivalent fractions occupy the same point on a number line, represent the same distance from zero, or represent the same distance between points.

Levels 0-7

Conceptual Development of
Fractions

(Battista, 2012, p. 10)

Level 5

Use Visual Representations of Fractions to Solve Simple Fraction Arithmetic Problems

Texas Essential Knowledge and Skills for Mathematics (TEKS)

(§§111.1 – 111.7)

TEKS 3.3

The student applies mathematical process standards to <u>represent and explain fractional units</u>. The student is expected to:

(H) Compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models.

Which statement is true for comparing fractions when the numerators are the same?

- The greater the denominator, the larger the fraction.
- The greater the denominator, the smaller the fraction.

Provide an example to support your choice.

The greater the denominator, the smaller the fraction. The denominator of a fraction represents the total number of equal parts the unit has been divided into. Increasing the denominator results in smaller parts because the unit has not changed.

An example is $\frac{2}{3}$ and $\frac{2}{7}$

The numerator represents that 2 parts of the unit are being represented. However, thirds are larger than sevenths because sevenths are divided into more equal parts.

Levels 0-7

Conceptual Development of Fractions

(Battista, 2012, p. 10)

Level 5

Use Visual Representations of Fractions to Solve Simple Fraction Arithmetic Problems

Texas Essential Knowledge and Skills for Mathematics (TEKS)

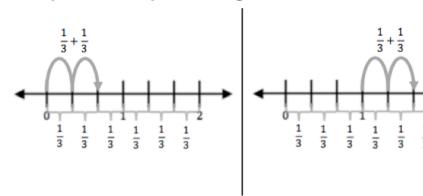
(§§111.1 – 111.7)

TEKS 4.3

The student applies mathematical process standards to <u>represent and generate fractions to solve</u> <u>problems</u>. The student is expected to:

(A) Represent a fraction a/b as a sum of fractions 1/b, where a and b are whole numbers and b > 0, including when a > b.

Model hops on top of the number line to show the joining of 2 intervals of $\frac{1}{3}$. Write an expression representing the sum.



Note: There is more than one way to model the expression. Accept any model that represents the sum of 2 one-third intervals.

Levels 0-7

Conceptual Development of Fractions

(Battista, 2012, p. 10)

Level 5

Use Visual Representations of Fractions to Solve Simple Fraction Arithmetic Problems

Texas Essential Knowledge and Skills for Mathematics (TEKS)

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TEKS 4.3

The student applies mathematical process standards to <u>represent and generate fractions to solve</u> problems. The student is expected to:

(G) Represent fractions and decimals to the tenths or hundredths as distances from zero on a number line.

Fraction	Decimal	Number Line Representation	Equation
7 10	0.7	←	$\frac{7}{10} = 0.7$

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 6 Understands Symbolic Computation of Fractions	TEKS 3.3 The student applies mathematical process standards to represent and explain fractional units. The student is expected to: (D) Compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts $1/b$. Identifying the fractional intervals within one unit (whole) helps to describe the distance from zero or the distance between two points on a number line. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ Note: There is more than one way to model the expression. Accept any model that represents the sum of 2 one-third intervals.		

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 6 Understands Symbolic Computation of Fractions	TEKS 4.3 The student applies mathematical problems. The student is expected to (B) Decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations.	Identify 3 different ways to write $\frac{5}{3}$ as an expression. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ $\frac{2}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ $\frac{2}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ $\frac{4}{3} + \frac{1}{3}$	

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)			
Level 6 Understands Symbolic Computation of Fractions	TEKS 4.3 The student applies mathematical problems. The student is expected to (E) Represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations.	Draw a model to represent this expression. $\frac{3}{7} + \frac{4}{7}$		
	C			

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)			
Level 6 Understands Symbolic Computation of Fractions	TEKS 5.3 The student applies mathematical propositive rational number computation student is expected to: (K) Add and subtract positive rational numbers fluently.			
		A. $\frac{4}{5}$	В.	4 6
		c. 1	D.	<u>5</u> 4
				Answer: D

