Changing the Way We Think About Instruction: Middle School Mathematics

Research to Practice Conference
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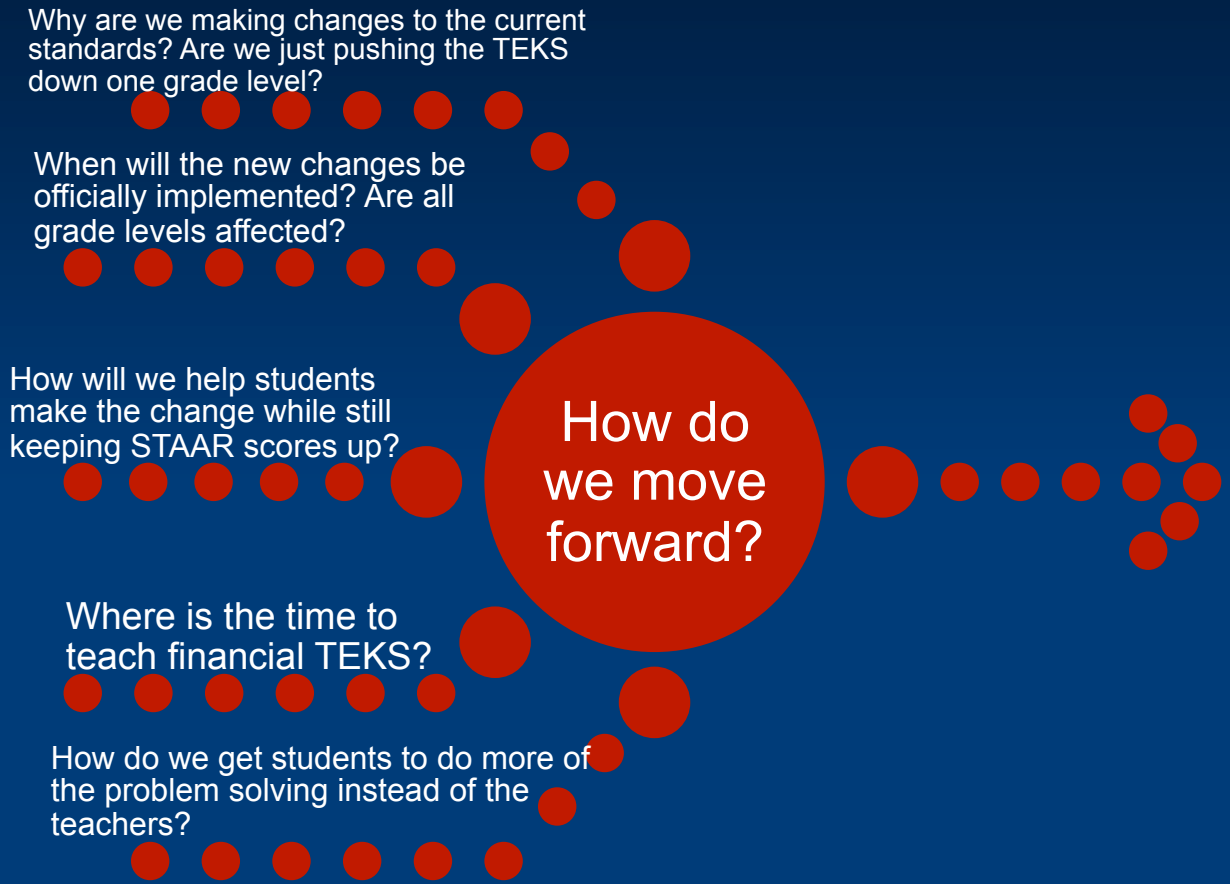
SMU | Annette Caldwell Simmons School of Education & Human Development

CHANGING MINDS


Why are we making changes to the current standards? Are we just pushing the TEKS down one grade level?
When will the new changes be officially implemented? Are all grade levels affected?
How will we help students make the change while still keeping STAAR scores up?
How do we move forward?
Where is the time to teach financial TEKS?
How do we get students to do more of the problem solving instead of the teachers?

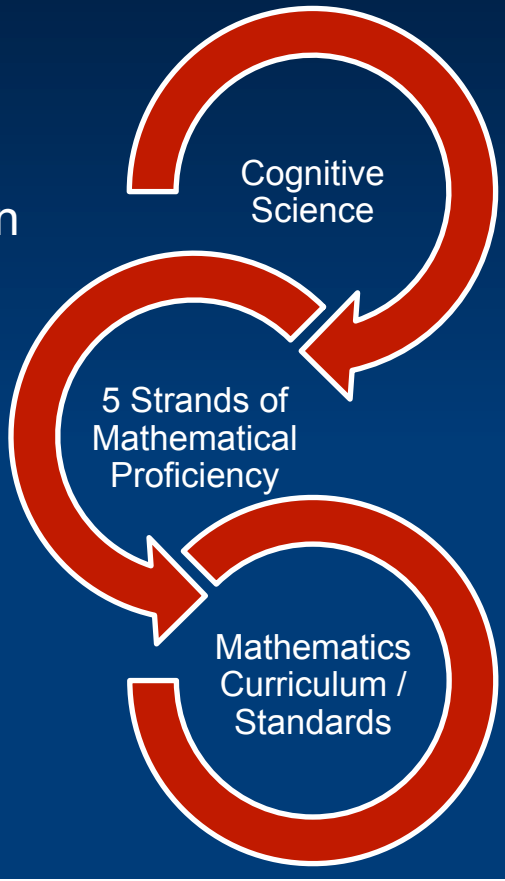
Tell Us What You've Seen…

Is the drawing accurate?

Does the artist's drawing show a clear understanding of the speaker's description?

"…teachers need to pay attention to the incomplete understandings, the false beliefs, and the naïve renditions of concepts that learners bring with them to a given subject…If students' ideas and initial beliefs are ignored, the understandings that they develop can be very different from what the teacher intends." (Bransford et al., 2000)

Studying how people learn



Conceptual Understanding
Procedural Fluency
Strategic Competence
Adaptive Reasoning
Productive Disposition

Process Standards
Content Standards

8.3(A)

The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

gen·er·al·ize *verb* \'jen-rə-ˌlīz, 'je-nə-\

: to make a general statement or form a general opinion; *especially* : to state an opinion about a larger group that is based on a smaller number of people or things within that group

: to apply (something specific, such as a theory or rule) to larger group

Connecting Cognitive Science to Mathematical Proficiency

Cognitive Science

- **Learning with Understanding**
  - Improves retention
  - Facilitates learning related material
- **Usable knowledge makes connections and shows evidence of transfer**
- **Expert knowledge is organized around "big ideas"**

Mathematical Proficiency

- **Conceptual Understanding**
  - The student is expected to create and use representations to organize, record, and communicate mathematical ideas. (6.1E, 7.1E, 8.1E)
  - The student is expected to analyze mathematical relationships to connect and communicate mathematical ideas. (6.1F, 7.1F, 8.1F)

Conceptual Understanding

8.3(A)

The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

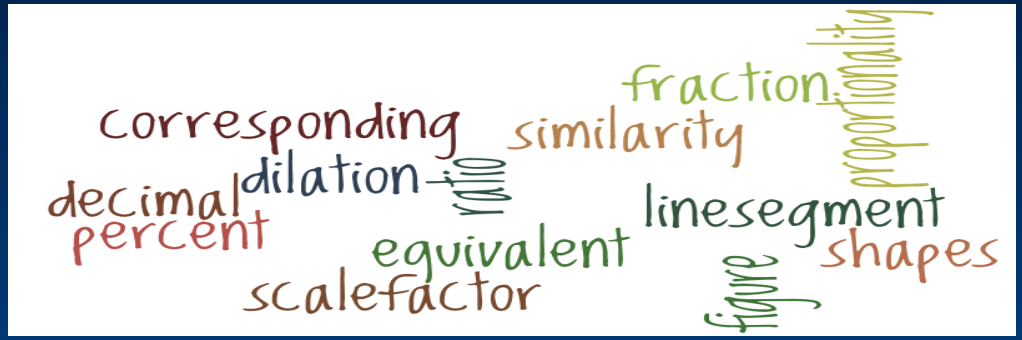
Questions to consider:

- How can this idea be represented in multiple ways?
- Do students understand the purpose of each representation?
- What evidence determines students' comprehension of the models, symbols, or formulas related to the concept?
- What types of connections to other knowledge should students make?
- What content vocabulary do students need to understand?

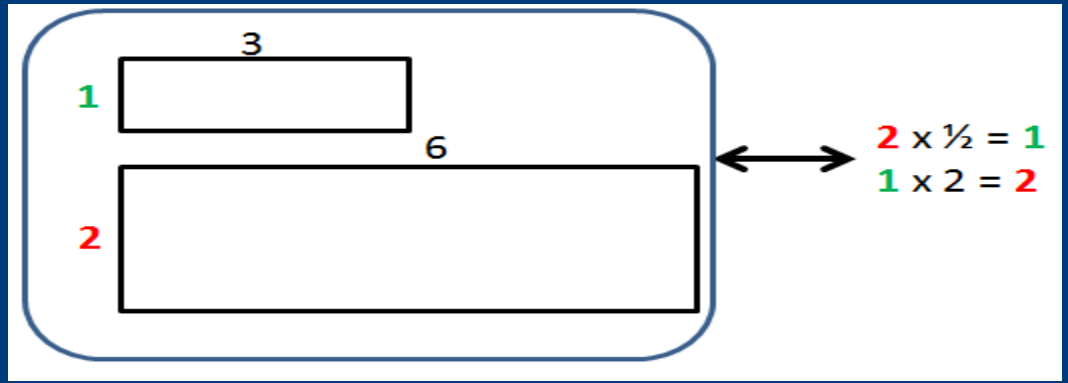
Big Idea: Representing, applying, and analyzing proportional relationships

Students extend their understanding of proportionality to include representations on a coordinate plane and applications, including slopes of lines. They contrast proportional relationships with relationships that are not proportional.

- Vocabulary



- Representations and Symbols



Connecting Cognitive Science to Mathematical Proficiency

Cognitive Science

- **Learning with Understanding**
  - Promotes fluency

Mathematical Proficiency

- **Procedural Fluency**
  - The student is expected to select…techniques, including mental math, estimation, and number sense as appropriate to solve problems. (6.1C, 7.1C, 8.1C)

Procedural Fluency

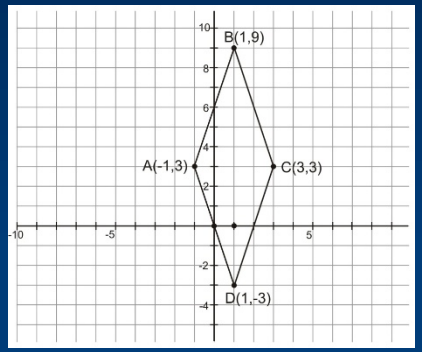
8.3(A)

The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

- Identifying the type of change (*how the change will take place*) on a coordinate plane
- Correctly reading ordered pairs in all four quadrants of the coordinate plane

Questions to consider:

- Which procedures, algorithms, and computational skills should students be able to use efficiently?
- Which concepts encourage students to estimate and/or determine a reasonable solution?
- Which mental strategies will be most efficient for students when studying this topic?
- Which mathematical properties should students be able to use?
- What computational errors do students commonly make when studying these concepts?



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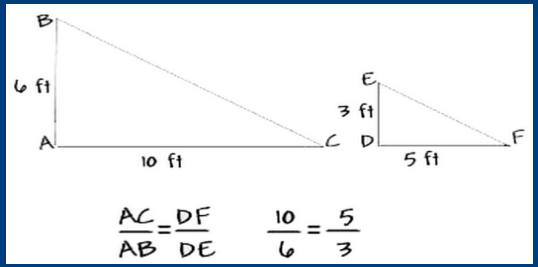
- Multiplication of rational numbers
- Determining equivalence of ratios
- Representing corresponding sides as a ratio



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Big Idea: Representing, applying, and analyzing proportional relationships

Students extend their understanding of proportionality to include representations on a coordinate plane and applications, including slopes of lines. They contrast proportional relationships with relationships that are not proportional.

Connecting Cognitive Science to Mathematical Proficiency

Cognitive Science

- **Metacognition**
  - "Experts have not only acquired knowledge, but are also good at retrieving knowledge that is relevant to a particular task."(Bransford et al., 2000)

Mathematical Proficiency

- **Strategic Competence**
  - The student is expected to select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate…(6.1C, 7.1C, 8.1C)
  - The student is expected to use a problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process. (6.1B, 7.1B, 8.1B)

Strategic Competence
8.3(A)

The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

Questions to consider:

- Will students be able to identify the problem in a variety of situations?
- What problem situations can the teacher use which will require students to access prior knowledge?
- Is the student able to construct and use an appropriate "problem model"?
- What strategies should students use to solve problems related to these ideas?
- What should it look like when students express this idea mathematically?
- Which representations can be used to help students develop mental models of a problem situation?

The coordinates of four right triangles are given below. Which two of the four right triangles are similar? Justify your answer.