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)			
Level 6 Understands Symbolic Computation of Fractions	TEKS 5.3 The student applies mathematical propositive rational number computation student is expected to: (L) Divide whole numbers by unit fractions and unit fractions by whole numbers.	ns in order to solve pro	blems with	
		Solve. 3	2	
		A. $\frac{3}{2}$	B.	1 6
		c . 6	D.	2 3
	Constitute @ 2014 Constitute Mathe dist			Answer: C

Levels 0-7

Conceptual Development of			
Fractions			
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(Battista, 2012, p. 10)

Level 7

Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works

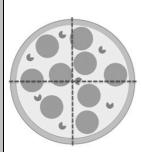
Texas Essential Knowledge and Skills for Mathematics (TEKS)

(§§111.1 – 111.7)

TEKS 3.3

The student applies mathematical process standards to <u>represent and explain fractional units</u>. The student is expected to:

(E) Solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8.



A group of 4 students are given a pizza to share evenly between them.

- · There is 1 pizza.
- There are 4 people in the group.
- 1 pizza divided between 4 people can be represented as 1 ÷ 4.
- If everyone receives the same amount of pizza, each person will receive of the pizza.

•
$$1 \div 4 = \frac{1}{4}$$

Ms. Jones has a pencil box and divides the pencils evenly among 8 students in her class. How much of the pencil box will each student receive?

Notice that this situation does not provide us with the number of pencils that are actually in the box. We can still determine how much of the pencil box each student received, even though we do not know the number of pencils in the box by dividing the box by 8 people.

What equation could we write to represent this situation?

$$1 \div 8 = \frac{1}{8}$$

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 7 Solve Fraction Arithmetic Problems and Explain Why	TEKS 4.3 The student applies mathematical proposed problems. The student is expected to	ocess standards to <u>represent and generate fractions to solve</u> :	
Fraction Computation Works	(F) Evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole.	Are the fractions $\frac{4}{4}$ and $\frac{4}{1}$ located at the same point on the number line? Explain why or why not. The fractions $\frac{4}{4}$ and $\frac{4}{1}$ represent different distances from zero on a number line. The fraction $\frac{4}{4}$ is equal to 1. Therefore, this fraction is actually 1 unit (or 4 one-fourth units) away from zero. The fraction $\frac{4}{1}$ is 4 units (or 4 one units) away from zero.	

Levels 0-7

Conceptual Development of Fractions

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Solve Fraction Arithmetic Problems and Explain Why **Fraction Computation Works**

Level 7

TEKS 5.3

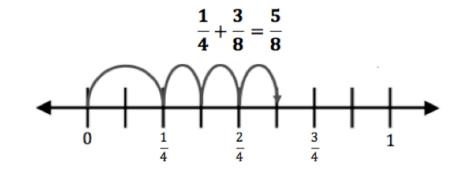
The student applies mathematical process standards to <u>develop</u> and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:

Texas Essential Knowledge and Skills for Mathematics (TEKS)

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(H) Represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations.

Construct a number line to solve this equation.



Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 7 Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works		ocess standards to <u>develop and use strategies and methods for</u> s in order to solve problems with efficiency and accuracy. The	
	multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models.	Construct a model to solve this equation. $\frac{1}{2} \times 3 = \square$ $\frac{\text{KEY:}}{1 \text{ whole}}$ $\frac{1}{2} \times 3 = 1\frac{1}{2}$	

Conceptual Development of Fractions (Battista, 2012, p. 10)	Texas Essential Knowledge and Skills for Mathematics (TEKS) (§§111.1 – 111.7)		
Level 7 Solve Fraction Arithmetic Problems and Explain Why Fraction Computation Works		Construct a number line to solve this equation. $\frac{1}{2} \div 3 = \boxed{}$ $\frac{1}{2} \div 3 = \frac{1}{6}$	

Presented by: Sharri Zachary and Dawn Woods

Effective Fraction Instruction

Levels 0-7

Battista, M.T. (2012). Cognition-based assessment & teaching of fractions: Building on students' reasoning. Portsmouth, N.H.: Heinemann

Texas Essential Knowledge and Skills for Mathematics, 19 Texas Administrative Code. §§ 111-111.1-111.7 (2012).