Triangle 1:	A (1,2)	B (1,1)	C (3,1)
Triangle 2:	A (1,9)	B (1,1)	C (13,1)
Triangle 3:	A (1,3)	B (1,1)	C (4,1)
Triangle 4:	A (1,6)	B (1,1)	C (9,1)

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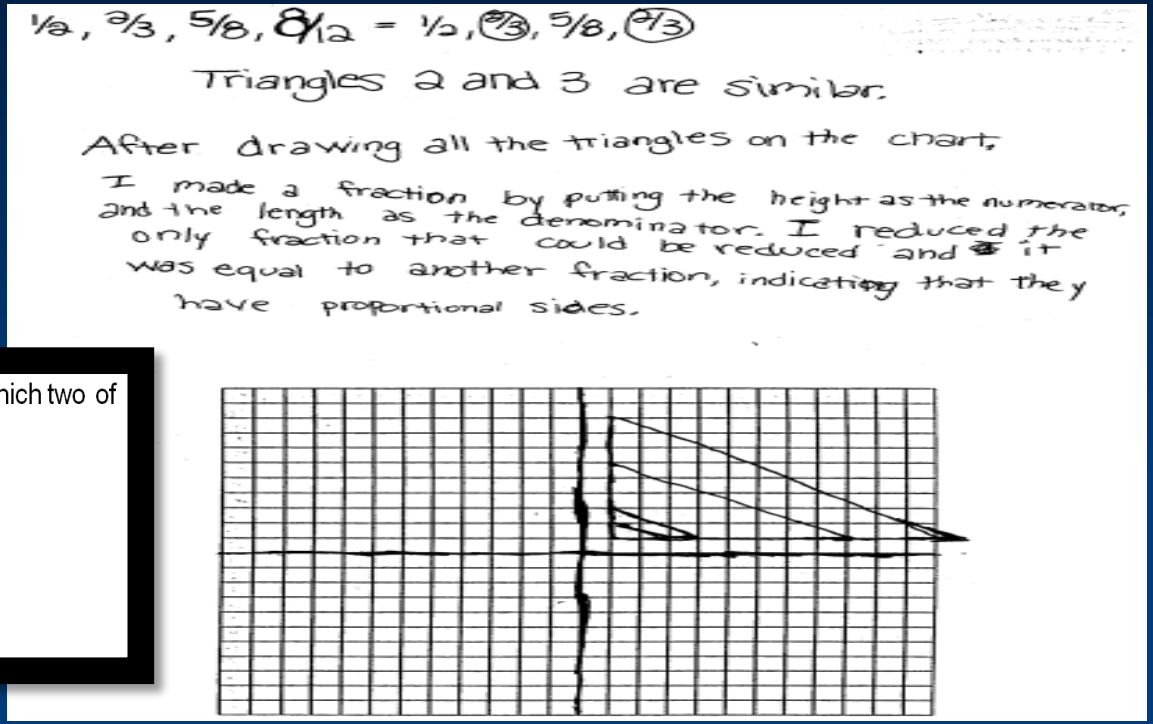

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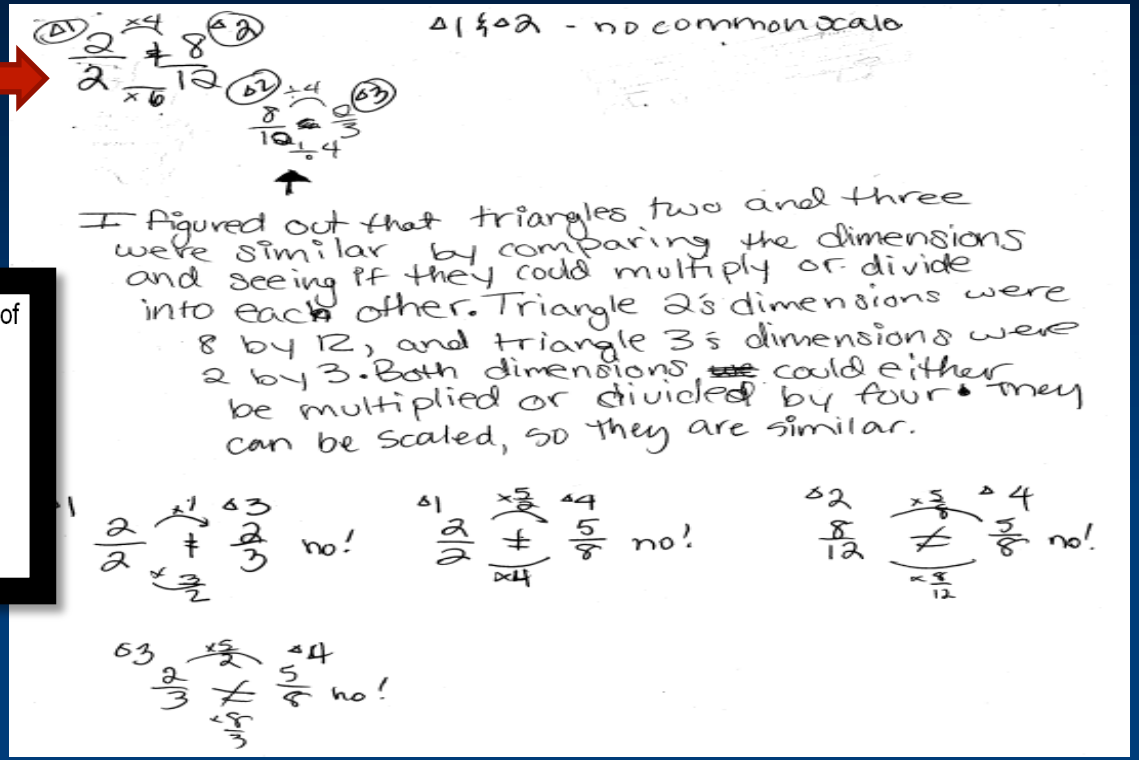

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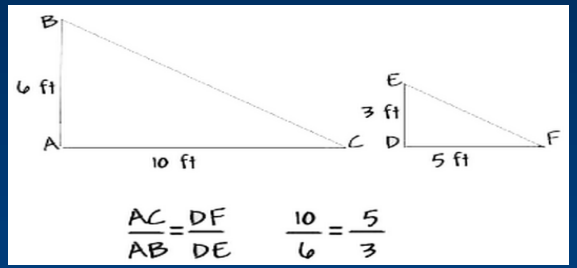
Strategic Competence (cont.)

- Representing corresponding sides as a ratio in multiple ways

$$DE/DF = AB/AC$$



- Strategies for finding ratio or determining similarity:
  - Compare side lengths (free standing shapes or on coordinate plane)
  - Examine relationship between ordered pairs
  - Examine slope of line segments
  - Given the perimeter of two similar shapes, can students find the scale factor / scaling ratio?
- Identify corresponding sides and scale factors when given angle measurements
- Make a table to show proportional relationships

Rethinking Problem Solving

- "A board was sawed into two pieces. One piece was two-thirds as long as the whole board and was exceeded in length by the second piece by four feet. How long was the board before it was cut?"

(Bransford et al., 2000)

Connecting Cognitive Science to Mathematical Proficiency

Cognitive Science

- **Metacognition**
  - Ability to monitor one's own understanding

Mathematical Proficiency

- **Adaptive Reasoning**
  - The student is expected to display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication. (6.1G, 7.1G, 8.1G)
  - The student is expected to use a problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process. (6.1B, 7.1B, 8.1B)
  - The student is expected to communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate. (6.1D, 7.1D, 8.1D)

Adaptive Reasoning

8.3(A)

The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

Questions to consider:

- Are students able to justify answers logically and with mathematical reasoning?
- Are students able to explain their thinking beyond the arithmetic and computation?
- Does student work demonstrate a strong knowledge base?
- Is the task understandable?

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- Why are these triangles similar?
- How could you determine similarity without plotting each ordered pair on a coordinate plane?
- How would you describe the transformational relationship between each triangle?

Connecting Cognitive Science to Mathematical Proficiency

Cognitive Science

- **Learning Influenced By Motivation**
- **Fixed vs. Malleable Intelligence**

Mathematical Proficiency

- **Productive Disposition**
  - The student is expected to apply mathematics to problems arising in everyday life, society and the workplace. (6.1A, 7.1A, 8.1A)

Productive Disposition

8.3(A)

The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

Questions to consider:

- What type of instructional opportunities will encourage students to "make sense" of the mathematics being taught?
- Which teacher behaviors can positively influence students' attitudes about math?

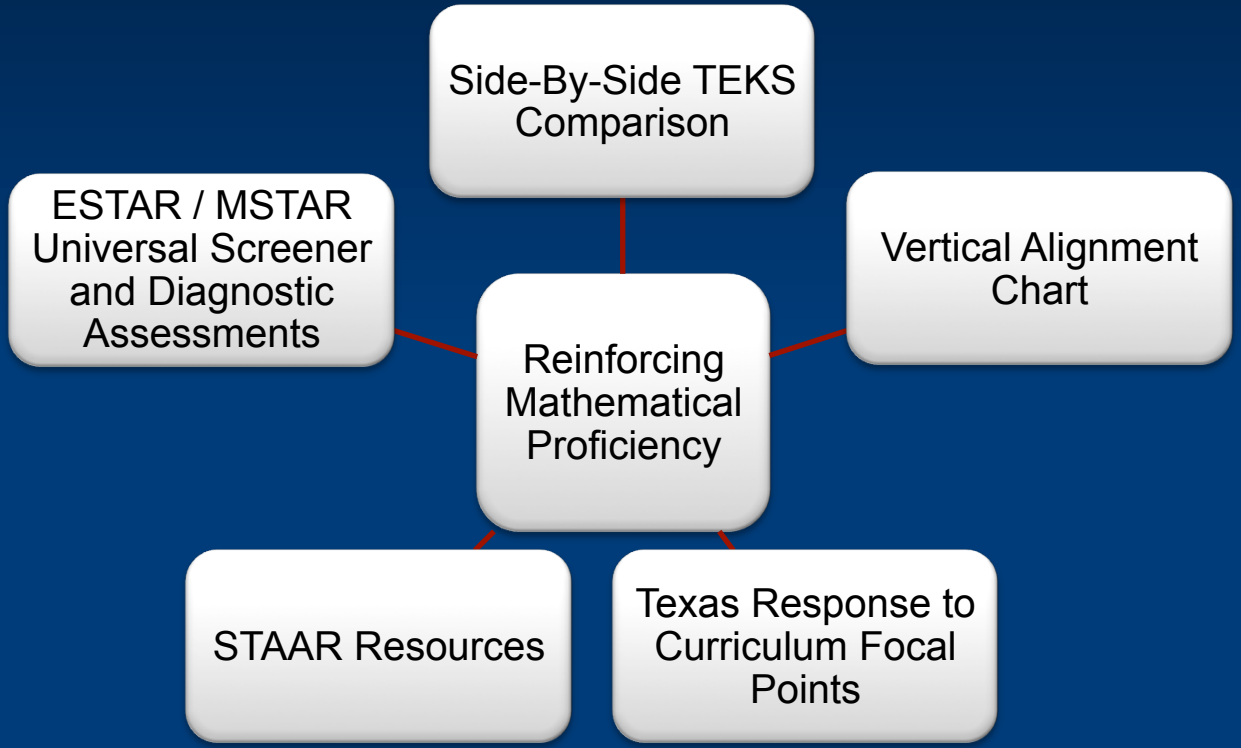
Big Idea: **Representing, applying, and analyzing proportional relationships**
Students extend their understanding of proportionality to include representations on a coordinate plane and applications, including slopes of lines. They contrast proportional relationships with relationships that are not proportional.

Rethinking Classroom Structure and Environment

- Organization
- Classroom Discussions
- Diagnostic Interviews
- Surveys and Observations
- Teacher Disposition towards instructional activities
- STEM projects which allow students to see the relevance of mathematics

"Although students appear to think mathematics is useful for everyday problems or important to society in general, it is not clear that they think it is important for them as individuals to learn a lot of mathematics" (Kilpatrick, Swafford, Findell, 2001)

Moving from Research to Practice



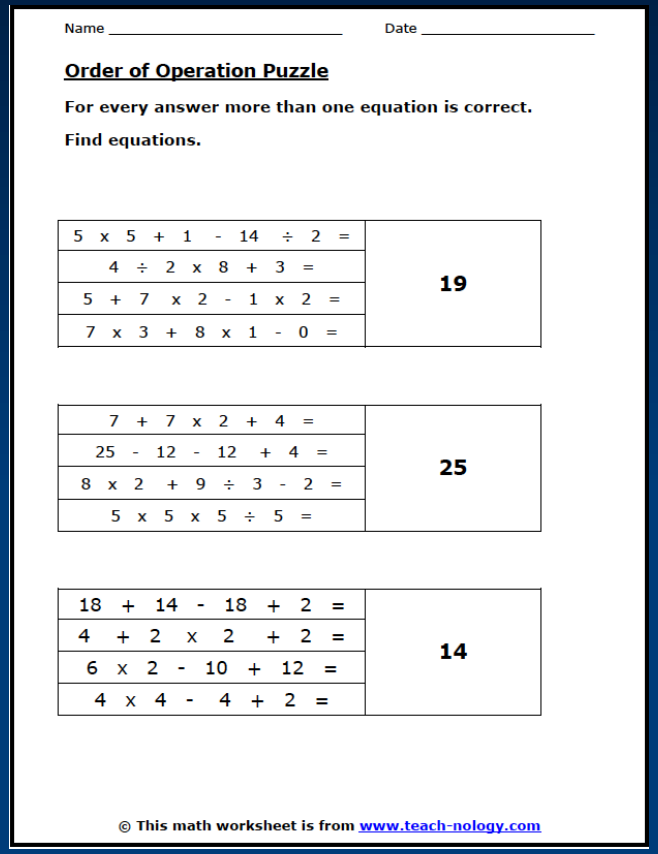
Group Activity - Scenario 1

Part 1 -The sixth grade teachers want to use the Order of Operations Puzzle as independent practice after teaching 6.7(A): *The student is expected to generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization*.

- Identify the instructional strengths and weaknesses of this activity as it relates to the standard
- Using the Mathematical Proficiency Planning Template, modify the activity with strategies that address each of the mathematical proficiency strands

Part 2

- How will you explain your reasoning when discussing your observations with your colleagues?
- What additional questions might you have for the sixth grade team?



Name _____ Date _____

Order of Operation Puzzle

For every answer more than one equation is correct.

Find equations.

5 x 5 + 1 - 14 ÷ 2 =	**19**
4 ÷ 2 x 8 + 3 =	
5 + 7 x 2 - 1 x 2 =	
7 x 3 + 8 x 1 - 0 =	

7 + 7 x 2 + 4 =	**25**
25 - 12 - 12 + 4 =	
8 x 2 + 9 ÷ 3 - 2 =	
5 x 5 x 5 ÷ 5 =	

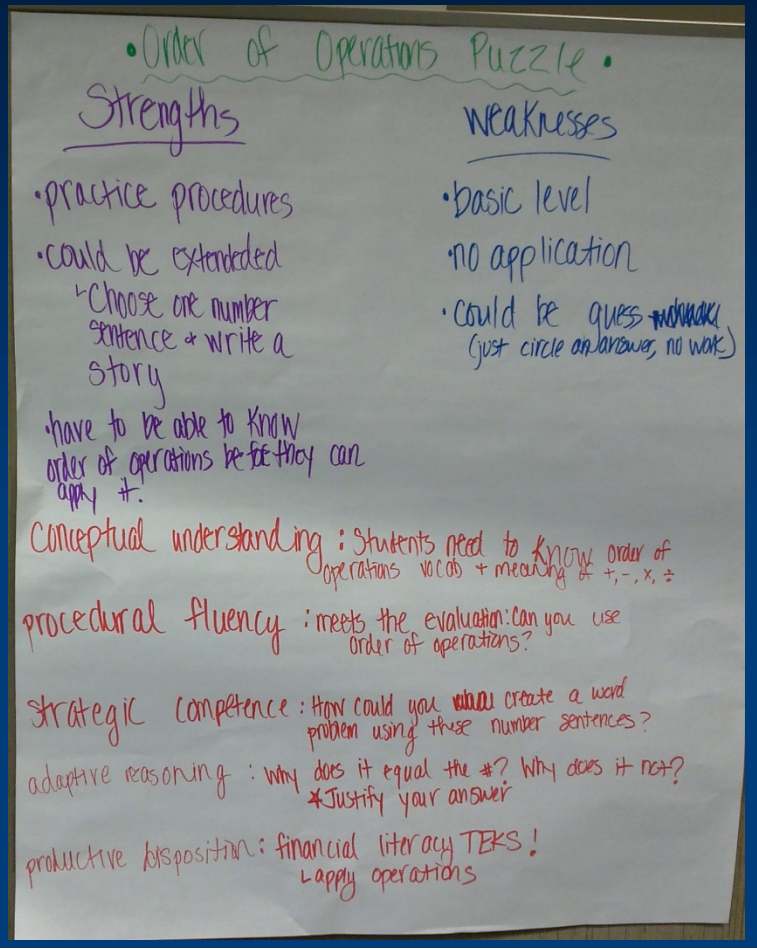
18 + 14 - 18 + 2 =	**14**
4 + 2 x 2 + 2 =	
6 x 2 - 10 + 12 =	
4 x 4 - 4 + 2 =	

© This math worksheet is from www.teach-nology.com

Scenario 1 Activity:

Order of Operations Handout

Results from Scenario 1



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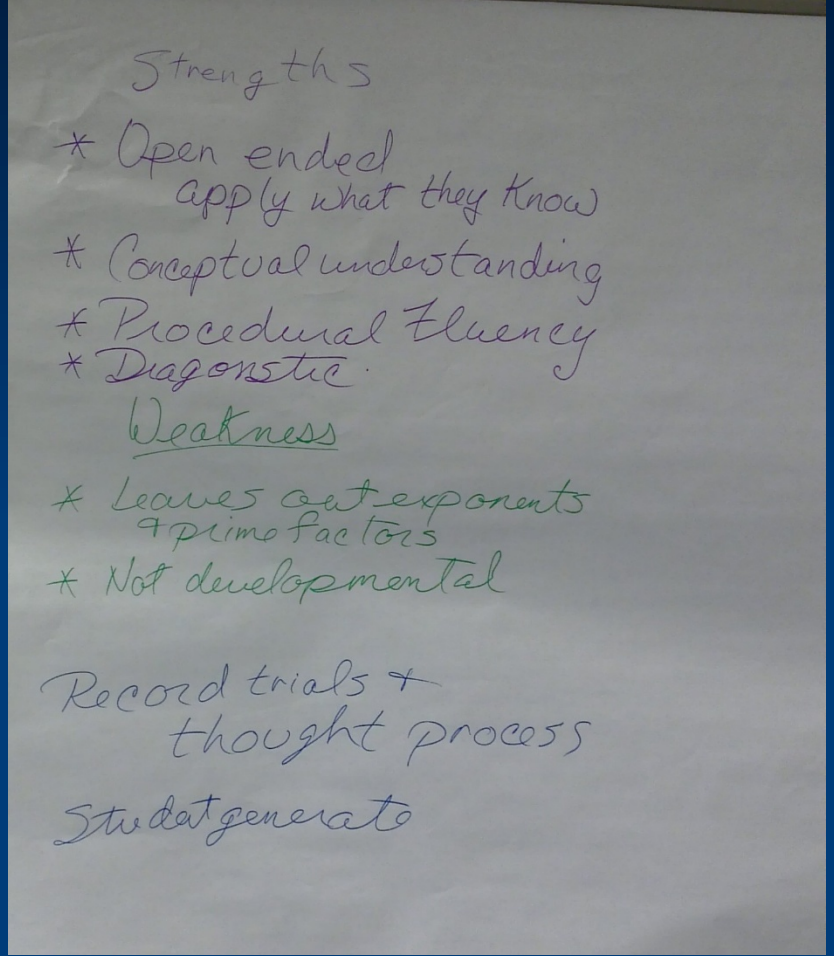

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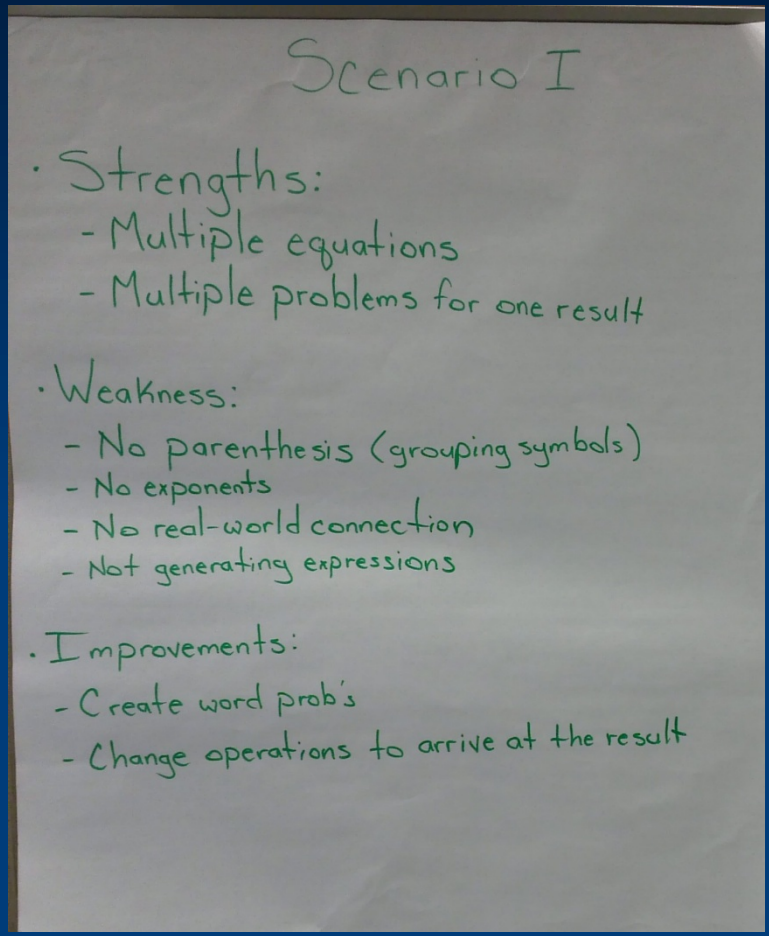

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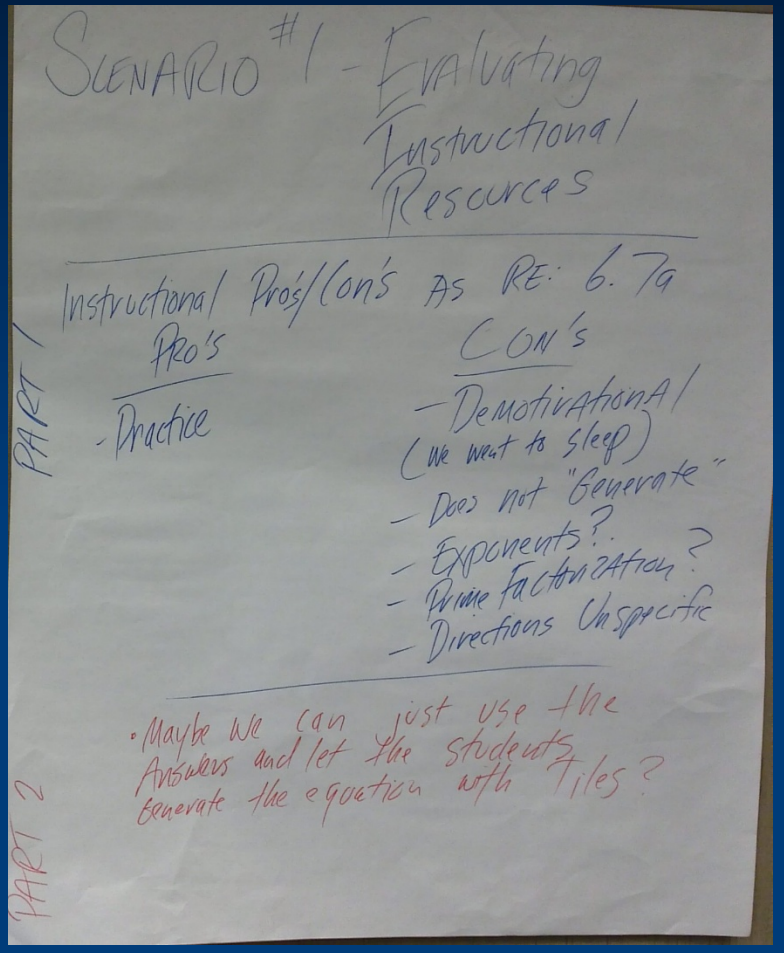

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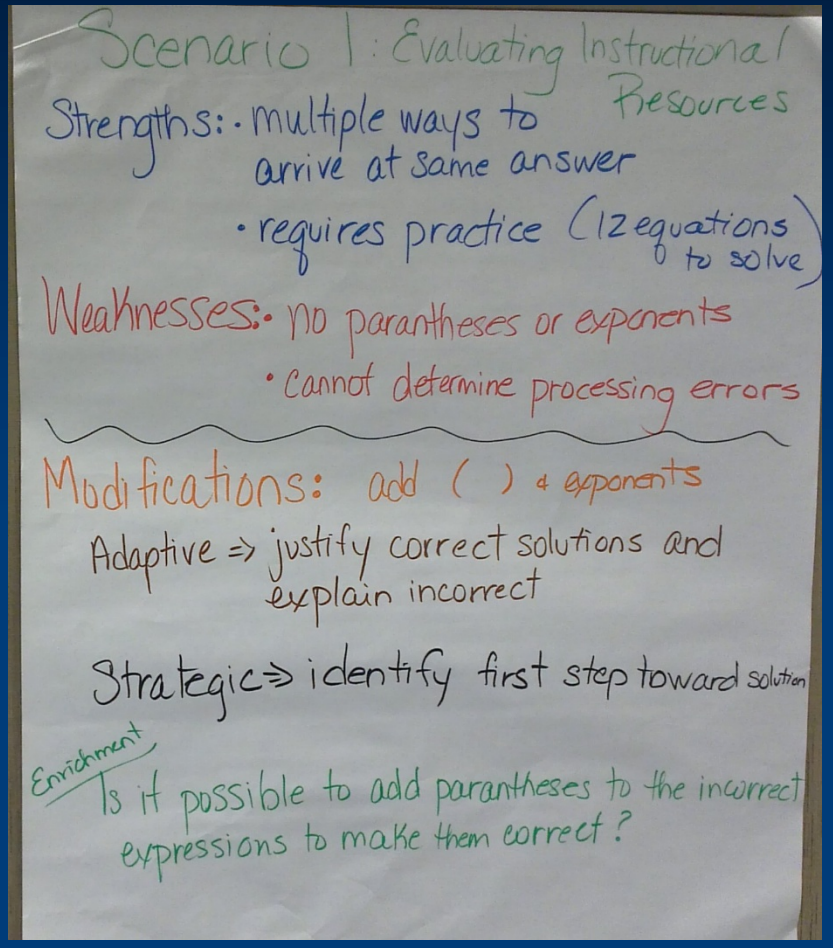

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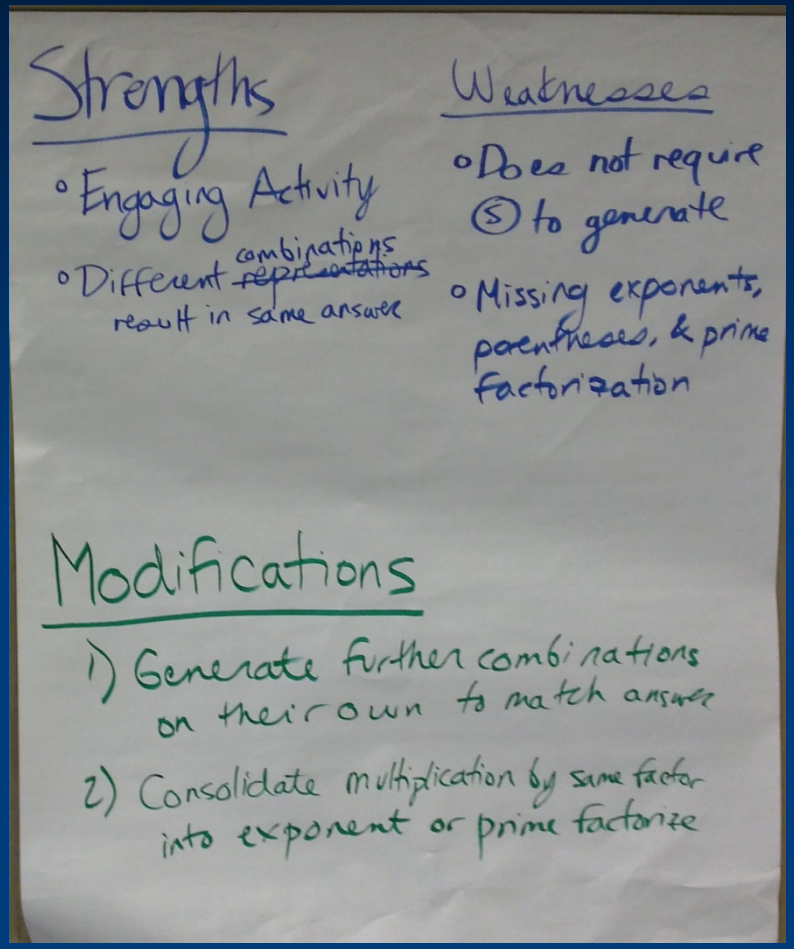

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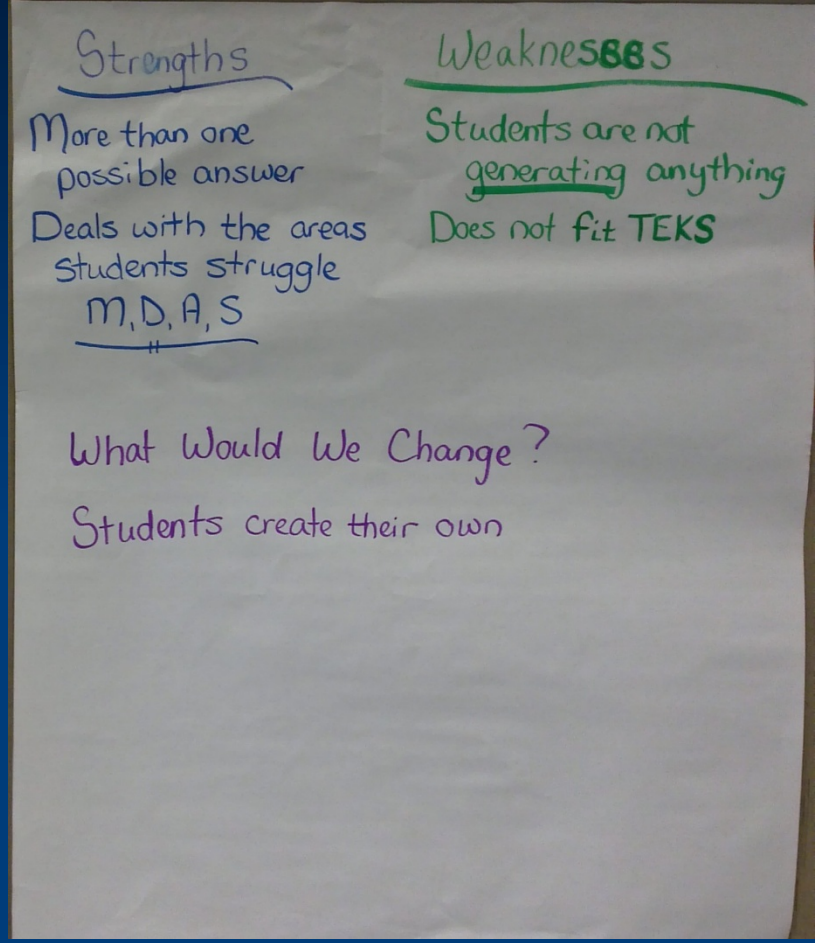

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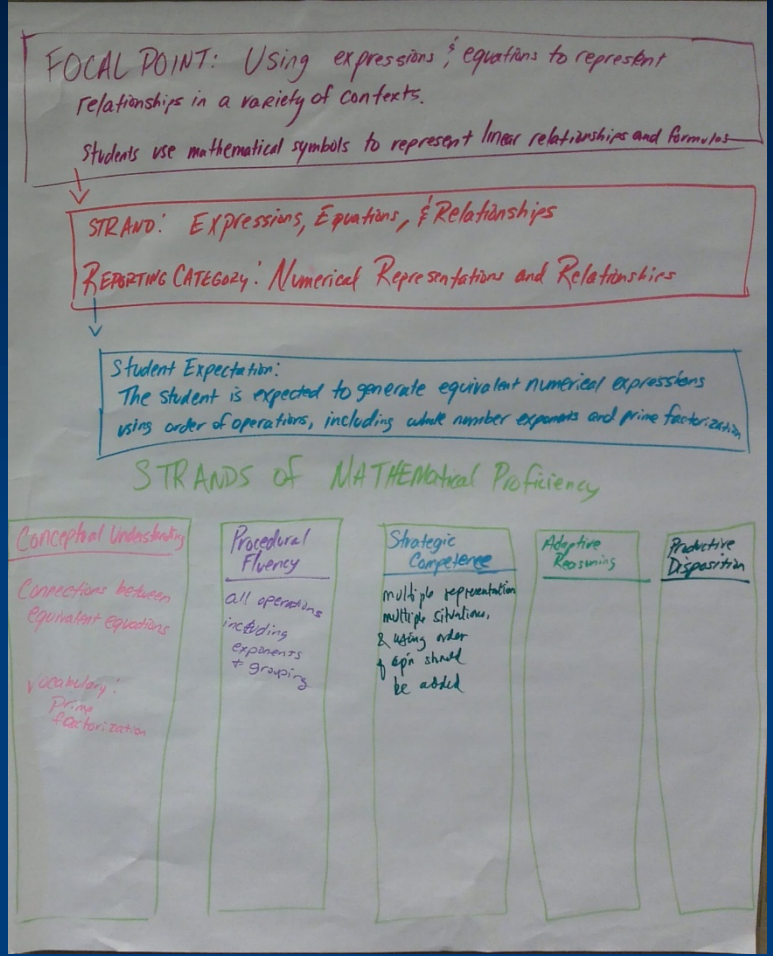

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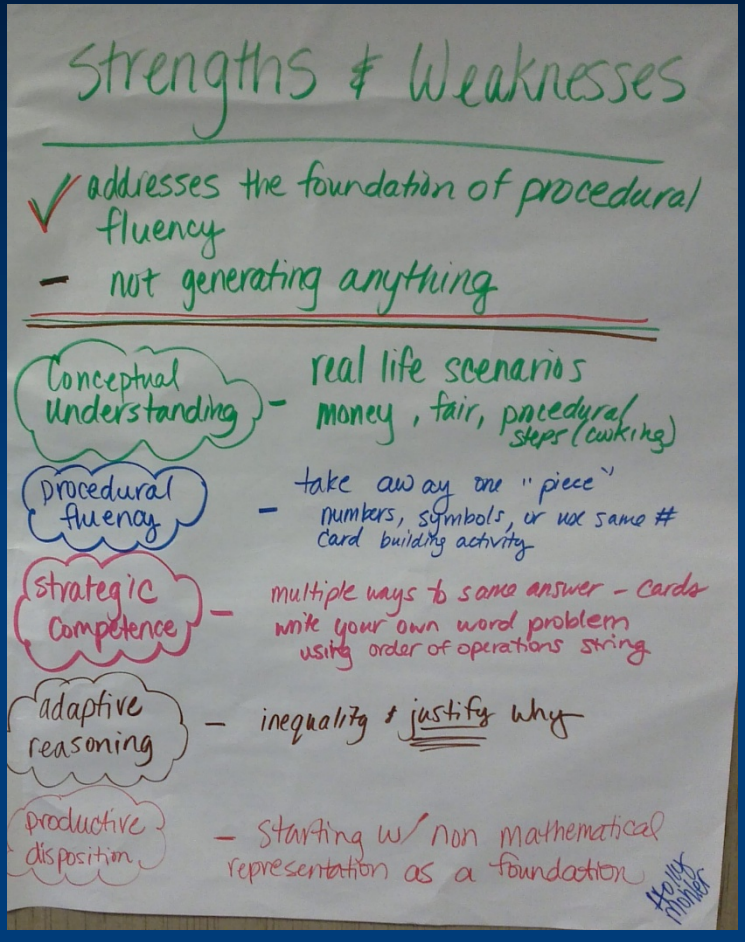

References

- Bransford J., Brown A., Cocking R. (2000). *How people learn: Brain, mind, experience, and school.* Washington, DC: The National Academy Press
- Generalize. 2014. In *Merriam-Webster.com.* Retrieved February 25, 2014, from http://www.merriam-webster.com/dictionary/generalize?show=0&t=1394119571
- Keeler, A. (2012). *Classifying Quadrilaterals*. Retrieved March 6, 2014 from http://www.ck12.org/user:YWtlZWxlckBhY2VsZnJlc25vLm9yZw../book/ACEL-Geometry-2012-2013/r2/section/6.3/
- Kilpatrick J., Swafford J., Findell B. (Eds.) (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.
- Order of Operation Puzzle. Retrieved February 25, 2014 from http://www.teach-nology.com/worksheets/math/order/ords2.html

References

- *Ratios and proportions – Similar figures – In depth*. Retrieved February 25, 2014 from http://www.math.com/school/subject1/lessons/S1U2L4DP.html

- Region IV Education Service Center. TEKS Questions and Sample Solutions. *Mathematics Benchmark Performance Assessments*. Retrieved February 25, 2014 from http://www.escweb.net/tx_bm/math/content/detail.aspx?grade=7&contentstrand=Geometry%20and%20spatial%20reasoning&grade_id=7&contentstrand_id=597&teks_id=445&teks_ide=7.6D

- Texas Education Agency (2013). *Introduction to the revised mathematics TEKS: Side-by-side TEKS comparison grade 6.* Retrieved from http://www.projectsharetexas.org/sites/default/files/resources/documents/Gr6_TEKS_SidebySide.pdf

- Texas Education Agency (2013). *Introduction to the revised mathematics TEKS: Vertical alignment chart grade 5 – Algebra 1*. Retrieved from http://www.projectsharetexas.org/sites/default/files/resources/documents/5-AlgebraIVAChart.pdf

- Texas Education Agency (2013). *TXRCFP: Texas response to curriculum focal points for K-8 mathematics revised 2013*. Retrieved from http://www.projectsharetexas.org/sites/default/files/resources/documents/TXRCFPrevised2013.pdf